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Bridging Economic Theory Models and the Cointegrated Vector Autoregressive Model

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Abstract:

Examples of simple economic theory models are analyzed as restrictions on the Cointegrated VAR (CVAR). This establishes a correspondence between basic economic concepts and the econometric concepts of the CVAR: The economic relations correspond to cointegrating vectors and exogeneity in the economic model implies the econometric concept of strong exogeneity for β . The economic equilibrium corresponds to the so-called long-run value (Johansen 2005), the comparative statics are captured by the long-run impact matrix, C ; and the exogenous variables are the common trends. Also, the adjustment parameters of the CVAR are shown to be interpretable in terms of expectations formation, market clearing, nominal rigidities, etc. The general-partial equilibrium distinction is also discussed.

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1 Introduction

The purpose of this paper is to facilitate the formulation of economic theory models as restrictions on the Cointegrated Vector Autoregressive (CVAR) model¹.

It is well-known that macroeconomic time series often exhibit persistence that can be modelled as the *integrated* type, $I(1)$, which makes the CVAR the relevant econometric model (Granger 1981, Engle and Granger 1987, and Johansen 1996). It is also well-known that, in spite of their diversity, most economic theory models involve the *same* basic concepts, such as behavioral relations, comparative statics, the equilibrium condition, the endogenous-exogenous dichotomy.

Given the purpose at hand, it therefore seems useful to relate such basic concepts of economic models to the statistical concepts of the CVAR, such as cointegrating relations, common trends, loadings matrix, etc. (Johansen 1996). To do this, I shall consider a few examples of simple theory models, and suggest how they translate into restrictions on a VAR, when the data can be *approximated* as $I(1)$. This establishes a simple framework, within which one can discuss and analyze interesting economic hypotheses about market clearing, nominal rigidities, expectations, partial and general equilibrium etc..

To keep the exposition accessible it seems useful to begin with static - and simple dynamic models, rather than the "state-of-the-art" Dynamic Stochastic General Equilibrium (DSGE) model, since the fundamental assumptions are similar in form. The idea is to suggest a simple framework that can be modified and extended for various purposes. Though the examples are simple, their form represents a wide range of theory models, such as competitive partial- and general equilibrium models, the IS-LM- and AS-AD models, the Wage- and Price setting models etc..

The methodological approach follows the Cointegrated VAR Methodology (Juselius 2006, Hoover, Juselius, and Johansen 2007). This implies, that theory models are viewed as sub-models embedded in a "larger" well-specified statistical model (Johansen 2006), here the *unrestricted* VAR, in which all variables are modelled (are endogenous) from the outset. Second, it also means that the $I(1)$ -, or unit root assumption is generally viewed as a *statistical approximation*, used to obtain useful inference on relationships between persistent series. For the type of theory models I am considering, the order of integration is not important for the theory model to hold, and hence, can be determined by statistical testing.

In the next section, I summarize the basic concepts for the type of economic theory models considered here. A simple supply- and demand model illustrates. The notion of persistence and the econometric concepts of the CVAR are then described briefly in section 3. Section 4.1 collects the threads by suggesting a set of restrictions on a VAR, consistent with the simple static supply- and demand model from section 2, *and* the persistence of the data cf. section 3. This establishes a correspondence between the economic -, and statistical concepts, upon which I

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shall elaborate. A few generalizations of the empirical model, under which the correspondence holds, are considered in section 4.2. To consider explicit hypotheses about the adjustment parameters of the CVAR, section 4.3 analyzes a simple dynamic model building on expectations formation. As an extension of the basic framework, a general equilibrium example is analyzed in section 4.4.1, and the potential relation to DSGE models is then briefly discussed (section 4.4.2). Discussion and further generalizations are found in section 5, while section 6 concludes.

2 Some basic concepts of economic theory models

The purpose of most economic models is to explain a set of variables, the *endogenous variables*, as a function of the *exogenous variables*². The generation of the latter is, by construction of the model, not explained. In the present paper *exogeneity* is referred to as "*economic exogeneity*", in order not to confuse it with the econometric concepts of weak- and strong exogeneity (Engle, Hendry, and Richard 1983): *A variable is economically exogenous if it is not influenced at any point in time by any other variable in the system under study, including other exogenous variables.*

The economic model contains *behavioral relations* for the endogenous variables. These may be plans *contingent* on either *observed outcomes*, or *expectations*, and may be regarded as solutions to optimization problems. An *equilibrium condition* is imposed to secure a solution with no inherent inconsistency between the plans of different agents, and hence no tendency for the system to change. This solution defines the *economic equilibrium*.

In *static* theory models the so-called *comparative static analysis* is the study of the effects on the endogenous variables in economic equilibrium from hypothetical changes in the exogenous variables (Samuelson 1941, Intriligator 1983). Hence, static models ignore the process of transition between the involved equilibria.

As a basic example, building on the above concepts consider the static supply- and demand model,

$$Q^d = a_0 - a_1P + a_2W, \quad (1)$$

$$Q^s = b_0 + b_1P - b_2Z, \quad (2)$$

$$Q^s = Q^d, \quad (3)$$

where Q^d and Q^s are, respectively, demanded and supplied quantity, P , the price level, W , wage income and Z , the price of an input used in the production of Q . All parameters are positive, and all variables are in logarithms. The endogenous variables are Q^d , Q^s and P , while W and Z are economically exogenous. The equations (1) and (2), define the two behavioral relations, and (3) is the equilibrium condition. The economic equilibrium is,

$$Q^* = \frac{b_1(a_0 + a_2W) + a_1(b_0 - b_2Z)}{a_1 + b_1}, \quad P^* = \frac{(a_0 + a_2W) - (b_0 - b_2Z)}{a_1 + b_1}, \quad (4)$$

²For more on the concepts of economic models and related issues see e.g. Intriligator (1983).

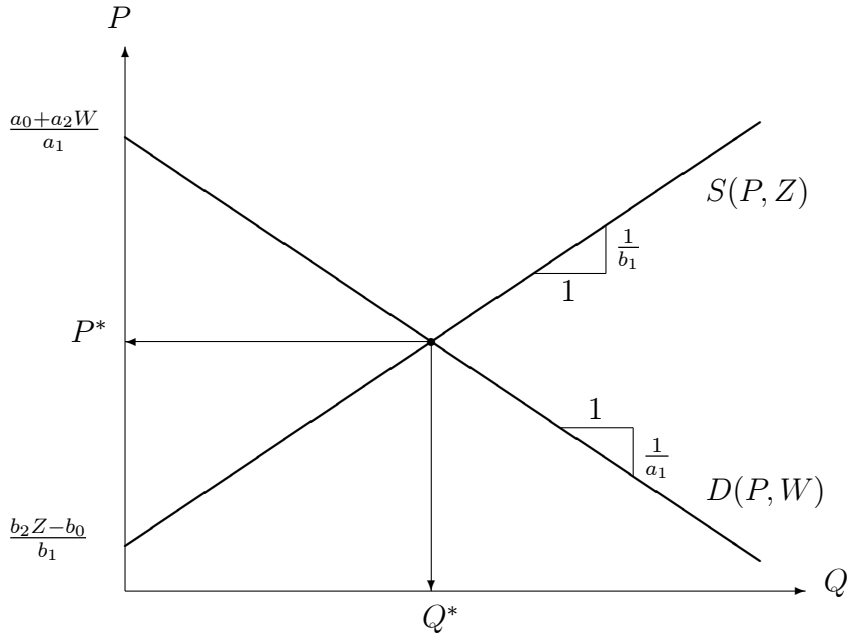


Figure 1: The economic cross of supply and demand.

