On the Effects of Selective Below-Cost Pricing in a Vertical Differentiation Model

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Abstract

We analyse the effects of predation in a vertical differentiation model, where the high-quality incumbent is able to price discriminate while the low-quality entrant sets a uniform price. The incumbent may act as a predator, that is, it may price below its marginal costs on a subset of consumers to induce the rival’s exit. We show that the entrant may adopt an aggressive attitude to make predation unprofitable for the incumbent. In this case predation does not occur and the equilibrium prices are lower than the equilibrium prices which would emerge in a contest of explicitly forbidden predation. Moreover, we show that when the incumbent may choose whether to price discriminate or not before the game starts, if the quality cost function is sufficiently convex, there always exists a parameter space on which the incumbent prefers to commit not to price discriminate.

JEL: D43, L12, L41

Keywords: Vertical differentiation; selective below-cost pricing; predation; price discrimination

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1. Introduction

Predatory prices are said to occur when a firm sets prices at a level which implies the sacrifice of short-run profits in order to reduce competition and obtain higher long-run profits (Motta, 2004). Moving from theory to practice, predatory prices are usually defined as prices which are below the marginal costs (Areeda and Turner, 1975), the average variable costs (Areeda and Turner, 1975), the average total costs (Joskow and Klevorici, 1979), the average avoidable costs (Baumol, 1996), and the average incremental costs (Bolton et al., 2000). Following the influential article by McGee (1958), the mere existence of predatory pricing has been debated for a long time. Nowadays, several theories explaining the rationale of predatory pricing have been developed, and economists are well convinced that predation may emerge as a complete rational choice of firms (Motta, 2004).

This paper is not about the rationale of predation, but concerns the effects of predation. Notwithstanding the importance of this issue, quite surprisingly the literature about the effects of predatory pricing is scarce. To be convinced about this, one may look at the analysis of predatory pricing – presumably, the most complete one – by Bolton et al. (2000), which is very extensive about the rationale of predation, but is totally lacking in considering the effects of predation. Similarly, one may look at three recent surveys about price discrimination (Armstrong, 2006 and 2008, and Stole, 2007), where no theory concerning the effects of predatory selective price cuts is mentioned. On the same line is Spector (2005).

Taking for granted predation rationality, our paper investigates on the effects of predation within a very simple vertical differentiation framework, where an incumbent firm faces the threat of the entrance by another firm. The incumbent is assumed to be able to price discriminate between consumers, while the entrant (if enters) has to set a uniform price to all consumers. This assumption can be rationalised noticing that in order to price discriminate a firm must have a quite deep knowledge of the market in which it operates. In this sense, it appears reasonable to assume that the incumbent has a better knowledge of the market than the entrant, due to the fact that it is in the market when the game starts while the entrant is outside the market. Moreover, the incumbent may act as a predator in the sense of Areeda and Turner (1975), i.e. it may set below-marginal cost prices on a subset of consumers. We show that there exists a range of parameters over which the threat of predation induces an aggressive attitude by the entrant which ultimately determines no predation and lower equilibrium prices with respect to the case in which predation is a priori impossible. Moreover, we show that when the incumbent may choose whether to price discriminate or not before the game starts, if the quality cost function is sufficiently convex, there always exist conditions on which the incumbent prefers to commit not to price discriminate in order to assure the entrant that predation will not be tempted in case of entrance. Finally, in a \( T \)-periods model, conditions are derived for the equilibrium prices to increase over time until they stabilize at the level that would result in absence of predation.

This paper is largely indebted with the fast-growing literature on price discrimination (see Liu and Serfes, 2005, Choudary et al. 2005, Encaoua and Hollander, 2007, for

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1Adoption of discriminatory pricing by the incumbent reveals information about buyers’ reservation prices that the entrant cannot possess. Then entrant is more likely – at least initially – to set a uniform price, or divide consumers into fewer classes for pricing purposes than the incumbent” (Encaoua and Hollander, 2007, p.15).
recent contributions on vertical price discrimination, as well as the surveys we mentioned above). However, we want to stress that the focus of this paper is not on price discrimination, but on the predatory use of price discrimination, an issue which has been largely neglected by theory. An exception is represented by a recent paper by Karlinger and Motta (2007). The authors develop a horizontal differentiation model in which an incumbent and an entrant compete by offering a network good to asymmetric buyers. They compare the exclusionary impact of three different pricing schemes (uniform pricing, second-degree price discrimination and third-degree price discrimination), and conclude that the scheme inducing the lower equilibrium prices has also the highest exclusionary power. Our paper differs in many aspects from the work by Karlinger and Motta (2007). Just the mention the most relevant ones, we adopt a vertical differentiation setup and firms’ asymmetry instead of consumers’ asymmetry. Moreover, second-degree price discrimination is left aside.

