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# A Note on Updating Forecasts When New Information Arrives between Two Periods

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## Abstract

In this note the author discusses the problem of updating forecasts in a time-discrete forecasting model when information arrives between the current period and the next period. To use the information that arrives between two periods, he assumes that the process between two periods can be approximated by a linear interpolation of the time-discrete forecasting model. Based on this assumption the author drives the optimal updating rule for the forecast of the next period when new information arrives between the current period and the next period. He demonstrates by theoretical arguments and empirical examples that this updating rule is simple, intuitively appealing, defendable and useful.

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# 1 Introduction to the Problems

Time discrete models are the most popular models used in economic forecasting. Updating forecasts based on newly arrived information is a routine work. While a lot is written about how to update forecasts for the next period when new information arrives at this period, little is discussed about how to update the forecasts for the next period when new information arrives between the current period and the next period. This question is however highly relevant for daily forecasting business. Suppose that we have a time-discrete forecasting model with data of a certain frequency, say quarterly data. In this case forecasts for the future quarters are calculated based on the known information up to the current quarter. However, before the new information for the next quarter comes in, there is already information at a higher frequency, say monthly, weekly or daily available. In many circumstances, the information at the higher frequencies can be informative for the forecasts of the next quarter. Since the forecasting model only uses data at the lower frequency, the information at other higher frequencies cannot be directly used to update the forecasts. Updating forecasts based on the higher frequency information is either ignored or is carried out on an ad hoc basis. For instance, for some variables with random walk property, the forecast for the next period will be updated by the newest observation. For stock variables it will be checked if the monthly observation lies between the forecast and the last quarterly observation; if this is not the case some adjustment of the forecast may be done; for flow variables it will be inspected whether the accumulation of the monthly flow amounts roughly to the forecast for the next period; if this is not the case the forecast will be adjusted, ect.

The need of updating the forecasts for the future periods when new information arrives between the current period and the next period is grounded on the assumption that the underlying economic process is continuous and the time-discrete forecasting model is an approximative description of the continuous process at discrete time points. Therefore the new information that arrives between the current period and the next period is informative for the forecasts of the next period.

Given the need of updating forecasts when information arrives between periods, we ask the question: how can we update the forecasts in a time-discrete model using the information that arrives between the current period and the next period?

This problem seems to be related to the issue of forecasting using data of different frequencies. In the literature there are several approaches that address this issue. One approach is to apply Kalman-filter technique to model the relation between the data of different frequencies. Mittnik and Zadrozny (2004) and Chen and Zadrozny (2008) are two examples of this approach. Mixed Data Sampling(MIDAS) regression as presented by Andreou, Ghysels, and Kourtellos (2007) provides another approach to address this issue. EM algorithm as applied in Stock and Watson (2002) can be applied to solve the problem estimation using data at different frequencies. Bridge equation models are a further alternative to solve this problem. The technique can be found in Krolzig and Hendry (2001). Marcellino and Schumacher (2008) and Barhoumi, Benk, Cristadoro, Reijer, Jakaitiene, Jelonek, Rua, Ruestler, Ruth, and Nieuwenhuyze (2008) are two examples of this approach. The common feature of these approaches is an explicit modeling of the relation between data with different frequencies. This implies that an additional modeling work has to be done and there may exist conflict between the original forecasting model and the additional model.

Furthermore, not matter at which high frequency the explicit modeling may be, it is impossible to exclude that some information at an even higher frequency will be useful for forecasting the future. Therefore these approaches do not really solve the original problem of *updating forecasts when new information arrives between two periods*.

In this note we suggest how to use the existing model structure to update forecasts when information arrives between the current period and the next period. This is done by formulating assumptions on the arriving information between the two periods, so that we can formalize the updating rules. We show that these assumptions are simple, intuitively appealing and they are defendable.

# 2 The Updating Rule

## 2.1 A Forecasting Model and Assumptions

We formulate an econometric forecasting model as follows

$$Y_{t+1} = E(Y_{t+1}|\Omega_t) + \epsilon_{t+1|t} \qquad \text{for } t = 1, 2, ....T. \tag{1}$$

where $\epsilon_{t+1|t}$ is the one-step ahead forecast error with $E(\epsilon_{t+1|t}) = E(\epsilon_{t+1}|\Omega_t) = 0$, $E(\epsilon_{t+1|t}\epsilon'_{t+1|t}|\Omega_t) = \Sigma$ and $\epsilon_{t+1|t} \sim$ i.i.d. $N(0, \Sigma)$. $\Omega_t$ is the information set known at time point $t$.