with comparative static effects,

$$\frac{\partial Q^*}{\partial W} = \frac{b_1 a_2}{a_1 + b_1}, \quad \frac{\partial Q^*}{\partial Z} = -\frac{a_1 b_2}{a_1 + b_1}, \quad \frac{\partial P^*}{\partial W} = \frac{a_2}{a_1 + b_1}, \quad \frac{\partial P^*}{\partial Z} = \frac{b_2}{a_1 + b_1}. \quad (5)$$

The model is illustrated as the famous economic cross in Figure 1, where $D(P, W)$ and $S(P, Z)$ denote the demand- and supply curves respectively. P is on the vertical axis and Q on the horizontal following the convention in economics. A similar cross will be used below to facilitate the interpretation of the CVAR.

The simple *dynamic* models discussed here, resemble the static ones, but they also describe movements *outside* equilibrium. Many theoretical assumptions result in dynamics, for example dynamic optimization, learning, or expectations formation. Here, the focus is on expectations formation.

The purpose is now to suggest a set of restrictions on a VAR, consistent with simple theory models like the above, when data are persistent. However, first, a precise notion of persistence and the econometric tools of the CVAR analysis are needed.

3 The persistence of macroeconomic data and the CVAR

The type of economic theory models under study can be written as sub-models of the general linear p - dimensional model,

$$Ax_t = B_1 x_{t-1} + \dots B_k x_{t-k} + B_0 D_t + u_t, \quad (6)$$

where D_t is a $d \times 1$ term of d deterministic components, the initial values, x_{1-k}, \dots, x_0 , are fixed, A has full rank and represents a normalization, $u_t \sim i.i.N(0, \Sigma)$ with Σ diagonal, and B_i are

unrestricted³. The corresponding *reduced form VAR(k)* model is,

$$x_t = \Pi_1 x_{t-1} + \dots + \Pi_k x_{t-k} + \Phi D_t + \varepsilon_t, \quad (7)$$

with $\varepsilon_t \equiv A^{-1}u_t$, $\Phi \equiv A^{-1}B_0$, and $\Pi_i = A^{-1}B_i$ for $i = 1, \dots, k$. This can be reparameterized in the Error- (or Equilibrium-) Correction-Mechanism form (ECM) as,

$$\Delta x_t = \Pi x_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \Phi D_t + \varepsilon_t, \quad (8)$$

where $\Pi \equiv \sum_{i=1}^k \Pi_i - I_p$ and $\Gamma_i \equiv -\sum_{j=i+1}^k \Pi_j$. For later, define $\Gamma \equiv I - \sum_{i=1}^{k-1} \Gamma_i$.

The dynamic properties are summarized in the *roots*, z , of the *characteristic equation* corresponding to (8),

$$|A(z)| = 0, \quad (9)$$

where $|\cdot|$ denotes the determinant and where,

$$A(z) \equiv (1 - z)I - \Pi z - \sum_{i=1}^{k-1} \Gamma_i (1 - z)z^i. \quad (10)$$

In practice, we typically have a *relatively short* sample of time series which yield a set of estimated roots, \hat{z} , all with $|\hat{z}| > 1$, but *some close to 1*, and where $|\cdot|$ denotes the modulus. I refer to such time series as being *persistent*. To conduct inference, assumptions about the underlying Date Generation Process (DGP) are needed, so that asymptotic distributions can be used as approximations of the unknown finite sample distributions of estimators and statistics. In this case, the choice is between assuming that all roots have $|z| > 1$, or, that some are at 1 while the rest have $|z| > 1$. Under the first assumption *asymptotics* are standard Gaussian based. However, when some roots are close to 1, as suggested by the estimates, the asymptotic distributions will be poor approximations for typical sample lengths, implying unreliable inference (See e.g. Johansen 2006). From a statistical inferential point of view, it is then probably more useful to impose $z = 1$ for some roots, as an approximation, and use the corresponding asymptotic inference theory for cointegrated I(1) processes described in Johansen (1996), cf. the second assumption.

However, though a useful statistical approximation, this unit root restriction may, or may not, contradict the economic model. We can distinguish between three cases:

First, if the economic theory predicts unit roots, we of course impose them and continue the analysis, to find out whether these are generated in the manner according to the theory.

Second, the theory model may instead involve a steady state, implying a stationary VAR model. Given the persistence and the sample at hand, it is however, not possible to conduct inference on the steady state relations and multipliers to a satisfactory extent. As a result, the price of valid inference is that we are forced to give up the stationarity assumption of the model,

³For more on the technical details and applications of the CVAR see Johansen (1996) and Juselius (2006) respectively.

hopefully in order to learn about other assumptions of the model. In this case, one would not necessarily claim that the data are incompatible with the underlying stationary "theory-VAR", but simply that inference under such an assumption is not useful.

Third, it may also be the case that the assumption of stationarity or non-stationarity is not necessarily implied by the theory model. This is the case for the type of theory models considered here: The economically exogenous variables cause the endogenous variables, but not vice versa. As a result, persistence in the system variables must originate from the generation of the former. As mentioned in section 2, this is outside the theory model, implying that imposing $z = 1$, i.e. estimating a CVAR, is not contradicting the theory model, and since it delivers better inference, it is the obvious thing to do (see section 4.1).

Whichever of the three cases, we see from (9) and (10), that imposing a root at 1, means $|A(1)| = |-\Pi| = 0$, and therefore imposing reduced rank on Π , which can be parameterized as,

$$\Pi = \alpha\beta', \quad (11)$$

using a singular value decomposition, and where the matrices α and β are $p \times r$, α being the *adjustment coefficients*, and β , the *r cointegrating vectors*.

The model (8), under the restriction on Π in (11), but otherwise *unrestricted* parameters, including α and β , and $r < p$, is thus a sub-model of the VAR, and is called a cointegrated I(1) model, denoted by $H(r)$. The theory models considered below are viewed as sub-models of the I(1) models.