The paper proceeds as follows. In section 2 we describe the model. In section 3 we solve the model and we illustrate the main result. In section 4 we consider the price policy choice by the incumbent. In section 5 the main result is generalized to a T-periods framework. Section 6 concludes.

2. The model

The framework we adopt is inspired by Tirole (1988). There is a continuum of consumers, differing in their tastes, described by the parameter ϑ which is assumed to be uniformly distributed on the interval [0,1] with density 1. Suppose to have two firms, H (the incumbent) and L (the entrant). Firm H produces a good of quality $s_H$, while firm L, if it enters, produces a good of quality $s_L$. Assume: $1 \geq s_H > s_L \geq 0$: that is, firm H is the high-quality firm, while firm L is the low-quality firm. Firm H is able to price discriminate between the consumers, while firm L is not able. Define with $p^H$ the price schedule set by firm H. The term “price schedule” has the same meaning as in Encaoua and Hollander (2007): it refers to a positive valued function $p^H(\cdot)$ defined on [0,1] that specifies the price $p^H(\vartheta)$ at which firm H is willing to sell one unit to consumer $\vartheta$.

Define with $p^L$ the uniform price set by firm L. Each consumer buys at most one unit of the good. The utility of a consumer $\vartheta$ when he buys from firm H is given by:

$$u = v + s_H^\vartheta - p^H,$$

while his utility when he buys from firm L is given by:

$$u = v + s_L^\vartheta - p^L.$$ 

Assume variable costs of quality improvement, represented by $c(s_j)$, where $j = H, L$, with $c'(\cdot) \geq 0$ and $c''(\cdot) > 0$. In what follows, we use the simplified notation $c_H$ for $c(s_H)$ and $c_L$ for $c(s_L)$. We make the following assumptions on the parameters of the model:

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2 This assumption is rooted in Lehmann-Grube (1997) article, where the author shows that in a sequential game where a leader chooses quality, then a follower chooses quality, and finally firms simultaneously set prices, the leader chooses the higher quality; that is, there is an incentive for the firm that enters first in the market to be the high-quality firm since this allows to obtain higher profits.

3 Variable costs of quality improvement arise when quality improvement depends on more skilled labour or more expensive materials (see for example Gal-Or, 1983; Motta, 1993; Crampes and Hollander, 1995; Encaoua and Hollander, 2007).
Assumption 1: \( c_L > c_H - 2(s_H - s_L) \)
Assumption 2: \( v > (c_H + c_L)/2 \)

Assumption 1 guarantees firm \( H \) has positive profits in the non-predatory duopoly, while assumption 2 guarantees that in equilibrium market is covered (see footnote 7).

The timing of the game is the following. At time 0 firm \( L \) decides whether to enter the market or stay out. There are no entrance costs. If firm \( L \) enters, firms compete for two periods, period 1 and period 2. At the end of period 1 firm \( L \) leaves the market if it obtains non-positive profits, while firm \( H \) has no such financial constraint\(^4\). In period 2, firms compete if firm \( L \) is still in the market, otherwise firm \( H \) acts as a monopolist.

Following the traditional approach in price discrimination literature with asymmetric firms, we assume that in each period first firm \( L \) (if it is present) sets its uniform price, and then firm \( H \) sets its price schedule\(^5\). The sub-game Nash equilibrium concept is used in solving the game.

### 3. Solution of the model

We start from period 2. First, consider the case in which firm \( L \) is still in the market (duopoly). In this case, predation by firm \( H \) is not a relevant issue: firm \( H \) has no incentive to prey, since there are no periods left to take advantage from the monopolistic position deriving from predation. Let define \( p^L_2 \) as the price set by firm \( L \) in period 2. The best price schedule firm \( H \) can set is such to serve as many consumers as possible without pricing below marginal costs. That is, the price schedule of firm \( H \) is obtained by solving: \( v + \vartheta s_H - p^D,H_2 = v + \vartheta s_L - p^L_2 \) and imposing \( p^D,H_2 \geq c_H \)\(^6\). It follows:

\[
p^D,H_2 = \vartheta(s_H - s_L) + p^L_2 \geq c_H
\]

(1)

Solving for \( \vartheta \) we get the consumer which is indifferent between the two firms:

\[
\hat{\vartheta} = \frac{c_H - p^L_2}{s_H - s_L}
\]

(2)

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\(^4\) Things do not change if firm \( L \) has a (limited) access to credit. What matters is that firm \( H \) has better access to credit than firm \( L \). There are many possible explanations for this asymmetry. For example, banks have a better knowledge of firm \( H \) than firm \( L \) (firm \( H \) has entered the market first), and therefore they are more prompt to give credit to firm \( H \) than to firm \( L \). Alternatively, given that in the non-predatory equilibrium firm \( H \) obtains larger profits than firm \( L \), firm \( H \) has larger collateral than firm \( L \). For more about the credit issue in predation models, see Motta (2004).