Equation (1) is a usual formulation of a forecasting model. It implies that we want our model to be the best model in terms of mean square errors and the residuals be homoscedastic.

By setting $t = 1, 2, ....T$ the model is set out to be a time-discrete model and nothing is said about what happens between two periods. On the one hand, this time-discrete model setting makes it impossible to use directly any information that is available between two periods to update forecasts; on the other hand, this allows to make any assumptions on the process between two periods, as far as they are not contradict to the process at the discrete time points as specified in the original model.

Since non-contradiction with the original model at the discrete time points is only a very weak restriction on the assumptions on the process between periods, it is crucial to make reasonable assumptions in order to obtain a useful updating rule for the forecast.

Generally the model process between two periods can be formulated as the sum its forecast and the associated forecast errors:

$$Y_{t+\Delta t|t} = E(Y_{t+\Delta t}|\Omega_t) + \epsilon_{t+\Delta t|t} \qquad \text{for } 0 < \Delta t < 1 \tag{2}$$

By specifying $E(Y_{t+\Delta t}|\Omega_t)$ and $\epsilon_{t+\Delta t|t}$, we pinpoint the process between two periods. One set of possible assumptions are now given as follows.

**Assumption 2.1**

- *A1*

$$\frac{E(Y_{t+\Delta t}|\Omega_t) - Y_t}{\Delta t} = \frac{E(Y_{t+1}|\Omega_t) - Y_t}{1} \qquad \textit{for } 0 < \Delta t < 1$$

- *A2*

$$\epsilon_{t+1|t} = \Sigma^{\frac{1}{2}}(P(t+1) - P(t)) \qquad \textit{for } t = 1, 2, 3...,$$

$$\epsilon_{t+\Delta t|t} = \Sigma^{\frac{1}{2}}(P(t+\Delta t) - P(t)) \qquad \textit{for } \leq \Delta t \leq 1,$$

*where $P(t)$ is a process with independent zero-mean increment and*

$$Var(P(t+\Delta t) - P(t)) = \Delta t I.$$

A1 say the the conditional expectation is a simple linear "interpolation" between $Y_t$ and $E(Y_{t+1}|\Omega_t)$. A2 says that the information set used to forecast $Y_{t+1}$ is increasing with $t$ and the variance of the forecast error increases linearly with the time span of the forecast. As information is getting close to $t+1$ the forecast uncertainty becomes smaller and smaller. This set of assumptions assures that the process between periods is consistent with model (1) at discrete time points:

- for $\Delta t \to 0$ we have $E(Y_{t+\Delta t}|\Omega_t) \to Y_t$ and $\epsilon_{t+\Delta t|t} \to 0$ in probability; and

- for $\Delta t \to 1$ we have $E(Y_{t+\Delta t}|\Omega_t) \to E(Y_{t+1}|\Omega_t)$ and $\epsilon_{t+\Delta t|t} \to \epsilon_{t+1|t}$ in probability.

Assumption 2.1 pins down a process between two periods based on which we can derive an optimal updating rule for the forecast of the next period when information arrives between the current period and the next period.

**Lemma 2.2** *Under Assumption 2.1 the minimal variance forecast updating rule for model (1) is given in the following equation.*

$$E(Y_{t+\Delta t}|\Omega_t) = Y_t + (E(Y_{t+1}|\Omega_t) - Y_t)(1 - \Delta t) \tag{3}$$

$$Var(\epsilon_{t+\Delta t|t}) = \Delta t \Sigma \tag{4}$$

$$E(Y_{t+1}|\Omega_{t+\Delta t}) = Y_{t+\Delta t} + (E(Y_{t+1}|\Omega_t) - Y_t)(1 - \Delta t) \tag{5}$$

$$Var(\epsilon_{t+1|t+\Delta t}) = (1 - \Delta t)\Sigma \tag{6}$$

Proof: (3) and (4) follow directly from A1 and A2. To prove (5) we have:

