On the effects of selective below-cost pricing in a vertical differentiation model

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Abstract

We analyse the effects of predation in a vertical differentiation model, where the high-quality incumbent is able to price discriminate while the low-quality entrant sets a uniform price. The incumbent may act as a predator, that is, it may price below its marginal costs on a subset of consumers to induce the rival’s exit. We show that the entrant may adopt an aggressive attitude to make predation unprofitable for the incumbent. In this case predation does not occur and the equilibrium prices are lower than the equilibrium prices which would emerge in a contest of explicitly forbidden predation. Moreover, we show that when the incumbent may choose whether to price discriminate or not before the game starts, if the quality cost function is sufficiently convex, there always exists a parameter space on which the incumbent prefers to commit not to price discriminate.

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1. Introduction

Predatory prices are said to occur when a firm sets prices at a level which implies the sacrifice of short-run profits in order to reduce competition and obtain higher long-run profits (Motta, 2004). Moving from theory to practice, predatory prices are usually defined as prices which are below the marginal costs (Areda and Turner, 1975), the average variable costs (Areda and Turner, 1975), the average total costs (Joskow and Klevioric, 1979), the average avoidable costs (Baumol, 1996), and the average incremental costs (Bolton et al., 2000). Following the influential article by McGee (1958), the mere existence of predatory pricing has been debated for a long time. Nowadays, several theories explaining the rationale of predatory pricing have been developed, and economists are well convinced that predation may emerge as a complete rational choice of firms (Motta, 2004).

This paper is not about the rationale of predation, but concerns the effects of predation. Notwithstanding the importance of this issue, quite surprisingly the literature about the effects of predatory pricing is scarce. To be convinced about this, one may look at the analysis of predatory pricing – presumably, the most complete one – by Bolton et al. (2000), which is very extensive about the rationale of predation, but is totally lacking in considering the effects of predation. Similarly, one may look at three recent surveys about price discrimination (Armstrong, 2006 and 2008, and Stole, 2007), where no theory concerning the effects of predatory selective price cuts is mentioned. On the same line is Spector (2005).

Taking for granted predation rationality, our paper investigates on the effects of predation within a very simple vertical differentiation framework, where an incumbent firm faces the threat of the entrance by another firm. The incumbent is assumed to be able to price discriminate between consumers, while the entrant (if enters) has to set a uniform price to all consumers. This assumption can be rationalized noticing that in order to price discriminate a firm must have a quite deep knowledge of the market in which it operates. In this sense, it appears reasonable to assume that the incumbent has a better knowledge of the market than the entrant, due to the fact that it is in the market when the game starts while the entrant is outside the market. As Encaoua and Hollander (2007) argue: “adoption of discriminatory pricing by the incumbent reveals information about buyers’ reservation prices that the entrant cannot possess. Then entrant is more likely – at least initially – to set a uniform price, or divide consumers into fewer classes for pricing purposes than the incumbent” (p.15). Moreover, the incumbent may act as a predator in the sense of Areda and Turner (1975), i.e. it may set below-marginal cost prices on a subset of consumers. We show that there exists a range of parameters over which the threat of predation induces an aggressive attitude by the entrant which ultimately determines no predation and lower equilibrium prices with respect to the case in which predation is a priori impossible. Moreover, we show that when the incumbent may choose whether to price discriminate or not before the game starts, if the quality cost function is sufficiently convex, there always exist conditions on which the incumbent prefers to commit not to price discriminate in order to assure the entrant that predation will not be tempted in case of entrance.

This paper is largely indebted with the fast-growing literature on price discrimination (see Liu and Serfes, 2005, Choudary et al. 2005, Encaoua and Hollander, 2007, for recent contributions on vertical price discrimination, as well as the surveys we mentioned above). However, we want to stress that the focus of this paper is not on
price discrimination, but on the predatory use of price discrimination, an issue which has been largely neglected by theory. An exception is represented by a recent paper by Karlinger and Motta (2007). The authors develop a horizontal differentiation model in which an incumbent and an entrant compete by offering a network good to asymmetric buyers. They compare the exclusionary impact of three different pricing schemes (uniform pricing, second-degree price discrimination and third-degree price discrimination), and conclude that the scheme inducing the lower equilibrium prices has also the highest exclusionary power. Our paper differs in many aspects from the work by Karlinger and Motta (2007). Just the mention the most relevant ones, we adopt a vertical differentiation setup and firms’ asymmetry instead of consumers’ asymmetry. Moreover, second-degree price discrimination is left aside.