As alluded to above, a relevant assumption about the DGP is that,

$$\text{The roots of (9) have } |z| > 1 \text{ or } z = 1. \quad (12)$$

Under (12) and,

$$|\alpha'_\perp \Gamma \beta_\perp| \neq 0, \quad (13)$$

where α_\perp and β_\perp are the orthogonal complements, the I(1) model can be represented in *Moving Average (MA) form*,

$$x_t = C \sum_{i=1}^t (\Phi D_i + \varepsilon_i) + C(L)(\Phi D_t + \varepsilon_t) + C_0, \quad (14)$$

where $C \equiv \beta_\perp (\alpha'_\perp \Gamma \beta_\perp)^{-1} \alpha'_\perp$ is the *long-run impact matrix*, $C(L)$, a convergent lag polynomial, and C_0 depends on initial values, with $\beta' C_0 = 0$ (Theorem 4.2, Johansen 1996).

The long-run movement of the series is described by the $p - r$ dimensional vector of *common (stochastic) trends*, (CT_t) , given by,

$$CT_t \equiv \alpha'_\perp \sum_{i=1}^t \varepsilon_i. \quad (15)$$

Usually, the stochastic trend, $C \sum_{i=1}^t \varepsilon_i$, is decomposed into CT_t , and the so-called *loadings*

matrix, given by,

$$L \equiv \beta_{\perp}(\alpha'_{\perp}\Gamma\beta_{\perp})^{-1}, \quad (16)$$

which tells us how each of the $p - r$ common trends affect the individual variables.

Below, the focus is primarily on the VAR with one lag. This keeps the analysis simple while still illustrating the main points clearly. More importantly, the VAR(1) has a particular status since any VAR(k) can be rewritten as a VAR(1), using the *companion form* (See the appendix in Johansen 2005).

I shall also assume that the deterministic term, ΦD_t , is a constant term, which is restricted, so that it does not produce a trend in the series. Again, this is to keep it simple and generalizing deterministic (trends, indicator variables etc.), does not affect the conclusions, but merely blurs the illustrations. Hence, I assume that,

$$D_i = 1 \text{ and } \Phi = \alpha s, \quad (17)$$

in (8), where s is $r \times 1$. The resulting CVAR(1) can therefore be written as,

$$\Delta x_t = \alpha(\beta' x_{t-1} + s) + \varepsilon_t, \quad (18)$$

which is used repeatedly below.

In a VAR(1), $\Gamma = I$, and the condition in (13) reduces to,

$$|\alpha'_{\perp}\beta_{\perp}| \neq 0, \quad (19)$$

and the MA representation becomes,

$$x_t = C \sum_{i=1}^t \varepsilon_i + \sum_{i=0}^{\infty} C_i^* (\alpha s + \varepsilon_{t-i}) + C x_0, \quad (20)$$

with $C = \beta_{\perp}(\alpha'_{\perp}\beta_{\perp})^{-1}\alpha'_{\perp}$, $C_i^* = \alpha(\beta'\alpha)^{-1}(I_r + \beta'\alpha)^i\beta'$.

Under the assumption that $r(\Pi) = r < p$, the assumptions, (12), and (19) *together*, are equivalent to $A(z)$ having exactly $p - r$ roots at $z = 1$, while the rest have $|z| > 1$. Either of these equivalent conditions imply that the eigenvalues of the matrix $I_r + \beta'\alpha$, all have modulus less than 1, or equivalently that,

$$\rho(I_r + \beta'\alpha) < 1, \quad (21)$$

where $\rho(\cdot)$ is the spectral radius, which in turn implies that $\beta'\alpha$ has full rank, r . From these assumptions one can then establish the identity,

$$\alpha(\beta'\alpha)^{-1}\beta' + \beta_{\perp}(\alpha'_{\perp}\beta_{\perp})^{-1}\alpha'_{\perp} = I_p, \quad (22)$$

which can be used to derive the expression in (20). Given (20) the *impulse response function*

is,

$$\frac{\partial E(x_{t+h} | x_t)}{\partial x_t} = \frac{\partial E(x_{t+h} | x_t)}{\partial \varepsilon_t} = C + C_h^* \rightarrow C, \text{ for } h \rightarrow \infty, \quad (23)$$

where $C_h^* \rightarrow 0$ follows from (21).

The so-called *attractor set*, for the VAR(1), is usually defined as,

$$\mathcal{A} = \{x \in R^p \mid \beta'x = 0\} = sp(\beta_{\perp}). \quad (24)$$

Finally, two concepts of econometric exogeneity are needed, *weak*- and *strong exogeneity*, (See Engle, Hendry, and Richard 1983). They are both defined with respect to the parameters of interest, which is β in this context. These concepts are usually discussed in connection with efficient estimation and forecasting from partial models respectively (Ericsson, Hendry, and Mizon 1998). Here, the focus is on their relation to (and distinction from) the above concept of economic exogeneity. A variable is said to be *weakly exogenous* for β if it has a zero row in α implying that the variable does not react to equilibrium errors (Johansen 1992). This in turn implies that the cumulation of shocks to this variable is a common trend. If, in addition, this variable is not *Granger Caused* by the endogenous variables, the variable is said to be *strongly exogenous* for β (Johansen 1992). Partitioning x_t as $(x'_{1t}, x'_{2t})'$, and correspondingly the matrices in (8), under the restriction, (11), as,

$$\alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}, \text{ and } \Gamma_i = \begin{pmatrix} \Gamma_{11,i} & \Gamma_{12,i} \\ \Gamma_{21,i} & \Gamma_{22,i} \end{pmatrix}, \quad (25)$$

weak exogeneity of x_{2t} for β , is the restriction that $\alpha_2 = 0$, while strong exogeneity requires $\Gamma_{21,i} = 0$, in addition. For the VAR(1) the two concepts coincide.

4 Analyzing theory models in the CVAR model

4.1 A static theory model

Suppose that a VAR(1) describes the variation in the time series (Q_t, P_t, W_t, Z_t) , corresponding to the variables in section 2. Assume that these series are *persistent*, cf. section 3. Under this assumption, a set of restrictions on the VAR, consistent with the simple demand- and supply model (1) - (3) is now suggested.

In general, the equations defining an economic model involve *latent* constructs, such as expectations and plans. Hence, they are not directly empirically implementable. Here, the relations, (1) and (2), are the plans involving the latent variables, Q^s and Q^d . In particular, introducing a time index, I shall assume that these relations are *contingent plans*, conditional on *observed* outcomes (see e.g. Hendry 1995): For example, for the demand relation, (1), $Q_t^d = a_0 - a_1 P_t + a_2 W_t$, denotes *demand at time t*, where the plan, Q_t^d , is unobserved while P_t and W_t are *realized* values. This is a *point* on the *demand curve at time t*, which is denoted by $D(P, W_t) \equiv a_0 - a_1 P + a_2 W_t$.