$$\begin{aligned}
E(Y_{t+1}|\Omega_{t+\Delta t}) &= E(E(Y_{t+1}|\Omega_t) + \epsilon_{t+1|t}|\Omega_{t+\Delta t}) \\
&= E(Y_{t+1}|\Omega_t) + E(\epsilon_{t+1|t}|\Omega_{t+\Delta t}) \\
&= E(Y_{t+1}|\Omega_t) + \Sigma^{-1/2}E(P(t+1) - P(t+\Delta t) - (P(t+\Delta t) - P(t))|\Omega_{t+\Delta t}) \\
&= E(Y_{t+1}|\Omega_t) - Y_t + Y_t + \Sigma^{-1/2}(P(t+\Delta t) - P(t)) \\
&= (E(Y_{t+1}|\Omega_t) - Y_t)(1 - \Delta t) + (E(Y_{t+1}|\Omega_t) - Y_t)\Delta t + Y_t + \epsilon_{t+\Delta t|t} \\
&= (E(Y_{t+1}|\Omega_t) - Y_t)(1 - \Delta t) + E(Y_{t+\Delta t}|\Omega_t) + \epsilon_{t+\Delta t|t} \\
&= (E(Y_{t+1}|\Omega_t) - Y_t)(1 - \Delta t) + Y_{t+\Delta t}.
\end{aligned}$$

Then (6) follows from A2. □

Beside the consistence and simplicity, additional justifications for this set of assumptions can be seen in a class of models that encompass many often used linear forecasting models in economics. We will show this in the next section.

## 2.2 The Updating Rule in Some Special Cases

### 2.2.1 AR(1) Cases

AR(1) process as given in (7) is an often used forecast model for many economic time series.

$$y_{t+1} = \rho y_t + \sigma \epsilon^*_{t+1} \qquad \text{for } t = 1, 2, ...., \tag{7}$$

where $\epsilon^*_{t+1}$ is a standard normal random variable. The variance minimal forecast of $y_{t+1}$ is

$$\hat{y}_{t+1} = E(y_{t+\Delta t}|\Omega_t) = \rho y_t. \tag{8}$$

A natural way to consider the process between two time points in a time-discrete model is to assume that the time-discrete model is a model fitted to a time-continuous underlying process that is only sampled at the discrete time points.

If this assumption holds, how does the underlying continuous process for AR(1) look like? It can be shown that the continuous process is a linear diffusion process[1] as follows.

$$dy_t = -\kappa y_t dt + \sigma dW_t \qquad \text{for any } t \geq 0. \tag{9}$$

where $W_t$ is a standard Wiener process. The solution of the stochastic differential equation (9) is a continuous process:

$$y_{t+\Delta t} = e^{-\kappa \Delta t} y_t + \sigma \int_t^{t+\Delta t} e^{-\kappa(t+\Delta t - s)} dW_s.$$

The second term on the right hand side is an independent normal zero-mean increment process. The first term gives the conditional mean of $y_{t+\Delta t}$. For $t = 0, 1, 2, ...$ and $\Delta t = 1$ we have:

$$y_{t+1} = e^{-\kappa} y_t + \sigma \int_t^{t+\Delta t} e^{-\kappa(t+1-s)} dW_s$$

Comparing the equation above with (7) we have $\rho = e^{-\kappa}$ and $\epsilon^*_{t+1} = \int_t^{t+\Delta t} e^{-\kappa(t+1-s)} dW_s$. Indeed equation (9) gives the underlying continuous process of an time-discrete AR(1).

Applying Taylor expansion to $e^{-\kappa \Delta t}$ and taking the first linear term we have: $e^{-\kappa \Delta t} \approx 1 - \kappa \Delta t$. The solution of the continuous stochastic differential equation above can be then approximated as follows

$$y_{t+\Delta t} \approx (1 - \kappa \Delta t) y_t + \sigma \int_t^{t+\Delta t} e^{-\kappa(t+\Delta t - s)} dW_s. \tag{10}$$

The approximated process (10) can be used now to show that Assumption 2.1 hold for this process and the optimal forecast updating rule for this process is exactly the same as given in (5).

[1]See Kloeden and Platen (1995, P.117) for details.