The paper proceeds as follows. In section 2 we describe the model. In section 3 we solve the model and we illustrate the main result. In section 4 we consider the price policy choice by the incumbent. Section 5 concludes.

2. The model

The framework we adopt is inspired by Tirole (1988). There is a continuum of consumers, differing in their tastes, described by the parameter $\mathcal{G}$ which is assumed to be uniformly distributed on the interval $[0,1]$ with density 1. Suppose to have two firms, $H$ (the incumbent) and $L$ (the entrant). Firm $H$ produces a good of quality $s_H$, while firm $L$, if it enters, produces a good of quality $s_L$. Assume: $1 \geq s_H > s_L \geq 0$; that is, firm $H$ is the high-quality firm, while firm $L$ is the low-quality firm. Firm $H$ is able to price discriminate between the consumers, while firm $L$ is not able. Define with $p^H(\mathcal{G})$ the price schedule set by firm $H$. The term “price schedule” has the same meaning as in Encaoua and Hollander (2007): it refers to a positive valued function $p^H(\cdot)$ defined on $[0,1]$ that specifies the price $p^H(\mathcal{G})$ at which firm $H$ is willing to sell one unit to consumer $\mathcal{G}$. In what follows, we use the simplified notation $p^H$ to indicate that the price set by firm $H$ is a function of the consumer’s location. Define with $p^L$ the uniform price set by firm $L$. Each consumer buys at most one unit of the good. The utility of a consumer $\mathcal{G}$ when he buys from firm $H$ is given by: $u = v + \mathcal{G}s_H - p^H$, while his utility when he buys from firm $L$ is given by: $u = v + \mathcal{G}s_L - p^L$. There are no production costs, while there are variable costs of quality improvement, represented by $c(s_j)$, where $j = H, L$, with $c(0) = 0$, $c'(\cdot) \geq 0$ and $c''(\cdot) > 0$. We make the following assumption on the parameters of the model:

1 The implications of endogenous quality choice are briefly discussed in section 5.
2 This assumption is rooted in Lehmann-Grube (1997) article, where the author shows that in a sequential game where a leader chooses quality, then a follower chooses quality, and finally firms simultaneously set prices, the leader chooses the higher quality; that is, there is an incentive for the firm that enters first in the market to be the high-quality firm since this allows to obtain higher profits.
3 Variable costs of quality improvement arise when quality improvement depends on more skilled labour or more expensive materials (see for example Gal-Or, 1983; Motta, 1993; Crampes and Hollander, 1995; Encaoua and Hollander, 2007). Another relevant stream of literature considers fixed costs of quality improvement (see for example Bonanno, 1986; Lutz et al., 2000; Lambertini and Tedeschi, 2007; Liao,
Assumption 1: \( c(l) < \min[v,2] \)

Assumption 1 guarantees both that firm \( H \) has positive profits in the non-predatory duopoly and that in equilibrium market is covered (see later footnote 8). In what follows, we use the simplified notation \( c_H \) for \( c(s_H) \) and \( c_L \) for \( c(s_L) \). Finally define the discount factor with \( \delta \in (0,1) \).

The timing of the game is the following. At time 0 firm \( L \) decides whether to enter the market or stay out. There are no entrance costs. If firm \( L \) enters, firms compete for two periods, period 1 and period 2. At the end of period 1 firm \( L \) leaves the market if it obtains non-positive profits, while firm \( H \) has no such financial constraint\(^4\). In period 2, firms compete if firm \( L \) is still in the market, otherwise firm \( H \) acts as a monopolist. Following the traditional approach in price discrimination literature with asymmetric firms, we assume that in each period first firm \( L \) (if it is present) sets its uniform price, and then firm \( H \) sets its price schedule\(^5\). The sub-game Nash equilibrium concept is used in solving the game.