In contrast, the VAR model is formulated in the observables. As a consequence, a mapping relating the latent variables and plans to the observables is needed. Usually, such mappings come in the form of an *observation equation*, for Q_t outside equilibrium, and an *adjustment equation* for P_t outside equilibrium⁴.

Consider the *price adjustment mapping*. I assume that it has the general form,

$$\Delta P_t = g(Q_{t-1}, Q_{t-1}^d, Q_{t-1}^s), \quad (26)$$

where $g()$ is a continuous and locally differentiable function. It seems reasonable that the adjustment in prices from period $t - 1$ to t depends on what is learned or observed in period $t - 1$: Say, at the end of period $t - 1$, firms realize the reduction in inventories and the increased willingness to buy. As a consequence, they probably charge higher prices the next period.

Suppose, that (26) has the specific, though still general, form,

$$\Delta P_t = g(Q_{t-1} - Q_{t-1}^d, Q_{t-1} - Q_{t-1}^s), \quad (27)$$

where prices adjust as a result of the discrepancy between plans and realizations for both consumers and producers. Compared to the equations often used in the literature (see e.g. Laroque and Salanie 1995), the mapping in (27) allows for *different* adjustment processes for demand- and supply deviations respectively, which seems empirically relevant, as these processes may involve different sets of agents.

As an equilibrium represents a state with no change, it is natural to assume that,

$$g(0, 0) = 0, \quad (28)$$

so that a simple Taylor expansion of $g()$ in the vicinity of the equilibrium can be used to obtain the mapping, i.e.,

$$\Delta P_t \simeq g'_1(0, 0)(Q_{t-1} - Q_{t-1}^d) + g'_2(0, 0)(Q_{t-1} - Q_{t-1}^s), \quad (29)$$

where (28) is used, and g' is a partial derivative. A special case of (29), in which, excess demand causes prices to rise, follows from assuming $g'_1(0, 0) < 0$ and $g'_2(0, 0) = -g'_1(0, 0)$, since this implies $\Delta P_t \simeq g'_1(0, 0)(Q_{t-1}^s - Q_{t-1}^d)$, resembling equation A6.11 in Hendry (1995).

The Taylor approximation is only useful provided that $Q_t - Q_t^d$ and $Q_t - Q_t^s$ are stationary. But, as we shall see, this is exactly what cointegration means in this case (See eq. 43, and section 5).

As the form of the *observation equation* is similar to (29), this is given by,

$$\Delta Q_t \simeq h'_1(0, 0)(Q_{t-1} - Q_{t-1}^d) + h'_2(0, 0)(Q_{t-1} - Q_{t-1}^s), \quad (30)$$

⁴For a general treatment of macroeconomic models and these related concepts see fx. Hendry (1995), p. 781 ff.

where $h(\cdot)$ is the function corresponding to $g(\cdot)$ etc..

The partial derivatives, h'_1, h'_2 and g'_1, g'_2 , are evaluated in the equilibrium. They are thus constants and henceforth they are denoted as, α_{11}, α_{12} and α_{21}, α_{22} , respectively.

The price- and quantity adjustment in equations (29) and (30), represent the systematic, or anticipated part of the change from one period to the next. It seems reasonable to add the error terms, ε_{Pt} and ε_{Qt} , respectively in these equations, representing *unanticipated* and *unmodelled* influences. Their stochastic properties are given below.

As argued in section 3 the economically exogenous variables, W and Z , are the source of persistence, and since the theory model is not concerned with *how* these are generated, it seems uncontroversial to *empirically model* them as I(1) processes. Assume therefore that,

$$W_t = W_{t-1} + \varepsilon_{Wt}, \quad (31)$$

$$Z_t = Z_{t-1} + \varepsilon_{Zt}. \quad (32)$$

As argued below, this is where the persistence is approximated by imposing the unit roots.

Finally, it is assumed, as is usual, that,

$$\varepsilon_t \sim i.i.N(0, \Omega), \quad (33)$$

where $\varepsilon_t = (\varepsilon_{Qt}, \varepsilon_{Pt}, \varepsilon_{Wt}, \varepsilon_{Zt})'$ and Ω is diagonal. The shocks, ε_W and ε_Z , are referred to as demand- and supply shocks respectively, not to be confused with ε_Q and ε_P .

Collecting all this, the system, (29) - (32) can be written as the following CVAR(1)

$$\begin{aligned} \Delta Q_t &= \alpha_{11}(Q_{t-1} - (a_0 - a_1P_{t-1} + a_2W_{t-1})) + \alpha_{12}(Q_{t-1} - (b_0 + b_1P_{t-1} - b_2Z_{t-1})) + \varepsilon_{Qt}, \\ \Delta P_t &= \alpha_{21}(Q_{t-1} - (a_0 - a_1P_{t-1} + a_2W_{t-1})) + \alpha_{22}(Q_{t-1} - (b_0 + b_1P_{t-1} - b_2Z_{t-1})) + \varepsilon_{Pt}, \\ \Delta W_t &= \varepsilon_{Wt}, \\ \Delta Z_t &= \varepsilon_{Zt}, \end{aligned} \quad (34)$$

or in the compact notation from section 3,

$$\Delta x_t = \alpha(\beta'x_{t-1} + s) + \varepsilon_t, \quad (35)$$

with $x'_t = (Q_t, P_t, W_t, Z_t)$, and matrices given by,

$$\alpha = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 1 \\ a_1 & -b_1 \\ -a_2 & 0 \\ 0 & b_2 \end{pmatrix}, \quad \text{and } s = \begin{pmatrix} -a_0 \\ -b_0 \end{pmatrix}, \quad (36)$$

and the corresponding orthogonal complements,

$$\alpha_{\perp} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \beta_{\perp} = \begin{pmatrix} \frac{a_2}{1+\frac{a_1}{b_1}} & \frac{-b_2}{1+\frac{b_1}{a_1}} \\ \frac{a_2}{b_1+a_1} & \frac{b_2}{b_1+a_1} \\ 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (37)$$

Under the assumption (12), the model generates I(1) variables only, since $|\alpha'_{\perp}\beta_{\perp}| = 1$. The MA representation thus implies the following components,

$$CT_t = \begin{pmatrix} \sum_{i=1}^t \varepsilon_{W_i} \\ \sum_{i=1}^t \varepsilon_{Z_i} \end{pmatrix}, L = \begin{pmatrix} \frac{a_2}{\frac{a_1}{b_1}+1} & -\frac{b_2}{\frac{b_1}{a_1}+1} \\ \frac{a_2}{a_1+b_1} & \frac{b_2}{a_1+b_1} \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \text{ and } C = \begin{pmatrix} 0 & 0 & \frac{a_2}{\frac{a_1}{b_1}+1} & -\frac{b_2}{\frac{b_1}{a_1}+1} \\ 0 & 0 & \frac{a_2}{a_1+b_1} & \frac{b_2}{a_1+b_1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (38)$$

using equations (15) and (16). Note that, whereas $|\alpha'_{\perp}\beta_{\perp}| = 1$ is implied by the theory model, (12) is not.