According to (10) we have:

$$E(y_{t+\Delta t}|\Omega_t) \approx y_t - \kappa y_t \Delta t = y_t + ((1-\kappa)y_t - y_t)\Delta t \approx y_t + (E(y_{t+1}|\Omega_t) - y_t)\Delta t. \quad (11)$$

This is the equation given by A1 in Assumption 2.1. And the forecast variance can be calculated as follows

$$Var\left(\Sigma^{\frac{1}{2}} \int_t^{t+\Delta t} e^{-\kappa(\Delta t - s)} dW_s\right) = \Sigma \frac{1 - e^{-2\kappa\Delta t}}{2\kappa} \approx \Sigma\Delta t$$

This is given in A2. According to (10) we have

$$\begin{aligned}
E(Y_{t+1}|\Omega_{t+\Delta t}) &= E(E(Y_{t+1}|\Omega_t) + \epsilon_{t+1|t}|\Omega_{t+\Delta t}) \\
&= E(Y_{t+1}|\Omega_t) + E(\epsilon_{t+1|t}|\Omega_{t+\Delta t}) \\
&= E(Y_{t+1}|\Omega_t) + \epsilon_{t+\Delta t|t} \\
&= (E(Y_{t+1}|\Omega_t) - Y_t)(1 - \Delta t) + (E(Y_{t+1}|\Omega_t) - Y_t)\Delta t + Y_t + \epsilon_{t+\Delta t|t} \\
&= (E(Y_{t+1}|\Omega_t) - Y_t)(1 - \Delta t) + Y_t(1 + (1 - e^{-\kappa})\Delta t) + \epsilon_{t+\Delta t|t} \\
&\approx (E(Y_{t+1}|\Omega_t) - Y_t)(1 - \Delta t) + Y_t e^{-\kappa\Delta t} + \epsilon_{t+\Delta t|t} \\
&= (E(Y_{t+1}|\Omega_t) - Y_t)(1 - \Delta t) + Y_{t+\Delta t},
\end{aligned}$$

where the approximation in the sixth row above is: $(1 + (1 - e^{-\kappa})\Delta t) \approx (1 + \kappa\Delta t) \approx e^{-\kappa\Delta t}$. This is exactly the updating rule given in (5).

In this example of an AR(1) process we demonstrate that if the time-discrete mode is a model fitted to a time-continuous process that only sampled at discrete time points, Assumption 2.1 gives the linear approximation of the best forecast of the underlying continuous process between the discrete time points.

This conclusion can be easily extended to multivariate cases of first order vector autoregressive processes VAR(1).

### 2.2.2 Multivariate VAR(1) Cases

The multivariate counterpart of AR(1) is the first order vector auto regression process VAR(1) that is a often used to forecast economic time series jointly. A VAR(1) model can be written as follows.

$$Y_{t+1} = RY_t + \Sigma^{\frac{1}{2}} \epsilon^*_{t+1} \qquad \text{for } t = 1, 2, ...., \quad (12)$$

where $\epsilon^*_{t+1}$ is a standard normal random variable with independent components. The forecast of $Y_{t+1}$:

$$\hat{Y}_{t+1} = E(Y_{t+\Delta t}|\Omega_t) = RY_t$$

is the variance minimal forecast. Similar to the case of AR(1), we can view VAR(1) as a model fitted to an underlying continuous process which is only sampled at discrete time points. The multivariate underlying process is specified by the following stochastic differential equation.

$$dY_t = -KY_t dt + \Sigma^{\frac{1}{2}} dW_t \qquad \text{for any } t \geq 0. \quad (13)$$

where $W_t$ is a standard Wiener process. The solution of the continuous process (13) is

$$Y_{t+\Delta t} = e^{-K\Delta t} Y_t + \Sigma^{\frac{1}{2}} \int_t^{t+\Delta t} e^{-\kappa(t+\Delta t - s)} dW_s$$

The second term is a normal zero mean random variable whose variance depend linearly on $\Delta t$.

The solution can be approximated as follows

$$Y_{t+\Delta t} \approx (I - K\Delta t)Y_t + \Sigma^{\frac{1}{2}}\sqrt{\Delta t}\epsilon^*_{t+\Delta t}, \tag{14}$$

where $\epsilon^*_{t+\Delta t}$ is a standard normally distributed random variable with independent components.