3. Solution of the model

We start from period 2. First, consider the case in which firm \( L \) is still in the market (duopoly). In this case, predation by firm \( H \) is not a relevant issue: firm \( H \) has no incentive to prey, since there are no periods left to take advantage from the monopolistic position deriving from predation. Let define \( p^L_L \) as the price set by firm \( L \) in period 2. The best price schedule firm \( H \) can set is such to serve as many consumers as possible without pricing below marginal costs. We assume without loss of generality that if the utility of the consumer is the same when he buys from the discriminating firm and when he buys from the non discriminating firm, the consumer buys from the discriminating firm\(^6\). Therefore, the price schedule of firm \( H \) is obtained by solving:

\[
v + g_H - p^{D,H}_{s,2} = v + g_L - p^L_L \quad \text{and imposing} \quad p^{D,H}_{s,2} \geq c_H.\]

It follows:

\(^2\) Assuming variable costs instead of fixed costs is likely to generate different results. Therefore, the results we obtain under the variable costs assumption may not be generalized to the case of fixed costs of quality improvement.

\(^4\) Things do not change if firm \( L \) has a (limited) access to credit. What matters is that firm \( H \) has better access to credit than firm \( L \). There are many possible explanations for this asymmetry. For example, banks have a better knowledge of firm \( H \) than firm \( L \) (firm \( H \) has entered the market first), and therefore they are more prompt to give credit to firm \( H \) than to firm \( L \). Alternatively, given that in the non-predatory equilibrium firm \( H \) obtains larger profits than firm \( L \), firm \( H \) has larger collateral than firm \( L \). (for more about the credit issue in predation models, see Motta, 2004). Therefore, since firm \( L \) has less financial resources than firm \( H \), the exit of firm \( L \) after the first period of non positive profits is from a forward-looking agent that expects additional future periods of losses that it cannot sustain while firm \( H \) can sustain (we really thank one anonymous referee for providing this helpful comment).

\(^5\) See, among the others, Thisse and Vives (1988), De Fraja and Norman (1993), Tabuchi (1999) and Liu and Serfes (2005). As Tabuchi (1999) argues: “such a leader-follower relationship may be justified by the flexibility of the price schedule used by the discriminatory pricing firm since it could easily cut the price at each location in secret if it were profitable” (p. 619).

\(^6\) This assumption is necessary to avoid the technicality of \( \varepsilon \)-equilibria, and it can be easily rationalized noting that the discriminating firm can always offer to the consumer a utility which is strictly larger than
\[ p_{D,2}^{H} = \mathcal{G}(s_{H} - s_{L}) + p_{2}^{L} \geq c_{H} \]  

(1)

Solving for \( \mathcal{G} \) we get the “threshold” consumer:

\[ \mathcal{G} = \frac{c_{H} - p_{2}^{L}}{s_{H} - s_{L}} \]  

(2)

Given that consumers are uniformly distributed, the demand of firm \( H \) is \( 1 - \mathcal{G} \), while the demand of firm \( L \) is \( \mathcal{G} \). The profit functions of the two firms are respectively:

\[ \Pi_{2}^{D,H} = \int_{0}^{1} (p_{D,2}^{H} - c_{H})d\mathcal{G} = \frac{(s_{H} - s_{L} + p_{2}^{L} - c_{H})^{2}}{2(s_{H} - s_{L})} \]  

(3)

\[ \Pi_{2}^{L} = (p_{2}^{L} - c_{L})\mathcal{G} = \frac{(p_{2}^{L} - c_{L})(c_{H} - p_{2}^{L})}{s_{H} - s_{L}} \]  

(4)

Consider now firm \( L \). It chooses \( p_{2}^{L} \) in order to maximize \( \Pi_{2}^{L} \). The equilibrium uniform price is:

\[ p_{2}^{L*} = \frac{c_{H} + c_{L}}{2} \]  

(5)

Substituting (5) into (1) we get the equilibrium discriminatory price schedule of firm \( H \):

\[ p_{S,2}^{D,H} = \mathcal{G}(s_{H} - s_{L}) + \frac{c_{H} + c_{L}}{2} \]  

(6)

Substituting (5) into (2) we get:

the utility he receives from the non-discriminating firm simply by setting a price equal to \( \hat{p}_{g} - \epsilon \), where \( \hat{p}_{g} \) is the discriminatory price which makes the consumer \( \hat{g} \) indifferent between the two firms and \( \epsilon \) is a positive small number. See for example Eber (1997).

