

Report on

A data-reconstructed fractional volatility model

by R. Vilela Mendes and M.J. Oliveira

The paper proposes a procedure for modeling the volatility process of returns on financial assets. The work is motivated by the fact that the widely adopted (geometric) Brownian motion (Bm) blatantly violates commonly observed empirical characteristics of financial return series. The paper is particularly motivated by the well-known volatility-clustering properties. Specifically, the paper introduces the particular volatility process, namely fractional Brownian motion (fBm), under consideration; it illustrates empirical evidence; and, finally, presents results on option valuation when volatility follows an fBm process.

General Comments

The paper addresses an important topic in theoretical and empirical finance. Conventional models are commonly based on simplifying assumptions, such as Bm-type data generating processes, in order to facilitate analytical and or statistical tractability. Especially with respect to the question of modeling volatility of asset-return processes, numerous modeling strategies, allowing for the observed empirical regularities, have been suggested in the literature. Clearly, a somewhat unifying approach would be highly desirable in this line of research; and it may well be the fBm-type models could be a step in that direction.

Unfortunately, neither the theoretical motivation nor the empirical analyses reported in the paper provide sufficient informational value, so the reader cannot assess the relevance of the results (see specific comments below). Especially, the empirical findings are poorly described and, thus, are irreproducible—a minimum requirement in scientific work. In view of the shortcomings in the presentation of the proposed volatility process, it is difficult to judge the value of the results on option pricing.

Specific Comments

1. It is difficult for the reader to place the results presented in the paper in the broader context of the existing literature in this field. For example, there has been quite extensive work on the use fBm in the finance literature. Also, how do the results relate to the class of fractionally integrated GARCH models? In short, a suitable review of the related literature in financial econometrics and quantitative finance would be highly desirable.
2. The variance process is defined in Eqn. (5) as a limiting process, and it is suggested that (5) can be used to derive the variance process empirically. It amounts to the commonly used unconditional moving-window sample estimate. However, when implemented according to the definition, σ_t^2 is defined in a forward-looking fashion, representing future dispersion.

3. Instead of (5), the authors suggest to estimate the variance of the log-price process, Eqn. (6), rather than the squared difference of log-price process, since the former may yield more reliable estimates. However, the justification is not clear. In fact, Eqn. (2) is the data generating process, it is not clear if the log-price variance is finite. Most empirical evidence, suggests that it is difficult to reject the infinite-variance hypothesis for log asset prices – and Eqn. (9) suggest this is here also the case.
4. The difficulties raised in the previous comment may be the reason why the authors perform a detrending procedure prior to their analysis. They fit a polynomial in time (presumably to the log-price series) to induce stationarity and apply their procedure to the residuals of that fit. It is not clear in what way the detrending step affects the results of the subsequent analysis. By design, the assumed data generating process has a stochastic trend, but the authors remove a deterministic trend. How should one interpret the deterministic polynomial trend? Does it mean stock prices follow a deterministic pattern and are slightly perturbed by some additive noise?

The authors do not report what degree the polynomial tends to have. The criterion “no longer well conditioned” (p. 4) is vague and suggests a lack of robustness. It is to be suspected, that the fitted polynomial varies greatly when applied to different subsamples of the data. Also, isn't there a mismatch in the concepts of “mathematical simplicity” and “no longer well conditioned”?

Given that the polynomial detrending typically induces strong temporal dependence in the residuals, it is questionable what we really learn about the memory properties of the data. Also, given that the detrending step involves the whole sample, it is not plausible to refer to the residual variance as a *local* variance estimate.

5. The reported estimate for the Hurst coefficient for the NYSE index is $H \approx 0.8$ (p. 9). In addition to the point estimate, the confidence interval should be reported. Moreover, the variation of the H-estimate with respect to different polynomial-trend specifications should be reported.
6. A particular parameter setting is used to generate Figure 2 (p. 8). Are the choices for H , k and β the result of an estimation from data or a, more or less, ad-hoc choice?
7. The authors speak of “a reasonable fit” when referring to Figure 2. In the statistics literature there are numerous goodness-of-fit measures for density estimation. Results of such measures would be more informative for the reader. More importantly, it is of little value to see in an isolated fashion how the authors' approach compares to the naïve normal model. Many alternative models for capturing distributional and memory properties have been suggested in the literature since Mandelbrot (1963) and are being applied in practice. Only in comparison to existing alternatives to geometric Bm-type models the reader can assess the usefulness of the proposed strategy. For example, a comparison with

FIGARCH-type models seems to be in order here. Ideally, out-of-sample forecasting comparisons should be conducted.

8. Better definitions and descriptions of what the figures really display would be extremely helpful.