The model in (35) and (36) is an identified sub model of the I(1) model, $H(2)$. It can be tested in $H(2)$ by jointly imposing the zero restrictions on α , the normalization, and the corresponding two *generically identifying* zero restrictions on β (See Johansen 1996). This can be done using the software CATS in RATS (Dennis, Hansen, and Juselius 2006).

From (36), it is seen that the consequence of the I(1) approximation is that the theoretical parameters of interest, the a_i and b_i , should be modeled as cointegrating parameters, and that the assumption of economic exogeneity translates into the econometric concept of strong exogeneity *in this case*.

As alluded to above, the I(1) -, or unit root approximation corresponds to the *empirical modelling* of the exogenous variables, in that (31) and (32) can be interpreted as approximations of the processes, $W_t = \rho_w W_{t-1} + \varepsilon_{Wt}$ and $Z_t = \rho_z Z_{t-1} + \varepsilon_{Zt}$, with $\rho_j < 1$ but close to 1, respectively. In such a case, instead of the unit roots, corresponding to (35) and (36), the true underlying process has two roots $\frac{1}{\rho_w}$ and $\frac{1}{\rho_z}$, both close to, but above 1, while the rest are the same, depending on α and β parameters only, also with $|z| > 1$, provided that (12) applies. The true process is therefore a stationary "near unit root" process generating persistent series. Approximating ρ_j by 1, is thus equivalent to approximating the two borderline unit roots, $\frac{1}{\rho_w}$ and $\frac{1}{\rho_z}$, by 1. Moreover, as opposed to the parameters a_i and b_i , ρ_w and ρ_z are *not theoretical parameters of interest*, and hence, it is clear that we should set them to 1, when data are persistent. This illustrates the argument put forward in section 3.

The model is illustrated in Figure 2. This resembles Figure 1 drawn for a given period, t : The intersection of the demand curve at time t , $D(P, W_t)$, and the supply curve at time t , $S(P, Z_t)$, determines the equilibrium at time t , (Q_t^*, P_t^*) , which is (4) with $W = W_t$ and $Z = Z_t$. In the diagram, the *realized* point in period t , (Q_t, P_t) , differs from the equilibrium.

The equilibrium at time t acts as a *pulling force* on the observed point, in the sense that, in

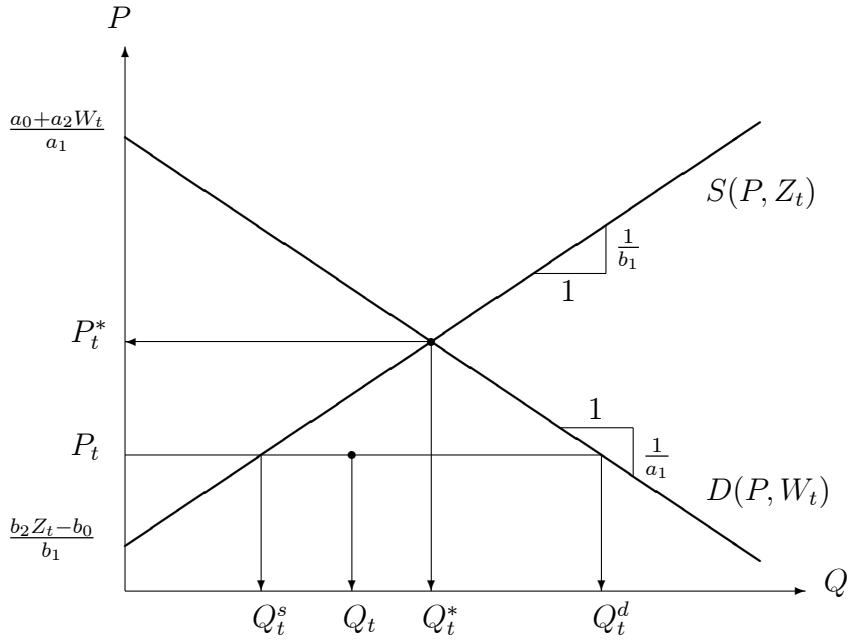


Figure 2: The supply and demand schedules at time t , i.e. given the values of the exogenous variables, W_t and Z_t .

the hypothetical absence of any other shocks from period $t + 1$ and onwards, the (Q, P) -point would converge towards (Q_t^*, P_t^*) , starting in (Q_t, P_t) . To see this, we simply set $\varepsilon_\tau = 0$ for $\tau \geq t + 1$, so that the values of W and Z are given at W_t and Z_t . Using the identity in (22) it can be shown that,

$$x_{t+h} = \alpha(\beta'\alpha)^{-1}(I_r + \beta'\alpha)^h(\beta'x_t + s) + Cx_t - \alpha(\beta'\alpha)^{-1}s, \quad (39)$$

for $h \geq 0$. Since (21) implies that $(I_r + \beta'\alpha)^h \rightarrow 0$, for $h \rightarrow \infty$, the limit of (39) is,

$$Cx_t - \alpha(\beta'\alpha)^{-1}s \equiv x^*. \quad (40)$$

This expression implies that $x^* = (Q_t^*, P_t^*, W_t, Z_t)'$, and since $W_{t+h} = W_t$ and $Z_{t+h} = Z_t$ this shows the convergence of (Q_{t+h}, P_{t+h}) towards the "economic equilibrium at time t ", (Q_t^*, P_t^*) . Since $x^* = x_{\infty,t} \equiv \lim_{h \rightarrow \infty} E[x_{t+h} | x_t]$ this economic equilibrium thus corresponds to the so-called *long-run value*, defined in Johansen (2005).

From (40) it follows that, $\frac{\partial x^*}{\partial x_t'} = C$, describing the long-run impact of unit changes in the variables. Thus, the C matrix in (38) captures the comparative static effects given in (5).

Starting from the point (Q_t, P_t) the expression for x_{t+h} in (39), tells us exactly where the (Q, P) -allocation is located in the diagram after h periods, in the absence of shocks. Premultiplying with β' in (39), we get an expression for the equilibrium error at time $t + h$,

$$\beta'x_{t+h} + s = (I_r + \beta'\alpha)^h(\beta'x_t + s), \quad (41)$$

and by writing $\beta'x_{t+h} + s$, as $(Q_{t+h} - Q_{t+h}^d, Q_{t+h} - Q_{t+h}^s)'$ we find that,

$$Q_{t+h}^s - Q_{t+h}^d = k'(I_r + \beta'\alpha)^h(\beta'x_t + s), \quad (42)$$

$k' = (1, -1)$. This shows that the assumption, $(I_r + \beta'\alpha)^h \rightarrow 0$, i.e. $\rho(I_r + \beta'\alpha) < 1$, has an economic interpretation of market clearing.