Using the same argument as in the case of AR(1) we can show that for the underlying continuous process of VAR(1) Assumption 2.1 holds up to the first order approximation, and the updating rule (5) gives approximately the best forecast of the underlying continuous process.

For some economic time series a forecasting model may need more lags. Hence, VAR(p) instead of VAR(1) is a more suitable class of models for forecasting. However, a VAR(p) model can always be stacked into a higher dimensional VAR(1) model, so that we can treat them as VAR(1) formally.

### 2.2.3 VAR(p) Cases

We demonstrate this in a VAR(2) example as follows[2].

$$y_{t+1} = A_1 y_t + A_2 y_{t-1} + \Omega^{\frac{1}{2}}\epsilon_{t+1} \tag{15}$$

Equation (15) is VAR(2) model in which the future variables depends not only on the current values of the variables but also on the values in the previous periods. Now we stack $y_t$ and $y_{t-1}$ into a vector, equation (15) can be rewitten as follows.

$$\begin{pmatrix} y_{t+1} \\ y_t \end{pmatrix} = \begin{pmatrix} A_1 & A_2 \\ I & 0 \end{pmatrix} \begin{pmatrix} y_t \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} \Omega^{1/2} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \epsilon_{t+1} \\ \epsilon_t \end{pmatrix} \tag{16}$$

Equation (16) can be written in matrix form.

$$Y_{t+1} = RY_t + \Sigma^{\frac{1}{2}}E_t, \tag{17}$$

where $Y_{t+1} = (y'_{t+1}, y'_t)'$, $\Sigma^{\frac{1}{2}} = \begin{pmatrix} \Omega^{\frac{1}{2}} & 0 \\ 0 & 0 \end{pmatrix}$ and $E_{t+1} = (\epsilon'_{t+1}, \epsilon'_t)'$.

The stacked model in (17) is formally a VAR(1) model. We can apply formula (5) to forecast $Y_{t+1}$ when new information arrives between the current period and the next period. The first component of $Y_{t+\Delta t}$ is the forecast of the VAR(2) model we need.

It is to note that although the updating formula (5) gives the variance minimal forecast for the underlying time-continuous model of the stacked VAR(1), it cannot be interpreted as giving the variance minimal forecast for the underlying time-continuous model of the original VAR(p). The reason is that a stacked VAR(1) model is equivalent to the original VAR(p) model only at discrete time points. Nevertheless, formula (5) provides a simple interpolation forecast that is consistent with the original VAR(p) model.

[2]See Hamilton (1994) p. 150 for general VAR(p) cases.

### 2.2.4 Remarks on General Cases

Since a forecasting model like (1) will not necessarily encompassed by a VAR(p) process, the possible underlying continuous process for the discrete forecasting model can be very general, such as

$$dy_t = \mu(Y_t, t)dt + \sigma(Y_t, t)dW.$$

This can be a very complicated diffusion process that may become untraceable. One often used method to study an untraceable diffusion process is local linearization[3], i.e. the general process is approximated locally piecewise by linear diffusion process.

$$dy_t = \kappa_t y_t dt + \sigma_t dW,$$

where the parameter $(\kappa_t, \sigma_t)$ is constant in the interval $(t, t+1)$, but they are generally time-varying from one interval to other interval. Note that Assumption 2.1 is a local assumption on the process within the interval $(t, t+1)$. Therefore, this assumption can be approximately satisfied by all locally linearized general continuous stochastic processes. In this sense we can apply the derive updating rule (5) approximatively to all general forecasting models.

## 2.3 Updating Forecast when only some Information arrives between two periods

One complication with multivariate cases is that often only for a subset of variables new information is available at higher frequency. This is in many cases the reason why the forecasting model is constructed as time-discrete model at the lower frequency. Then updating of a forecast must be done conditionally on this subset of variables. For this purpose we separate the variables into two parts, $Y_t = (Y_{1t}', Y_{2t}')$. $Y_{1t}$ represents those variables for which we have new information at $t + \Delta t$, while $Y_{2t}$ represents those variables for which we don't have new information. Using the train rule of expectation, we have

$$E(Y_{t+1}|\Omega_t, Y_{1,t+\Delta t}) = E((E(Y_{t+1}|\Omega_{t+\Delta t}))\,|\Omega_t, Y_{1,t+\Delta t})$$