\(^5\) See, among the others, Thisse and Vives (1988), De Fraja and Norman (1993) and Tabuchi (1999). As Tabuchi (1999) argues: “such a leader-follower relationship may be justified by the flexibility of the price schedule used by the discriminatory pricing firm since it could easily cut the price at each location in secret if it were profitable” (p.619)

\(^6\) The superscript \( D \) indicates that firm \( H \) is acting as a non-predator duopolist. Similarly, in what follows the superscripts \( M \) and \( P \) indicate respectively that firm \( H \) is acting as a monopolist and as a predator.
The demand of firm $H$ is $1 - \hat{\theta}$, while the demand of firm $L$ is $\hat{\theta}$. The profit functions of the two firms are respectively:

$$\Pi^D,H_2 = \int_{\hat{\theta}}^1 (p^{D,H}_2 - c_H) dx = \frac{(s_H - s_L + p^L_2 - c_H)^2}{2(s_H - s_L)} \quad (3)$$

$$\Pi^L_2 = (p^L_2 - c_L)\hat{\theta} = \frac{(p^L_2 - c_L)(c_H - p^L_2)}{s_H - s_L} \quad (4)$$

Consider now firm $L$. It chooses $p^L_2$ in order to maximize $\Pi^L_2$. The equilibrium uniform price is:

$$p^L_2 = \frac{c_H + c_L}{2} \quad (5)$$

Substituting (5) into (1) we get the equilibrium discriminatory price schedule of firm $H$:

$$p^{D,H}_2 = \theta(s_H - s_L) + \frac{c_H + c_L}{2} \quad (6)$$

Substituting (5) into (2) we get the equilibrium indifferent consumer:

$$\hat{\theta}^* = \frac{c_H - c_L}{2(s_H - s_L)} \quad (7)$$

Substituting (6) and (7) into equation (3) we get firm $H$' equilibrium duopolistic non-predatory profits:

$$\Pi^{D,H}_2 = \frac{(2s_H - 2s_L - c_H + c_L)^2}{8(s_H - s_L)} \quad (8)$$

Similarly, substituting (5) and (7) into equation (4) we get firm $L$' equilibrium duopolistic profits in case of no predation:

$$\Pi^L_2 = \frac{(c_H - c_L)^2}{4(s_H - s_L)} \quad (9)$$

Now, consider the case in which firm $L$ left the market at the end of period 1. Firm $H$ is a monopolist and it is able to extract the whole consumer surplus by setting the appropriate price schedule, which is given by:

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7 Firm $H^\prime$ demand is positive when $\hat{\theta}^* < 1$, which amounts to require $c_L > c_H - 2(s_H - s_L)$ (Assumption 1). Moreover, the market is covered when the consumer with the lowest taste for quality buys the good. This requires that $\nu - p^L_2 > 0$, or $\nu > (c_H + c_L)/2$ (Assumption 2).
\[ p_2^{M,H} = v + \delta s_H \] (10)

Equilibrium monopolistic profits of firm \( H \) follow from equation (10). We get:

\[ \Pi_2^{M,H} = \int_0^l (p_2^{M,H} - c_H) d\theta = v + \frac{s_H}{2} - c_H \] (11)

By using equation (8) and equation (11) we can calculate the future gains from predation. They are simply the difference between the monopolistic profits and the duopolistic profits. Therefore:

\[ B = \Pi_2^{M,H} - \Pi_2^{D,H} = \delta(v + \frac{s_H}{2} - c_H - \frac{(2s_H - 2s_L - c_H + c_L)^2}{8(s_H - s_L)}) \] (12)

where \( \delta \in (0,1) \) is the discount factor.

We move now to period 1. Consider firm \( H \). Given the price set by the rival in the first period, \( p_1^L \), firm \( H \) has two possibilities: on one hand it can price aggressively, in order to induce firm \( L \)’ exit at the end of the period; on the other hand, it can maximize profits in period 1.