The superscript \( D \) indicates that firm \( H \) is acting as a non-predator duopolist. Similarly, in what follows the superscripts \( M \) and \( P \) indicate respectively that firm \( H \) is acting as a monopolist and as a predator.

Firm \( H \)'s demand is positive when \( \mathcal{G}^{*} < 1 \), which amounts to require \( \frac{c_{H} - c_{L}}{s_{H} - s_{L}} < 2 \). Due to the convexity assumption, the left-hand side is increasing in the difference between the quality levels, and it is maximum when \( s_{H} = 1 \) and \( s_{L} = 0 \), which imply that the maximum level of the left-hand side is \( c(1) \).

Since by Assumption 1 it must be \( c(1) < 2 \), firm \( H \)'s demand is positive in the non-predatory equilibrium. Moreover, the market is covered when the consumer with the lowest taste for quality buys the good. This requires that \( v - p_{2}^{L*} > 0 \), or \( v > (c_{H} + c_{L})/2 \). Since the right-hand side of the inequality is increasing in the sum of the qualities, the right-hand side is maximum when \( s_{H} = 1 \) and \( s_{L} = 1 - \epsilon \), where \( \epsilon \) is a positive and infinitely small number. Therefore, disregarding \( \epsilon \), the maximum value of the right-hand side is \( c(1) \), which is always lower than \( v \) due to Assumption 1.
\[ \hat{\mathcal{G}}^* = \frac{c_H - c_L}{2(s_H - s_L)} \]  

(7)

Substituting (6) and (7) into equation (3) we get firm \( H \)’s equilibrium duopolistic second period non-predatory profits:

\[ \Pi_2^{D,H,*} = \frac{(2s_H - 2s_L - c_H + c_L)^2}{8(s_H - s_L)} \]  

(8)

Similarly, substituting (5) and (7) into equation (4) we get firm \( L \)’s equilibrium duopolistic second period profits in case of no predation:

\[ \Pi_2^{L,*} = \frac{(c_H - c_L)^2}{4(s_H - s_L)} \]  

(9)

Now, consider the case in which firm \( L \) left the market at the end of period 1. Firm \( H \) is a monopolist and it is able to extract the whole consumer surplus by setting the appropriate price schedule, which is given by:

\[ p_{\delta,2}^{M,H,*} = v + \mathcal{G}_H \]  

(10)

Equilibrium monopolistic profits of firm \( H \) follow from equation (10). We get:

\[ \Pi_2^{M,H,*} = \int (p_{\delta,2}^{M,H,*} - c_H) d\mathcal{G} = v + \frac{s_H}{2} - c_H \]  

(11)

By using equation (8) and equation (11) we can calculate the future gains from predation. They are simply the difference between the monopolistic profits and the duopolistic profits. Therefore:

\[ B = \Pi_2^{M,H,*} - \Pi_2^{D,H,*} = \delta(v + \frac{s_H}{2} - c_H - \frac{(2s_H - 2s_L - c_H + c_L)^2}{8(s_H - s_L)}) \]  

(12)

We move now to period 1. Consider firm \( H \). Given the price set by the rival in the first period, \( p_1^L \), firm \( H \) has two possibilities: on one hand it can price aggressively, in order to induce firm \( L \)'s exit at the end of the period; on the other hand, it can accommodate firm \( L \).

Suppose first that firm \( H \) acts in a predatory way. Firm \( H \) has to push firm \( L \)'s demand (given \( p_1^L \)) to zero: in this way firm \( L \) obtains zero profits and leaves the market. The equilibrium aggressive price schedule is obtained by solving the following condition:

\[ v + \mathcal{G}_H - p_{\delta,1}^{p,H} = v + \mathcal{G}_L - p_1^L \]  

from which we get:
\[ p_{s,1}^{p,H} = \mathcal{g}(s_H - s_L) + p_i^L \]  