For a given deviation from equilibrium, $(\beta'x_t + s)$, the expression (42) shows how (and how fast) the market clears. It may involve oscillations or smooth convergence, fast or slow, depending on the eigenvalues of $(I_r + \beta'\alpha)$. Thus, (42) offers a framework for formulating interesting hypotheses about the market clearing process, which could be formulated in terms of restrictions α given β , provided that that $\rho(I_r + \beta'\alpha) < 1$. It should be possible to formulate hypotheses of staggered price setting (Taylor 1979), or other Keynesian type of nominal -, or real rigidities, in this manner. For example, loosely illustrated in the right context of a simple AS-AD, with the same form as (1) - (2), a small value of α_{21} , combined with a large value of α_{11} , would describe little adjustment in prices while more adjustment in quantities, in the wake of a demand shock, i.e. "nominal rigidities".

The above "long run in the hypothetical absence of shocks" essentially resembles the theory model in pure form, but clearly, in each period unanticipated shocks hit all variables in the system: The demand- and supply curves are shifted by ε_W and ε_Z respectively, and in addition to the anticipated changes in Q and P , the shocks ε_Q and ε_P occur. An unanticipated realized position and an unanticipated equilibrium position have thus resulted, and adjustment in Q and P towards *this* equilibrium will take place *in the next period*, in which new shocks occur etc.. The economic equilibrium thus moves and corresponds to an *attractor*. This is also captured by the fact that *existence* of the economic equilibrium (4), requires $a_1 + b_1 \neq 0$, which is the requirement for the attractor set, $\mathcal{A} = sp(\beta_\perp)$, to exist, as seen from (37).

This moving equilibrium will induce lagged *error correction*, or, in other words, x_{t+1} will depend on the *equilibrium error at time t*, as is seen from,

$$\beta'x_{t+1} + s = (I_r + \beta'\alpha)(\beta'x_t + s) + \beta'\varepsilon_{t+1}, \quad (43)$$

resembling (41) for $h = 1$, when shocks occur. Under (21) this is (asymptotically) stationary, which supports the use of the Taylor expansion in (29) and (30), as $\beta'x_t + s = (Q_t - Q_t^d, Q_t - Q_t^s)'$.

As the demand- and supply shocks, ε_W and ε_Z , change the locations of respectively the demand- and the supply curves *permanently*, it is seen from the C matrix in (38), that they have a long-run impact on the endogenous variables. In contrast, the shocks, ε_Q and ε_P , have no long-run impact. This is essentially because they do not affect the positions of the curves: Starting from an equilibrium, an unanticipated price shock say, $\varepsilon_P < 0$, will introduce excess demand inducing upward price adjustment until the initial equilibrium is restored. It is therefore the cumulation of ε_W and ε_Z , and not ε_Q and ε_P that determines the long-run position of the endogenous Q and P , which is what CT_t , in (38) shows.

The loadings matrix, L , in (38) shows how these common trends affect the endogenous

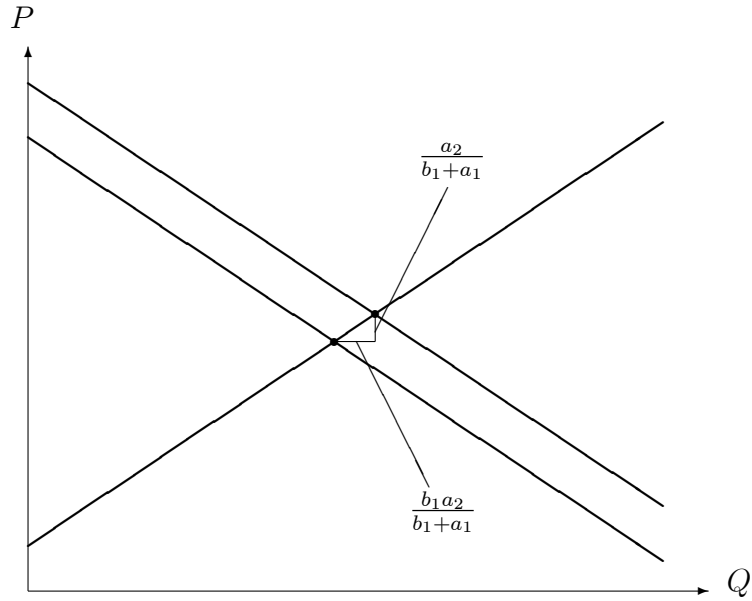


Figure 3: Illustration of the long run impact of a unit rise in ε_W (positive demand shock) on the endogenous variables, Q and P .

variables. The interpretation of the elements in L is facilitated by use of the demand- and supply diagram: Consider a unit rise in ε_{Wt} , which according to L in (38) will have a long-run impact of $\frac{a_2}{\frac{a_1}{b_1}+1}$ units on Q , and of $\frac{a_2}{b_1+a_1}$ units on P . In the economic cross, in Figure 3, this corresponds to a unit shock to wage earning, W_t , which shifts the demand curve upwards by $\frac{a_2}{a_1}$ units, eventually resulting in a rise in the equilibrium value of Q and P of the same magnitudes, $\frac{a_2}{1+\frac{a_1}{b_1}}$ and $\frac{a_2}{b_1+a_1}$ respectively.

Similarly, from L , we can see that the unit shock in W will have the full impact, $\frac{a_2}{a_1}$, on P , while no effect on Q , for $b_1 \rightarrow 0$. That is, when the supply curve is vertical, demand shocks will have no effect on quantity, while full effect on prices, e.g. as in a simple classical AS-AD model. In contrast, when $b_1 \rightarrow \infty$, corresponding to a horizontal supply curve, there is no effect on the price level from the demand shock, while the full effect is on Q , and equals a_2 , since the horizontal shift in the demand curve is the vertical shift, $\frac{a_2}{a_1}$, multiplied by the numerical inverse slope, a_1 .

Thus, the impact on the endogenous variables from demand and supply shocks is completely determined by the slopes (partial derivatives) of the curves, and this is exactly what the loadings matrix captures. Note that, since α'_\perp has the form, $(0, I_{p-r})$, $C = L\alpha'_\perp = (0, L)$, that is, L and C contain the same information when there are $p - r$ weakly exogenous variables.

4.2 Some generalizations of the empirical model

The simplified framework of the previous section is now generalized in three directions: First, since the mappings $g(\cdot)$ and $h(\cdot)$ are not part of the theory model, they are in some sense arbitrary. These are therefore generalized with respect to the lag of response, which is 1 above (see e.g. 27). Second, as the data often suggest more than one lag, I consider this case as well. Third, the endogenous variables may also be allowed to respond to current changes in the

exogenous variables.

The theory model is still the same, and hence, α , β , α_\perp and β_\perp are unaltered, and it is investigated whether the interpretations of the CVAR parameters, C and L , as describing comparative statics, can be retained.