$E(Y_{t+1}|\Omega_{t+\Delta t})$ can be calculated using the updating formula in (5) which in this case takes the following form.

$$E\left(\begin{array}{c} Y_{1,t+1} \\ Y_{2,t+1} \end{array} \middle| \Omega_{t+\Delta t}\right) = \left(\begin{array}{c} Y_{1,t+\Delta t} \\ Y_{2,t+\Delta t} \end{array}\right) + (1-\Delta t)\left(\begin{array}{c} E(Y_{1,t+1}|\Omega_t) - Y_{1,t} \\ E(Y_{2,t+1}|\Omega_t) - Y_{2,t} \end{array}\right) \tag{18}$$

Now we need to calculated

$$E\left(\begin{array}{c} Y_{1,t+\Delta t} \\ Y_{2,t+\Delta t} \end{array} \middle| \Omega_t, Y_{1,t+\Delta t}\right)$$

According to A1 in Assumption 2.1 the forecasting model in (1) can be written as follows.

$$\left(\begin{array}{c} Y_{1,t+\Delta t} \\ Y_{2,t+\Delta t} \end{array}\right) = \left(\begin{array}{c} Y_{1,t} \\ Y_{2,t} \end{array}\right) + \Delta t \left(\begin{array}{c} E(Y_{1,t+1}|\Omega_t) - Y_{1,t} \\ E(Y_{2,t+1}|\Omega_t) - Y_{2,t} \end{array}\right) + \left(\begin{array}{c} \epsilon_{1,t+\Delta t} \\ \epsilon_{1,t+\Delta t} \end{array}\right) \tag{19}$$

[3]See Ozaki (1992) for more details.

Premultiplying equation (19) with $\begin{pmatrix} I & 0 \\ -\Sigma_{21}\Sigma_{11}^{-1} & I \end{pmatrix}$ we eliminate the correlation between the forecast error of $\epsilon_{1,t+1}$ and that of $\epsilon_{2,t+1}$ and obtain:

$$Y_{2,t+\Delta t} = Y_{2,t}+\Delta t(E(Y_{2,t+1}|\Omega_t)-Y_{2,t})+\Sigma_{21}\Sigma_{11}^{-1}(Y_{1,t+\Delta t}-Y_{1,t}-\Delta t(E(Y_{1,t+1}|\Omega_t)-Y_{1,t}))+\epsilon^*_{2,t+\Delta}$$

where $\epsilon^*_{2,t+\Delta} = \epsilon_{2,t+\Delta} - \Sigma_{21}\Sigma_{11}^{-1}\epsilon_{1,t+\Delta}$ is uncorrelated with $\epsilon_{1,t+\Delta}$. Taking the expectation conditionally on $Y_{1,t+\Delta t}$ we have:

$$\hat{Y}_{2,t+\Delta t|Y_{1,t+\Delta t},\Omega_t} = Y_{2,t}+\Delta t(E(Y_{2,t+1}|\Omega_t)-Y_{2,t})+\Sigma_{21}\Sigma_{11}^{-1}(Y_{1,t+\Delta t}-Y_{1,t}-\Delta t(E(Y_{1,t+1}|\Omega_t)-Y_{1,t})) \tag{20}$$

Inserting $\hat{Y}_{2,t+\Delta t|Y_{1,t+\Delta t},\Omega_t}$ from (20) into (18) we obtain the updating rule for the case when only a part of the information is available between current period and the next period.

$$E\begin{pmatrix} Y_{1,t+1} \\ Y_{2,t+1} \end{pmatrix}\Big|\Omega_t, Y_{t+\Delta t}\Bigg) = \begin{pmatrix} Y_{1,t+\Delta t} \\ \hat{Y}_{2,t+\Delta t|Y_{1,t+\Delta t},\Omega_t} \end{pmatrix} + (1-\Delta t)\begin{pmatrix} E(Y_{1,t+1}|\Omega_t) - Y_{1,t} \\ E(Y_{2,t+1}|\Omega_t) - Y_{2,t} \end{pmatrix} \tag{21}$$

# 3 Two Application Examples

The formula in (5) can be used to update forecasts in a quarterly model using monthly new information. In this case $\Delta t$ assumes the value of 0.33 and 0.66 for the first and the second month respectively. For the third month $\Delta t = 1$, formula (5) becomes the usual updating formula for quarterly data.