Suppose first that firm \( H \) acts in a predatory way. Firm \( H \) has to push firm \( L \)’ demand (given \( p_1^L \)) to zero: in this way firm \( L \) obtains zero profits and leaves the market. The equilibrium aggressive price schedule is obtained through the indifference condition:

\[ v + \delta s_H - p_1^{P,H} = v + \delta s_L - p_1^L \]

from which it follows:

\[ p_1^{P,H} = \delta(s_H - s_L) + p_1^L \] (13)

Note that firm \( H \) may want to price below marginal costs, while this is excluded in a non-predatory situation (compare equation 13 with equation 1). Predatory profits of firm \( H \) are therefore:

\[ \Pi_1^{P,H} = \int_0^l (p_1^{P,H} - c_H) dx = \frac{s_H - s_L + 2p_1^L - 2c_H}{2} \] (14)

Suppose now that firm \( H \) does not prey firm \( L \). The equilibrium prices are never lower than the marginal costs and they coincide with the prices defined in equation (1):

\[ p_1^{D,H} = \delta(s_H - s_L) + p_1^L \geq c_H \] (15)

The non-predatory profits correspond to equation (3):
\[
\Pi_1^{D,H} = \frac{(s_H - s_L + p_1^L - c_H)^2}{2(s_H - s_L)}
\]  
(16)

Using equation (14) and (16) we can calculate the losses from predation, which amount to the reduction of current profits induced by the adoption of a sub-optimal discriminatory price schedule. Therefore:

\[
Y = \Pi_1^{D,H} - \Pi_1^{P,H} = \frac{(c_H - c_L)^2 + 4s_H (c_H + c_L - 2p_1^L) + b[8p_1^L - 4(c_H + c_L)]}{8(s_H - s_L)}
\]  
(17)

Equation (12) and equation (17) provide the necessary and sufficient condition for predation to occur (given \( p_1^L \)). Since predation occurs when future gains outweigh current losses, the following inequality must be satisfied in order to observe predation:

\[
B > Y \rightarrow p_1^L > \Gamma \equiv \frac{4\delta_s L^2 + (c_H - c_L)(1 + \delta) + 4(s_H - s_L)[c_H + c_L - \delta(2v - c_H - c_L)] - 4\delta s H s_L}{8(s_H - s_L)}
\]  
(18)

We state the following result:

**Result 1:**

1) When \( \Gamma > p_1^{L*} > c_L \), at the profit-maximizing uniform price \( p_1^{L*} \) predation is not convenient for firm \( H \). Therefore, the equilibrium prices are \( p_1^{L*} = (c_H + c_L)/2 \) and \( p_1^{D,H} = (p_1^{L*}) \), and no predation occurs. At time 0 firm \( L \) enters.

2) When \( p_1^{L*} > c_L > \Gamma \), the only prices which induce no predation are below the marginal costs of firm \( L \). Therefore, predation occurs if firm \( L \) enters. At time 0 firm \( L \) stays out, and in equilibrium firm \( H \) sets the monopolistic price schedule \( p_1^{M,H} = p_2^{M,H} = v + \delta s H \) in both periods.

3) When \( p_1^{L*} > \Gamma > c_L \), firm \( L \) can avoid predation. Since by avoiding predation firm \( L \) obtains positive profits in both periods, it has the incentive to avoid predation. It sets the highest uniform price which induces no predation by firm \( H \). Therefore the equilibrium prices of firm \( L \) and firm \( H \) are respectively \( \hat{p}_1^{L*} = \max[\Gamma, c_H + s_L - s_H] \) \(^9\) and \( p_1^{D,H} = (p_1^{L*}) \), and predation does not occur. At time 0 firm \( L \) enters.

The most interesting case is case 3). Firm \( L \) is aggressive (it sets a low price) in order to reduce the aggressiveness of firm \( H \) (firm \( H \) does not set predatory prices). Let call this strategy by firm \( L \) as a fight-to-survive strategy. The most striking consequence of the adoption of this strategy concerns the level of the equilibrium prices. By comparing the equilibrium prices under this strategy with respect to the non-predation case, we observe

\(^8\) It is immediate to note that for any price lower than \( p_1^{L*} \) the profits of firm \( L \) are increasing in price.