Consider the issue of prolonged response time, and suppose instead of (27), that the mapping $g(\cdot)$ were,

$$\Delta P_t = g(Q_{t-u} - Q_{t-u}^d, Q_{t-u} - Q_{t-u}^s), \quad (44)$$

for $u \geq 1$. Hence, the reaction to market disequilibrium takes place u periods later. I assume the same for quantity, i.e. for $h(\cdot)$. This implies, that instead of (35) we have,

$$\Delta x_t = \alpha(\beta' x_{t-u} + s) + \varepsilon_t, \quad (45)$$

where as before α and β are given by (36). As this can always be rewritten as,

$$\Delta x_t = \alpha(\beta' x_{t-1} + s) + \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \dots + \Gamma_{u-1} \Delta x_{t-(u-1)} + \varepsilon_t, \quad (46)$$

with the restrictions that, $\Gamma_i = -\alpha\beta'$, for $i = 1, \dots, u-1$, we find that $\Gamma = I + (u-1)\alpha\beta'$, which implies that C and L are unaltered as, $C = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1} \alpha'_\perp$, and $\alpha'_\perp \alpha = 0$, $\beta' \beta_\perp = 0$. Hence, the result that C and L can be interpreted as above, is invariant with respect to reaction time u . This is intuitively expected, as the *long-run* effect is the same as before. This holds even though P and Q react to equilibrium errors with *different* lags.

Now, suppose that the CVAR needs more than one lag. Since α_\perp and β_\perp are the same, we see from, $C = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1} \alpha'_\perp = L \alpha'_\perp$, that Γ needs to fulfill certain requirements, for the interpretations to be unaltered. Without loss of generality, consider the corresponding CVAR with two lags, written as,

$$\Delta x_t = \alpha(\beta' x_{t-1} + s) + \Gamma_1 \Delta x_{t-1} + \varepsilon_t. \quad (47)$$

Introduce the block matrix notation,

$$x_t = \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix}, \quad \Gamma_1 = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}, \quad \alpha_\perp = \begin{pmatrix} 0 \\ I_2 \end{pmatrix}, \quad \beta_\perp = \begin{pmatrix} B_{1\perp} \\ I_2 \end{pmatrix}, \quad (48)$$

where $x_{1t} = (Q_t, P_t)'$ and $x_{2t} = (W_t, Z_t)'$, and the definition of $B_{1\perp}$ follows from β_\perp in (37). From this, it follows that,

$$\alpha'_\perp \Gamma \beta_\perp = \alpha'_\perp (I_4 - \Gamma_1) \beta_\perp = I_2 - \Gamma_{22} - \Gamma_{21} B_{1\perp}, \quad (49)$$

which enters the expression for C .

Now, under economic exogeneity of W and Z , cf. section 2, these are unaffected by the lagged differences of the endogenous variables, Δx_{1t} , implying that $\Gamma_{21} = 0$, that is, that x_{1t} does not Granger cause x_{2t} . As α is unaltered, x_{2t} is thus strongly exogenous for β (See section 3).

Moreover, economic exogeneity also means that the exogenous variables are mutually unrelated, which amounts to Γ_{22} being diagonal. The requirement of economic exogeneity is thus stronger than strong exogeneity. Denoting the diagonal elements of Γ_{22} by γ_{ii} ,

$$(\alpha'_{\perp} \Gamma \beta_{\perp})^{-1} = \begin{pmatrix} \frac{1}{1-\gamma_{11}} & 0 \\ 0 & \frac{1}{1-\gamma_{22}} \end{pmatrix} \equiv D, \quad (50)$$

provided that $\gamma_{ii} \neq 1$, which is the condition in (13). This implies that,

$$C = \begin{pmatrix} 0 & B_{1\perp} D \\ 0 & D \end{pmatrix}. \quad (51)$$

So, the C matrix has in fact changed, but this is simply because the W and Z are now modelled as AR(2) I(1) variables, implying that the long-run impact of a unit rise in ε_W on W , say, which is what C shows, is no longer 1, but $(1-\gamma_{11})^{-1}$. Recalling that comparative statics are concerned with the effect of a unit change in W , we simply need to normalize the shock ε_W in order to produce a long run effect of 1 on W . So, if we change ε_{Wt} by $1-\gamma_{11}$ we obtain a unit change in W in the long run, resembling the comparative static experiment. As described in (Johansen 2005) we can add δ to the variables at time t , i.e. to ε_t , which gives the long-run impact $C\delta$. So, if we add $\delta = (0, 0, (1-\gamma_{11}), 0)'$ to the variables we essentially normalize the column in C showing the impact of ε_W shocks and get the same as before.

Hence, under economic exogeneity nothing substantial has changed, and provided we change the current values of the exogenous in order to produce a *long-run unit change* in them, C still captures the comparative statics.

As seen from (49) the short-run adjustment dynamics of the endogenous variables can be generalized arbitrarily by Γ_{11} and Γ_{12} , without affecting the conclusions.

Turn now to the issue of current effects. The econometric model in section (4.1) is the model in (6), with $k = 1$, and normalization $A = I$. That is, there are no current effects between the variables. Consider now the case of a more general normalization A (still with 1 as diagonal elements). It is sufficient to consider the CVAR(1). The corresponding so-called "structural" CVAR is,

$$A\Delta x_t = a(\beta' x_{t-1} + s) + u_t, \quad (52)$$

where $a \equiv A\alpha$, $u_t \equiv A\varepsilon_t$ distributed as *i.i.N*(0, Σ), Σ diagonal (Juselius 2006, chapter 15). From the MA representation the modified long-run impact matrix is,

$$\tilde{C} = CA^{-1}. \quad (53)$$

Hence, in the presence of current effects the C matrix will in general change. However, under economic exogeneity, A has a certain structure, which implies that we can still maintain the

interpretations from before. To see this, partition matrices again, i.e.,

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \text{ and } C = \begin{pmatrix} 0 & B_{1\perp} \\ 0 & I_2 \end{pmatrix}. \quad (54)$$

Under economic exogeneity we have that,

$$A_{21} = 0 \text{ and } A_{22} = I_2, \quad (55)$$

which in turn implies that,

$$A^{-1} = \begin{pmatrix} A_{11}^{-1} & -A_{11}^{-1}A_{12} \\ 0 & I_2 \end{pmatrix}. \quad (56)$$

Inserting this into (53) along with C from (54), we get,

$$\tilde{C} = C. \quad (57)$$

Hence, under economic exogeneity we can also generalize with respect to (identified) A_{11} and A_{12} parameters, without affecting the conclusion about C and L , and we can use the impulse response function (23), which is based on the reduced form, for "structural" impulse response analysis, i.e. of the propagation of the demand and supply shocks, ε_W and ε_Z .

