
This paper provides an interesting overview on the economic relevance of the Törnqvist price/quantity indices in index (number) theory. The current work complements (and partially overlaps with) previous publications of the author on the subject. I find this survey/overview useful also for economists that are non expert on the subject, it reviews the role of the Törnqvist indices within a number of frameworks of relevance for the measurement of real values and inflation.

Concerning the technical content of the paper I have no major comments, while I would like to provide some suggestions related to the exposition.

1- In particular the author provides, across the various sections, a number of results concerning the Törnqvist indices, often labelled as "Proposition" even though in some cases a proposition consists only of a displayed formula. My suggestion is to highlight systematically all these results, that are in my opinion the focus of the paper, making use of a consistent notation and provide self contained propositions as done for instance for the first and second Törnqvist approximation theorems at pages 10-11.

For instance it will be useful to formalize with a proposition the result in (3.26) at page 12, moreover my suggestion is to drop the terms "Proposition" before (4.8) and (4.9) at page 14 and instead to formalize with an unique self contained proposition the implied result illustrated in (4.10) which directly concerns the Törnqvist indices. In the same spirit I suggest to expand the propositions at page 15 in order to make them self contained and not just reduce them at the formulas (4.13) and (4.14). Similar comments hold for the pair of propositions at page 16 and 18 (that can also be reduced to only one per page) and the proposition at page 17.
Moreover, the author, when appropriate, should highlight the original works where are derived the results illustrated in the paper, for instance the first and second Törnqvist approximation theorems are similarly proved as an original contribution in Hillinger (2003) [The money metric, price and quantity aggregation and welfare measurement in Contributions in Macroeconomics, Vol. 3, Issue 1, Article 7].

2- There is a further aspect that I would like to highlight, it concerns a substantial overlap of the material in some sections of the paper with the earlier work in Hillinger (2003). To be more precise sections 4, 5, 6, 7, and 8 at pages 13-18 in the paper in large part overlap with the material presented in sections 3, 4, and 5 in Hillinger (2003) with exceptions of some integrations and adaptations in the text. I feel that these connections should at least be more emphasized in the current work. As far as I can see the only reference is at page 3 where the author writes: "... a precursor of the present paper is Hillinger (2003)."

3- To complete I’ll focus on some minor specific issues. (1) in the central part of page 4, just above (2.3) the sentence "The three conditions that do not are shown below" should be modified into "The three axioms that do not are.....". (2) At page 11 most of the material in the first paragraphs of the subsection "Third derivation of the Törnqvist index" have already been presented at page 8 I suggest to drop this material quoting the relevant formulas at page 11. (3) The symbols in (3.28) and (3.29) should be $>$ instead of $\geq$. 