\(^9\) From equation (2) follows that for firm \( L' \) prices lower than \( c_H + s_L - s_H \) the demand of firm \( H \) is zero. Therefore, firm \( L \) has never the incentive to decrease the price below \( c_H + s_L - s_H \).
that the adoption of the *fight-to-survive* strategy lowers the prices for all consumers. This is due to the fact that firm $L$ increases competition in order to reduce the incentive to prey by firm $H$. Note that when the threat of predation is absent, there is no need for a *fight-to-survive* strategy, and all the equilibrium prices would be higher. In this sense, the possibility to predation unambiguously improves the consumer surplus through the increase of competition it generates, provided that firm $L$ is able to resist to predation: if firm $L$ is too weak (or if the gains from predation are too high), predation occurs and consumer welfare decreases.

To gain insight, in what follows we investigate on the determinants of the *fight-to-survive* strategy. Assume that the cost function takes the following form: $c(s_j) = ks_j^2$, with $j = H, L$. By taking derivates of $\Gamma$ with respect to $v$ and $\delta$, it can be easily verified that $\Gamma$ is decreasing in $v$ and $\delta$. Since the marginal costs of firm $L$ are invariant in $v$ and $\delta$, it follows that the higher is the size of the market or the discount factor, the more stringent is the condition for the emerging of the *fight-to-survive* strategy. Therefore, predation is more likely to occur. The intuition is straightforward. The future gain from predation depends on the expected monopolistic profits, which in turn are affected positively by the dimension of the market. At the same time, whatever is the difference between monopolistic and duopolistic profits, such difference is more valued by firm $H$ when the discount factor is high. It turns out that firm $H$ is more prone to predation and the set of firm $L$’ marginal costs allowing for the *fight-to-survive* strategy shrinks. On the contrary, the derivative of $\Gamma$ with respect to $s_H$ is positive. It follows that the condition for the emerging of the *fight-to-survive* strategy is less stringent, i.e. predation is less likely to occur. The reason is that the *gains from predation* (equation 12) decrease with the level of quality while the *losses from predation* (equation 17) increase with the level of quality. It follows that the higher is the quality of the high-quality firm the less firm $H$ is induced to prey, and the *fight-to-survive* strategy is more likely to occur. Consider now parameter $k$ (the degree of convexity of the cost function). It can be shown that when $k$ increases, function $\Gamma$ increases too. However, the marginal costs of the low-quality firm increase with $k$ as well. Therefore, it is not obvious whether higher convexity implies more or less opportunity for predation. However, it can be proved that $\partial \Gamma / \partial k > s_L^2$, which implies that $\Gamma$ increases with respect to $k$ faster that $c_L$. Therefore, higher convexity of the cost function makes predation less sustainable, all else being equal\textsuperscript{10,11}.

### 4. Selecting the price policy

An interesting implication of the analysis developed in section 3 is that under the *fight-to-survive* strategy there is actually no predation in equilibrium. However, the

\textsuperscript{10} The sign of the derivatives and the comparison between $\partial \Gamma / \partial k$ and $s_L^2$ have been calculated using the software Mathematica.

\textsuperscript{11} The impact of $s_L$ (the quality level of the low-quality firm) is instead ambiguous. In fact, function $\Gamma$ initially decreases with $s_L$, then increases, and finally it decreases again, while the marginal costs of firm $L$ are obviously increasing in $s_L$. Therefore, an unambiguous relationship between $s_L$ and the likelihood of predation cannot be found.
possibility of predation induces the threaten firm to behave aggressively in order to
discourage the other firm from setting exclusionary prices. The result is that both firms
obtain lower equilibrium profits with respect to the case in which predation would be a
priori impossible\(^{12}\). In this case, the incumbent may find it profitable to commit not to
prey. Consider the following situation. Before that the game starts, firm \(H\) may decide
between the following pricing policies: committing not to discriminate (U) and not
committing (D)\(^{13}\).

When no commitment is taken (D), from Result 1 we get:

1) If \(\Gamma > p_l^* > c_L\), total profits of firm \(H\) are:
\[
\Pi_{NP}^{D} = \frac{(1+\delta)(2s_H - 2s_L - c_H + c_L)^2}{8(s_H - s_L)}
\]

2) If \(p_l^* > c_L > \Gamma\), total profits of firm \(H\) are:
\[
\Pi_{FTS}^{D} = \frac{4(s_H - s_L + \Gamma - c_H)^2 + \delta(2s_H - 2s_L - c_H + c_L)^2}{8(s_H - s_L)}
\]

3) If \(p_l^* > \Gamma > c_L\), total profits of firm \(H\) are:
\[
\Pi_{U} = \frac{(1+\delta)(2s_H - 2s_L - c_H + c_L)^2}{9(s_H - s_L)}
\]

Suppose instead that firm \(H\) commits to pricing uniformly (U). Total profits are the
following\(^{14}\):

\[
\Pi_{U} = \frac{(1+\delta)(2s_H - 2s_L - c_H + c_L)^2}{9(s_H - s_L)} \tag{19}
\]