Note that firm \( H \) may want to price below marginal costs, while this is excluded in a non-predatory situation (compare equation 13 with equation 1). First period predatory profits of firm \( H \) are therefore:

\[
\Pi_1^{p,H} = \int (p_{s,1}^{p,H} - c_H) d\mathcal{g} = \frac{s_H - s_L + 2p_i^L - 2c_H}{2} \tag{14}
\]

Suppose now that firm \( H \) does not prey on firm \( L \). The equilibrium prices are never lower than the marginal costs and they coincide with the prices defined in equation (1):

\[ p_{s,1}^{D,H} = \mathcal{g}(s_H - s_L) + p_i^L \geq c_H \tag{15} \]

The first period non-predatory profits correspond to equation (3):

\[
\Pi_1^{D,H} = \frac{(s_H - s_L + p_i^L - c_H)^2}{2(s_H - s_L)} \tag{16}
\]

Using equation (14) and (16) we can calculate the losses from predation, which amount to the reduction of current profits induced by the adoption of a sub-optimal discriminatory price schedule. Therefore:

\[
Y = \Pi_1^{D,H} - \Pi_1^{P,H} = \frac{(p_i^L - c_H)^2}{2(s_H - s_L)} \tag{17}
\]

Equation (12) and equation (17) provide the necessary and sufficient condition for predation to occur (given \( p_i^L \)). Since predation occurs when future gains outweigh current losses, the following inequality must be satisfied in order to observe predation:

\[
B > Y \rightarrow p_i^L > \Gamma = c_H - \sqrt{\delta[4s_H(s_L + 2v - c_H - c_L) - 4s_L^2 - (c_H - c_L)^2 - 4s_L(2v - c_H - c_L)]} \tag{18}
\]

We can state the following result:

**Result 1:**
1) When \( \Gamma > p_i^{L,*} > c_L \), at the profit-maximizing uniform price \( p_i^{L,*} \) predation is not convenient for firm \( H \). Therefore, the equilibrium prices are \( p_i^{L,*} = (c_H + c_L)/2 \) and \( p_{s,1}^{D,H} = (p_i^{L,*}) \), and no predation occurs. At time 0 firm \( L \) enters.
2) When \( p_i^{L,*} > c_L > \Gamma \), the only prices which induce no predation are below the marginal costs of firm \( L \). Therefore, predation occurs if firm \( L \) enters. At time 0 firm \( L \)
stays out, and in equilibrium firm $H$ sets the monopolistic price schedule $p_{s_1}^{xH} = p_{s_2}^{M,H} = v + g_{s_H}$ in both periods.

3) When $p^*_L > \Gamma > c_L$, firm $L$ can avoid predation. Since by avoiding predation firm $L$ obtains positive profits in both periods, it has the incentive to avoid predation. It sets the highest uniform price which induces no predation by firm $H$. Therefore the equilibrium prices of firm $L$ and firm $H$ are respectively $\hat{p}^L_1 = \max[\Gamma, c_H + s_L - s_H]$ and $\hat{p}^D_{s_1} = (\hat{p}_1^L)$, and predation does not occur. At time 0 firm $L$ enters.

The most interesting case is case 3). Firm $L$ is aggressive (it sets a low price) in order to reduce the aggressiveness of firm $H$ (firm $H$ does not set predatory prices). Let call this strategy by firm $L$ as a fight-to-survive strategy. The most striking consequence of the adoption of this strategy concerns the level of the equilibrium prices. By comparing the equilibrium prices under this strategy with respect to the non-predation case, we observe that the adoption of the fight-to-survive strategy lowers the prices for all consumers. This is due to the fact that firm $L$ increases competition in order to reduce the incentive to prey by firm $H$. Note that when the threat of predation is absent, there is no need for a fight-to-survive strategy, and all the equilibrium prices would be higher. In this sense, the possibility to predation unambiguously improves the consumer surplus through the increase of competition it generates, provided that firm $L$ is able to resist to predation: if firm $L$ is too weak (or if the gains from predation are too high), predation occurs and consumer welfare decreases.