**Example 1** Forecasting a unit-root variable.

Many economic time series such interest rates, exchange rates and stock market indices are modelled as random walk processes. It is well known that random walk processes are Markov and the best forecast for the future is the current value.

Suppose that we have a quarterly model for a certain interest rate that is formulated as a unit root process as follows.

$$r_{t+1} = \rho r_t + \epsilon_{t+1}.$$

where $\rho = 1$. We have $\hat{r}_{t+1|t} = E(r_{t+1}|\Omega_t) = r_t$. This model says the best forecast for the interest rate in next period is the current value of the interest rate. After one month we have new information about the interest rate $r_{t+0.33}$. According to updating formula (5) we have

$$\hat{r}_{t+1|t+\Delta t} = r_{t+\Delta t} + (1-\Delta t)(\hat{r}_{t+1|t} - r_t) = r_{t+\Delta t}.$$

This is the same what we would have done intuitively when we regard the interest rate follows a random walk.

**Example 2** Forecasting Consumption Growth

We consider an example of forecasting the Australian growth of private consumption expenditure using the consumer sentiment index. A VAR(5) model is fitted to the log difference of the real private consumption expenditure and log of the consumer sentiment index. [4]

$$\begin{pmatrix} y_{1,t+1} \\ y_{2,t+1} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \sum_{k=1}^{5} \begin{pmatrix} c_{11,k} & c_{12,k} \\ c_{21,k} & c_{22,k} \end{pmatrix} \begin{pmatrix} y_{1,t-k} \\ y_{2,t-k} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t+1} \\ \epsilon_{2,t+1} \end{pmatrix}, \tag{22}$$

where $y_{1,t}$ is the first difference of log of the private consumption expenditure and $y_{2,t}$ is the log of the consumer sentiment index. The estimated variance-covariance matrix of the residuals is

$$\hat{\Sigma} = \begin{pmatrix} 0.00003496 & 0.00003220 \\ 0.00003220 & 0.00641546 \end{pmatrix}$$

For the September quarter we have $y_{1,t} = 0.000571$, $y_{2,t} = 4.5240$. The forecast for the December quarter 2008 was $y_{1,t+1|t} = 0.005127$, $y_{2,t} = 4.5930$. The realization of the consumption growth in the December quarter was 0.000865. According to this realization the September forecast overestimated the December consumption growth. The data for the private consumption expenditure are available quarterly, while the data of the consumer sentiment index are available monthly. This is the case when only a part of the information arrives between the current period and the next period. Using the updating rule (21), we could update the forecast of the consumption growth for the December quarter, when the October observation of the consumer sentiment index was available. The index value was 82.0. According to the updating rule (21) we obtain $y_{1,t+1|t+\Delta t} = 0.003582$ and $y_{2,t+1|t+\Delta t} = 4.4523$. This was an improvement of the forecast.

Over the last 5 years the root mean square error (RMSE) of the quarterly one step ahead forecast of the consumption growth was 0.0051. The MSE of the updated quarterly forecast was 0.0047. This shows that the simple updating rule (5) can improve the forecasting quality in empirical cases.

# 4 Concluding Remarks

Time-discrete models are the most popular models used in forecasting economic time series. One often encountered problem is to update forecasts according the newest information. If models are set out to be time-discrete, new information that arriving between the current period and the next period cannot be directly used in the model. In this note we attempt to formalize an updating rule for forecasting when information arrives between periods in a time-discrete model. Using the existing model structure, our approach bridges the process between periods by linear interpolation. The interpolation formula provided has the following properties: (1) It is consistent with the original model at discrete time points; (2) If the original model is for VAR(1) the interpolation formula gives the variance minimal forecast of the underlying time-continuous model, when we taken the original VAR(1)/AR(1) model as result of a model fitted to a continuous process that only sampled at discrete time points. (3) For a general underlying continuous stochastic process, our updating rule gives the variance minimal forecast for the approximating process by local linearization.

[4] Source of data: Reserve Bank of Australia Australian Bureau of Statistics.

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