First, note that \(\Pi_{NP}^{D} > \Pi_{NP}^{D}\). That is, the predatory profits are always larger than the non-
predatory profits. This is obvious, because firm \(H\) would prefer to be monopolist in both
periods than competing in both periods. Moreover, note that \(\Pi_{NP}^{D} > \Pi_{FTS}^{D}\) and
\(\Pi_{NP}^{D} > \Pi_{U}\). That is, when predation is impossible because firm \(L\) engages in the fight-
to-survive strategy, it is better for firm \(H\) to convince firm \(L\) that no predation will be
tempted in case of entry in order to avoid the aggressive attitude of the entrant (first
inequality); at the same time, firm \(H\) prefers remaining in the advantaged position of
being able to discriminate instead than setting a uniform price (second inequality). Let
consider now the profits of firm \(H\) when it takes no commitment and firm \(L\) adopts the
fight-to-survive strategy (\(\Pi_{FTS}^{D}\)). Note that \(\Pi_{FTS}^{D}\) is strictly increasing in \(\Gamma\), with
\(\Gamma \in [c_H + s_L - s_H, (c_H + c_L)/2]\) being the equilibrium price set by firm \(L\) (Result 1). The
minimum value of \(\Pi_{FTS}^{D}\) is \(\Pi_{FTS}^{D} = [\delta(2s_H - 2s_L - c_H + c_L)^2]/8(s_H - s_L)\), while the

\(^{12}\) Instead, when the incumbent is able to induce the entrant to stay out excluding predation would be
beneficial for firm \(L\) (which enters and obtains positive profits) and would be detrimental for firm \(H\)
(which would prefer prey in the first period in order to be a monopolist in the second period).

\(^{13}\) Clearly firm \(A\) may prefer (if possible) a third pricing policy: committing not to prey while maintaining
the possibility to discriminate between the consumers. However, we are sceptical about the existence of such a commitment:
while commitment-not-to-discriminate strategies exist (see on this issue Corts, 1998,
and Liu and Serfes, 2004), we are not aware of the existence of commitment-not-to-prey strategies.

\(^{14}\) The equilibrium profits when both firms set uniform prices can be obtained by standard calculations.
See for example Tirole (1988).
maximum value of $\Pi_{FTS}^D$ is $\Pi_{FTS}^D = [(1 + \delta)(2s_H - 2s_L - c_H + c_L)^2]/4(s_H - s_L)$. It can be easily verified that: $\Pi_{FTS}^D < \Pi_U < \Pi_{FTS}^D$. Therefore, two situations are possible:

1) $\Pi_P^D > \Pi_{NP}^D > \Pi_{FTS}^D > \Pi_U$
2) $\Pi_P^D > \Pi_{NP}^D > \Pi_U > \Pi_{FTS}^D$

In the first case, firm $H$ always prefers not to commit. In contrast, in the second case firm $H$ may prefer to commit to uniform pricing. This occurs when firm $L$ engages in a particularly aggressive fight-to-survive strategy. The entrant lowers so much its uniform price in order to avoid predation that firm $H$ prefers to guarantee firm $L$ that predation will not occur by completely renouncing to the possibility to price discriminate. Figure 1 illustrates this case.

Figure 1

When $\Gamma$ is low, predation is both profitable and possible, since the lower bound to firm $L$ price, i.e. the marginal costs $c_L$, binds. Therefore, firm $H$ chooses D, which guarantees the highest possible profits. When $\Gamma$ is between $c_L$ and $(c_H + c_L)/2$ firm $H$ anticipates that firm $L$ will engage in the fight-to-survive strategy to avoid predation. For low values of $\Gamma$ ($\Gamma < \Gamma^*$) firm $H$ prefers to commit not do discriminate in order to avoid the aggressive reaction of firm $L$ and chooses $U$; for high values of $\Gamma$ ($\Gamma > \Gamma^*$) the aggressive reaction of firm $L$ is less severe, and firm $H$ prefers to accept the fight-to-survive strategy than renouncing to the ability to discriminate, and therefore it chooses

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15 This result better qualifies the statement of Encaoua and Hollander (2007, p.15): “an incumbent who discriminates prior to entry is more likely to deter entry than an incumbent who prices uniformly”. We suggest that when entrance cannot be deterred, the incumbent may prefer to renounce to discriminate in order to avoid that the entrant adopts a fight-to-survive strategy.
D. Finally, when $\Gamma > \frac{c_H + c_L}{2}$, both predation and the *fight-to-survive* strategy do not occur, and firm $H$ prefers to price discriminate than setting a uniform price; thereby it chooses $D$.