To gain insight, in what follows we investigate on the determinants of the fight-to-survive strategy. Assume that the cost function takes the following form: $c(s_j) = k s_j^2$, with $j = H, L$. By taking derivatives of $\Gamma$ with respect to $v$ and $\delta$, it can be easily verified that $\Gamma$ is decreasing in $v$ and $\delta$. Since the marginal costs of firm $L$ are invariant in $v$ and $\delta$, it follows that the higher is the reservation price of the consumers or the discount factor, the more stringent is the condition for the emerging of the fight-to-survive strategy. Therefore, predation is more likely to occur. The intuition is straightforward. The future gain from predation depends on the expected monopolistic profits, which in turn are affected positively by the reservation price of the consumers. At the same time, whatever is the difference between monopolistic and duopolistic profits, such difference is more valued by firm $H$ when the discount factor is high. It turns out that firm $H$ is more prone to predation and the set of firm $L$’s marginal costs allowing for the fight-to-survive strategy shrinks. Consider now parameter $k$ (the degree of convexity of the cost function). It can be shown that when $k$ increases, function $\Gamma$ increases too. However, the marginal costs of the low-quality firm increase with $k$ as well. Therefore, it is not obvious whether higher convexity implies more or less opportunity for predation. However, it can be proved that $\partial \Gamma / \partial k > s_L^2$, which implies

\[ \text{It is immediate to note that for any price lower than } p^*_L \text{ the profits of firm } L \text{ are increasing in price.} \]

\[ \text{From equation (2) it follows that for firm } L \text{’s prices lower than } c_H + s_H - s_L \text{ the demand of firm } H \text{ is zero. Therefore, firm } L \text{ has never the incentive to decrease the price below } c_H + s_L - s_H. \]
that \( \Gamma \) increases with respect to \( k \) faster that \( c_L \). Therefore, higher convexity of the cost function makes predation less sustainable, all else being equal\(^{11}\).

4. Selecting the price policy

An interesting implication of the analysis developed in section 3 is that under the fight-to-survive strategy there is actually no predation in equilibrium. However, the possibility of predation induces the threatened firm to behave aggressively in order to discourage the other firm from setting exclusionary prices. The result is that both firms obtain lower equilibrium profits with respect to the case in which predation would be \textit{a priori} impossible\(^{12}\). In this case, the incumbent may find it profitable to commit not to price discriminate.

Consider the following situation. Before that the game starts, firm \( H \) may decide between the following pricing policies: committing not to discriminate (\( U \)) and not committing (\( D \))\(^{13}\). When no commitment is taken (\( D \), from Result 1 we get:

1) If \( \Gamma > p_L^* > c_L \), total profits of firm \( H \) are: 
\[
\Pi_{NP}^D = \frac{(1 + \delta)(2s_H - 2s_L - c_H + c_L)^2}{8(s_H - s_L)}
\]

2) If \( p_L^* > c_L > \Gamma \), total profits of firm \( H \) are: 
\[
\Pi_{\delta}^D = (1 + \delta)(v + \frac{s_H}{2} - c_H)
\]

3) If \( p_L^* > \Gamma > c_L \), total profits of firm \( H \) are:
\[
\Pi_{FTS}^D = \frac{4(s_H - s_L + \Gamma - c_H)^2 + \delta(2s_H - 2s_L - c_H + c_L)^2}{8(s_H - s_L)}
\]

Suppose instead that firm \( H \) commits to pricing uniformly (\( U \)). Total profits are the following\(^{14}\):

\[
\Pi^U = \frac{(1 + \delta)(2s_H - 2s_L - c_H + c_L)^2}{9(s_H - s_L)}
\]  
(19)

First, note that \( \Pi_{\delta}^D > \Pi_{NP}^D \). That is, the predatory profits are always larger than the non-predatory profits. This is obvious, because firm \( H \) would prefer to be monopolist in both periods rather than competing in both periods. Moreover, note that \( \Pi_{NP}^D > \Pi_{FTS}^D \) and

\(^{11}\) The sign of the derivatives and the comparison between \( \partial \Gamma / \partial k \) and \( s_L^2 \) have been calculated using the software Mathematica.

\(^{12}\) Instead, when the incumbent is able to induce the entrant to stay out excluding predation would be beneficial for firm \( L \) (which enters and obtains positive profits) and would be detrimental for firm \( H \) (which would prefer prey in the first period in order to be a monopolist in the second period).