An interesting implication which follows directly from the observation of Figure 2 is the following: if $c_L < c_H + s_L - s_H$, a parameter space always exists on which committing not to discriminate is rational for firm $A$. Note that condition $c_L < c_H + s_L - s_H$ can be written as: $c_H - c_L - (s_H - s_L) > 0$, where the left term of the inequality is increasing in the degree of the convexity of the cost function. Therefore, we can conclude with the following result:

**Result 2:** for sufficiently convex cost function there always exists a parameter space on which the incumbent firm prefers to commit not to discriminate.

### Figure 2

Suppose now that firms compete for more than two periods. That is, the game lasts for $T$ periods, where $T$ is not constrained to be finite. Let index each period with $i = 1, 2, ..., T$.

At the end of each period, firm $L$ decides whether to stay in the market or leave. If firm $L$ leaves at the end of period $i$, in period $1 + i$ firm $H$ is the monopolist, otherwise in period $1 + i$ there is again a duopoly.

Suppose that in period $i$ firm $H$ acts as a predator. The future gains from predation are the following:

$$\Pi^{\text{H}} = (1 + \delta)(s_H - c_H + c_L)^2 \frac{\pi}{\delta(s_H - s_L)}$$

$$\Pi^{\text{L}} = (1 + \delta)(s_H - c_H + c_L)^2 \frac{\pi}{\delta(s_H - s_L)}$$

$\Gamma$ = $\frac{c_H + c_L}{2}$

### 5. Price dynamic over time

Suppose now that firms compete for more than two periods. That is, the game lasts for $T$ periods, where $T$ is not constrained to be finite. Let index each period with $i = 1, 2, ..., T$.

At the end of each period, firm $L$ decides whether to stay in the market or leave. If firm $L$ leaves at the end of period $i$, in period $1 + i$ firm $H$ is the monopolist, otherwise in period $1 + i$ there is again a duopoly.

Suppose that in period $i$ firm $H$ acts as a predator. The future gains from predation are the following:

$16$ Note that the monopolistic profits and the duopolistic non-predatory profits do not depend on the period. In fact, if predation has been successful, the monopolistic price schedule depends only on the market structure and on the marginal costs of firm $H$ (equation 10). Similarly, if firm $H$ has not engaged
\[ B_i = \sum_{t=i+1}^{T} \delta^{T-t-1} \Pi_{M,H}^{t} - \sum_{t=i+1}^{T} \delta^{T-t-1} \Pi_{D,H}^{t} = \frac{\delta(1-\delta^{T-i})}{1-\delta} (\Pi_{M,H}^{t} - \Pi_{D,H}^{t}) \]  

Note that \( B_i \) is decreasing in \( i \). This implies that if firm \( H \) decides to engage in predation, it must prey in \( i = 1 \). In order to find the necessary condition for predation to occur, we substitute equation (8) and equation (11) into \( B_i \). Solving the inequality \( B_i > Y \) with respect to the price we get:

\[ p_i^L > \tilde{\Gamma}(i,T) = \frac{1}{8} \left\{ 4(s_H-s_L)(c_H+c_L)+(c_H-c_L)^2 \right\} \frac{8\delta(1-\delta^{T-i})}{s_H-s_L} \left[ v + \frac{s_H}{2} - \frac{(2s_H-2s_L-c_H+c_L)^2}{8(s_H-s_L)} \right] \]

As for the two-period game, three cases are possible. If \( \tilde{\Gamma}(i,T) > p_i^L > c_L \), no predation occurs at the profit maximizing price of the entrant; if \( p_i^L > c_L > \tilde{\Gamma}(i,T) \), predation occurs whatever price firm \( L \) sets; if \( p_i^L > \tilde{\Gamma}(i,T) > c_L \) firm \( L \) can avoid predation by adopting the fight-to-survive strategy. Note that:

\[ \frac{\partial \tilde{\Gamma}}{\partial T} = -\frac{\partial \tilde{\Gamma}}{\partial i} = \left[ \delta^{T-i} \log(\delta) \right] \left[ v + \frac{s_H}{2} - \frac{(2s_H-2s_L-c_H+c_L)^2}{8(s_H-s_L)} \right] \]

The second term is positive since it is simply the difference between the monopolist profits and the duopolistic non-predatory profits (equation 12), while the first term is negative since \( 0 < \delta < 1 \). Therefore, \( \partial \tilde{\Gamma}/\partial T < 0 \) and \( \partial \tilde{\Gamma}/\partial i > 0 \). These inequalities imply that the longer is the horizon, the more likely is predation. This is due to the fact that a longer time horizon implies that the gains from predation last for more periods, and this, ceteris paribus, makes predation more profitable. Recall that firm \( H \) sets predatory prices only in period 1. Therefore, if \( p_i^L > c_L > \tilde{\Gamma}(1,T) \), firm \( H \) sets predatory prices in period 1 and firm \( L \) obtains non-positive profits. Instead, if \( \tilde{\Gamma}(1,T) > p_i^L > c_L \), predation does not occur in the first period even if firm \( L \) sets the profit-maximizing uniform price. Since predation does not occur in the first period when the incentive to prey is the highest, it does not occur in any subsequent period, and firms set the non-predatory

\[ \text{in predation in period } i, \text{ in each subsequent period the equilibrium price schedule will result from the intersection of the best-reply functions, and the equilibrium profits will coincide with equation (8).} \]

\[ \text{17 The intuition is straightforward. Write: } \Pi_{M,H}^{t} - \Pi_{D,H}^{t} = w > 0. \text{ Suppose } T = 4. \text{ If firm } H \text{ preys in period } 1, \text{ it gains from predation in periods } 2, 3 \text{ and } 4 \text{ obtaining } w(\delta + \delta^2 + \delta^3). \text{ If firm } H \text{ preys in period } 2, \text{ it remains a monopolist in periods } 3 \text{ and } 4 \text{ and obtains } w(\delta + \delta^2). \text{ If it preys in period } 3, \text{ it is a monopolist only in period } 4, \text{ and gets } w\delta. \text{ Instead, the losses from predation are current losses (i.e. they do depend neither on past periods nor on future periods).} \]

\[ \text{18 Note that the profit maximizing price of firm } L \text{ does not depend on } i, \text{ therefore in order to simplify notation we omit the subscript to the price.} \]
duopolistic equilibrium prices in each period. The most interesting case emerges when the fight-to-survive strategy arises in the first period, that is, when $p^L > \Gamma(1,T) > c_L$. The equilibrium prices of firm $L$ and firm $H$ are respectively $\hat{p}^L_1 \Gamma(1,T)$ and $\hat{p}^H_1 \Gamma(1,T) + \theta(s_H - s_L)$. At the end of the first period firm $L$ stays in the market. In the second period there are two possibilities: $\Gamma(2,T) > p^H > c_L$ or $p^L > \Gamma(2,T) > c_L$. In the first case, firm $L$ sets the profit maximizing price $p^L$ and no predation occurs; in the second case, firm $L$ engages again in the fight-to-survive strategy and sets $\hat{p}^L_2 = \Gamma(2,T)$, while firm $H$ sets $\hat{p}^H_2 = \hat{p}^L_1 + \theta(s_H - s_L)$. In the subsequent periods the story continues in the same way: at a certain period the future gains from predation become so low that firm $L$ can set the profit maximizing uniform price without the threat of being preyed. Let define $i^*$ as the first period in which engaging in the fight-to-survive strategy is not necessary for firm $L$ to avoid predation. Moving from $i = 1$ to $i^*$, the fight-to-survive strategy of the entrant becomes less aggressive, because the incentive to prey becomes less strong. Consequently both the entrant and the incumbent set higher and higher prices. After $i^*$, the threat of predation disappears and the standard equilibrium duopolistic prices emerge. Therefore, our model shows that equilibrium prices may be lower at the initial stages of the competition when the entrant has to discourage the incumbent to drive it out from the market, then equilibrium prices progressively increase, and, finally, they stabilize when predation is no more a threat.

6. Conclusions

In this paper we analysed the effects of predation in a vertical differentiation model, where the high-quality incumbent is able to price discriminate while the low-quality entrant sets a uniform price. The incumbent may act as a predator, that is, it may price below marginal costs to induce the rival’s exit. The most striking result is that, when predation is possible, the entrant may adopt an aggressive attitude to make predation unprofitable for the incumbent. In this case predation does not occur and the equilibrium prices are lower than the equilibrium prices which would emerge in a contest of explicitly forbidden predation. Moreover, in the case of sufficiently convex quality cost functions, the incumbent may prefer to commit not to price discriminate in order to avoid the aggressive behaviour by the entrant. Finally, in a $T$-periods model we show that equilibrium prices may be lower at the initial stages of the competition when the entrant has to discourage the incumbent to prey it and then progressively increase as long as predation becomes less appealing for the incumbent.
References


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