\(^{13}\) Clearly firm \( H \) may prefer (if possible) a third pricing policy: committing not to prey while maintaining the possibility to discriminate between the consumers. However, we are sceptical about the existence of such a commitment: while commitment-not-to-discriminate strategies exist (see on this issue Corts, 1998, and Liu and Serfes, 2004), we are not aware of the existence of commitment-not-to-prey strategies.

\(^{14}\) The equilibrium profits when both firms set uniform prices can be obtained by standard calculations. See for example Tirole (1988).
\( \Pi^D_{NP} > \Pi^U \). That is, when predation is impossible because firm \( L \) engages in the *fight-to-survive* strategy, it is better for firm \( H \) to convince firm \( L \) that no predation will be tempted in case of entry in order to avoid the aggressive attitude of the entrant (first inequality); at the same time, firm \( H \) prefers remaining in the advantaged position of being able to discriminate rather than setting a uniform price (second inequality). Let consider now the profits of firm \( H \) when it takes no commitment and firm \( L \) adopts the *fight-to-survive* strategy (\( \Pi^D_{FTS} \)). Note that \( \Pi^D_{FTS} \) is strictly increasing in \( \Gamma \), with \( \Gamma \in [c_H + s_L - s_H, (c_H + c_L)/2] \) being the equilibrium price set by firm \( L \) (Result 1). The minimum value of \( \Pi^D_{FTS} \) is \( \Pi^D_{FTS} = [\delta(2s_H - 2s_L - c_H + c_L)^2]/8(s_H - s_L) \), while the maximum value of \( \Pi^D_{FTS} \) is \( \Pi^D_{FTS} = [(1 + \delta)(2s_H - 2s_L - c_H + c_L)^2]/4(s_H - s_L) \). It can be easily verified that: \( \Pi^D_{FTS} < \Pi^U < \Pi^D_{FTS} \). Therefore, two situations are possible:

1) \( \Pi^D > \Pi^D_{NP} > \Pi^D_{FTS} > \Pi^U \)
2) \( \Pi^D > \Pi^D_{NP} > \Pi^U > \Pi^D_{FTS} \)

In the first case, firm \( H \) always prefers not to commit. In contrast, in the second case firm \( H \) may prefer to commit to uniform pricing. This occurs when firm \( L \) engages in a particularly aggressive *fight-to-survive* strategy. The entrant lowers so much its uniform price in order to avoid predation that firm \( H \) prefers to guarantee firm \( L \) that predation will not occur by completely renouncing to the possibility to price discriminate. Figure 1 illustrates this case.

**Figure 1**

When \( \Gamma \) is low, predation is both profitable and possible, since the lower bound to firm \( L \)'s price, i.e. the marginal costs \( c_L \), binds. Therefore, firm \( H \) chooses \( D \), which guarantees the highest possible profits. When \( \Gamma \) is between \( c_L \) and \( c_H + c_L \) \( \frac{\Gamma}{2} \), firm \( H \) anticipates that firm \( L \) will engage in the *fight-to-survive* strategy to avoid predation.
For low values of $\Gamma$ ($\Gamma < \Gamma^*$) firm $H$ prefers to commit not to discriminate in order to avoid the aggressive reaction of firm $L$ and chooses U\(^{15}\); for high values of $\Gamma$ ($\Gamma > \Gamma^*$) the aggressive reaction of firm $L$ is less severe, and firm $H$ prefers to accept the *fight-to-survive* strategy rather than renouncing to the ability to discriminate, and therefore it chooses D. Finally, when $\Gamma > \frac{c_H + c_L}{2}$, both predation and the *fight-to-survive* strategy do not occur, and firm $H$ prefers to price discriminate rather than setting a uniform a price: thereby it chooses D.

An interesting implication which follows directly from the observation of Figure 2 is the following: if $c_L < c_H + s_L - s_H$, a parameter space always exists on which committing not to discriminate is rational for firm $H$. Note that condition $c_L < c_H + s_L - s_H$ can be written as: $c_L + s_L - (s_H - s_H) > 0$, where the left-hand side of the inequality is increasing in the degree of the convexity of the cost function. Therefore, we can conclude with the following result:

**Result 2:** for sufficiently convex cost functions there always exists a parameter space on which the incumbent firm prefers to commit not to discriminate.

5. Conclusions and final remarks

In this paper we analysed the effects of predation in a vertical differentiation model, where the high-quality incumbent is able to price discriminate while the low-quality entrant sets a uniform price. The incumbent may act as a predator, that is, it may price

\(^{15}\) This result better qualifies the statement of Encaoua and Hollander (2007, p.15): “an incumbent who discriminates prior to entry is more likely to deter entry than an incumbent who prices uniformly”. We suggest that when entrance cannot be deterred, the incumbent may prefer to renounce to discriminate in order to avoid that the entrant adopts a *fight-to-survive* strategy.
below its marginal costs to induce the rival’s exit. The most striking result is that, when predation is possible, the entrant may adopt an aggressive attitude to make predation unprofitable for the incumbent. In this case predation does not occur and the equilibrium prices are lower than the equilibrium prices which would emerge in a contest of explicitly forbidden predation. Moreover, in the case of sufficiently convex quality cost functions, the incumbent may prefer to commit not to price discriminate in order to avoid the aggressive behaviour by the entrant.

In this paper we have kept qualities exogenous. One may wonder whether our conclusions would hold if endogenous quality choices are assumed\textsuperscript{16}. We can answer this question by extending our game to allow the two firms to choose the quality before competing in price. In particular, suppose that the following stages come before the timing outlined in section 2:

1) the incumbent chooses the quality
2) the incumbent chooses the pricing policy
3) the entrant (if enters) chooses the quality

Since qualities are chosen before prices, Result 1 does not change when the qualities are endogenous, since Result 1 is obtained under any possible couple of quality levels. Therefore, there may be conditions under which the entrant has the incentive to set a low price in order to discourage the incumbent from preying it. In this sense, the result that the entrant may use sufficiently aggressive pricing to prevent the incumbent from predatory pricing does not depend on the exogeneity assumption. Consider now the pricing policy choice by the incumbent. When the incumbent chooses the pricing policy, it anticipates the quality that will be rationally chosen by the entrant. Let define with \( s^D_l \) \ (* the equilibrium quality chosen by the entrant when the incumbent has chosen to discriminate at the pricing policy stage. The optimal quality level by the entrant may be such to induce the no-predation equilibrium at the profit maximizing uniform price (case 1 in Result 1) or the \textit{fight-to-survive} strategy (case 3 in Result 1) \textsuperscript{17}. The incumbent has the following choices: discriminate or not discriminate. If \( s^D_l \) induces the no-predation equilibrium at the profit maximizing uniform price, the incumbent chooses discrimination; if \( s^D_l \) induces the \textit{fight-to-survive} strategy, the incumbent chooses to renounce to price discriminate at the pricing policy stage when the aggressiveness of the equilibrium \textit{fight-to-survive} strategy is sufficiently high, otherwise it price discriminates. Therefore, the result that the incumbent may be willing to commit to uniform pricing in order to avoid a price war is maintained under endogenous quality choices.

\textsuperscript{16} We thank one anonymous reader for raising this question.

\textsuperscript{17} Clearly, it may also be possible that no quality level exists that allows the entrant to avoid predation: in this case the firm does not enter and the incumbent chooses discrimination at the pricing policy stage. Whether \( s^D_l \) induces the no-predation equilibrium at the profit maximizing uniform price or the \textit{fight-to-survive} equilibrium is not an obvious issue, once one takes into account that also \( \Gamma \) depends on the quality choice of the entrant. If the optimal quality level falls within the set of the quality levels inducing the no-predation equilibrium at the profit maximizing uniform price, the entrant will choose such quality level. However, it may be that in order to induce the no-predation equilibrium at the profit maximizing uniform price the entrant should choose a sub-optimal quality level. In this case, it cannot be said \textit{a priori} whether the entrant prefers a quality level inducing the no-predation equilibrium at the profit maximizing uniform price or the \textit{fight-to-survive} equilibrium, since the entrant may prefer to induce the \textit{fight-to-survive} equilibrium rather than inducing the no-predation equilibrium at the profit maximizing uniform price which requires a sub-optimal quality level.
References