4.3 A simple dynamic theory model based on expectations formation

If we are only interested in the "long run", static models may be sufficient as first approximations. We may take them to the (persistent) data in the way suggested above, and hypotheses about comparative statics can be formulated as hypotheses on the cointegrating vectors. However, theory models may also involve hypotheses about *dynamics of adjustment* (α and Γ), and to the extent that this is sluggish, these should concern us at least as much as equilibrium effects (β hypotheses).

Several assumptions make theory models dynamic. For example, related to the present context, ECMs have been derived from dynamic optimization involving quadratic loss functions (Nickell 1985)⁵. As mentioned, assumptions about expectations formation may also introduce dynamics in an otherwise static theory model, and this is considered now.

Economic theory models often assume that economic decisions depend on expectations. For example, simple macroeconomic models of wage formation usually involve trade unions and firms negotiating about the nominal wage based on the expected price level and labour market conditions (unemployment). Likewise, investment is based on expected sales, and the expected opportunity cost from tying up financial capital (the real interest rate). In both examples what matters for the decision variables chosen at t is really the value of the determinants in period t , in the sense that if there were no uncertainty at all, the relationships would be static. But

⁵For a survey of different interpretations of ECMs along these lines see Alogoskoufis and Smith (1991).

faced with the uncertainty of reality agents have to settle for the expected values, and it is likely that these depend, to some extent, on the past (are adaptive), resulting in dynamics.

Let us consider a simple model of "aggregate investment in an exporting sector for the small open economy". The behavioral relation for investment is,

$$I_t^p = c_0 - c_1 r_t^e + c_2 Y_t^e, \quad (58)$$

where I is the logarithm of investment purchase, r is the real interest rate, and Y the logarithm of aggregate international output, and where the superscripts, p and e , denote a plan, and an expected value respectively. I assume that r and Y are economically exogenous in the theory model. As before, given the persistence, they are thus modelled empirically as,

$$r_t = r_{t-1} + \varepsilon_{rt}, \quad (59)$$

$$Y_t = Y_{t-1} + \varepsilon_{Yt}. \quad (60)$$

The expectations formation of r is assumed to follow,

$$r_t^e = \lambda_1 r_t + \lambda_2 r_{t-1} + (1 - \lambda_1 - \lambda_2) r_{t-2}, \quad (61)$$

where $0 \leq \lambda_1 \leq 1$, and $0 \leq \lambda_1 + \lambda_2 \leq 1$. Although output expectations could be described similarly, the points can be illustrated assuming that,

$$Y_t^e = Y_{t-1}. \quad (62)$$

Finally, actual and planned investment are allowed to differ by an unsystematic unanticipated error, ε_{It} , i.e.,

$$I_t = I_t^p + \varepsilon_{It}. \quad (63)$$

Compared to the model structure in the previous section we now have expectations equations which, as seen by (61), map the latent r_t^e to the observables, r_t , r_{t-1} and r_{t-2} . An economic equilibrium in this model results in the absence of shocks, in which case, expectations become correct, i.e. are realized.

For illustration, I solve the model for the observables under three different assumptions: 1) $\lambda_1 = 0$ and $0 < \lambda_2 < 1$, which I call "Simple adaptive expectations", involving systematic expectational errors. 2) $\lambda_1 = 0$ and $\lambda_2 = 1$, i.e. "Rational expectations", as r_t^e becomes the mathematical expectation, $E(r_t | x_{t-1}, \dots, x_0)$. Finally, I consider the (even more) unrealistic case, 3) $\lambda_1 = 1$ and $\lambda_2 = 0$, i.e. "Perfect foresight". In all three cases the expectations of Y are rational, but I study only the dynamics of investment in response to changes in the real interest rate. Note that I use the term "Rational expectations" in spite of the prevalent controversies about its meaning. For a discussion of expectations and in particular the meaning of "Rational expectations", see e.g. Hendry (1995), chapter 6.

4.3.1 Simple adaptive expectations

In this case, $r_t^e = \lambda_2 r_{t-1} + (1 - \lambda_2)r_{t-2}$, and solving the model for the observables, gives,

$$\begin{aligned}\Delta I_t &= -(I_{t-1} - (c_0 - c_1 r_{t-1} + c_2 Y_{t-1})) + \gamma \Delta x_{t-1} + \varepsilon_{It}, \\ \Delta r_t &= \varepsilon_{rt}, \\ \Delta Y_t &= \varepsilon_{Yt},\end{aligned}\tag{64}$$

where $\gamma = c_1(1 - \lambda_2)$, or,

$$\Delta x_t = \alpha(\beta' x_{t-1} + s) + \Gamma_1 \Delta x_{t-1} + \varepsilon_t,\tag{65}$$

with matrices,

$$\alpha = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, \beta = \begin{pmatrix} 1 \\ c_1 \\ -c_2 \end{pmatrix}, s = -c_0, \Gamma_1 = \begin{pmatrix} 0 & \gamma & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\tag{66}$$

and MA components,

$$C\Gamma_t = \begin{pmatrix} \sum_{i=1}^t \varepsilon_{ri} \\ \sum_{i=1}^t \varepsilon_{Yi} \end{pmatrix}, L = \begin{pmatrix} -c_1 & c_2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } C = \begin{pmatrix} 0 & -c_1 & c_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.\tag{67}$$

There is one characteristic root at 1, and $|\alpha'_\perp \Gamma \beta_\perp| = 1$. So, in this case, (12) is also implied by the theory model. Note that, compared with the model in section 4.1, not only β , but also the adjustment parameters, α and Γ_1 , are now related to the theoretical parameters of interest. The interpretations of L and C , from the last section still apply, in the sense that in economic equilibrium, where expectations are realized, the effects from the economically exogenous variables on the endogenous are given by L and C .

The equilibrium error, or expectational error,

$$\beta' x_t + s = (I_r + \beta' \alpha)(\beta' x_{t-1} + s) + \beta' \Gamma_1 \Delta x_{t-1} + \beta' \varepsilon_t,\tag{68}$$

becomes,

$$I_t - (c_0 - c_1 r_t + c_2 Y_t) = c_1(1 - \lambda_2) \Delta r_{t-1} + \beta' \varepsilon_t,\tag{69}$$

as $I_r + \beta' \alpha = 0$. Hence, the deviation between actual investment, I_t , and the "optimal" ex post investment level, $c_0 - c_1 r_t + c_2 Y_t$, has an *anticipated* component, $c_1(1 - \lambda_2) \Delta r_{t-1}$, i.e. it is known in period t , and it is possible to get closer to the optimal level by using this information. Hence, as long as $\lambda_2 < 1$, systematic expectational errors will take place.

This is illustrated in Figure 4. The downward sloping investment demand curve is shifted up and down by the random walk Y , while the interest rate (also as a random walk) shifts the vertical line, which can be thought of as the supply curve. Depending on how the expectations

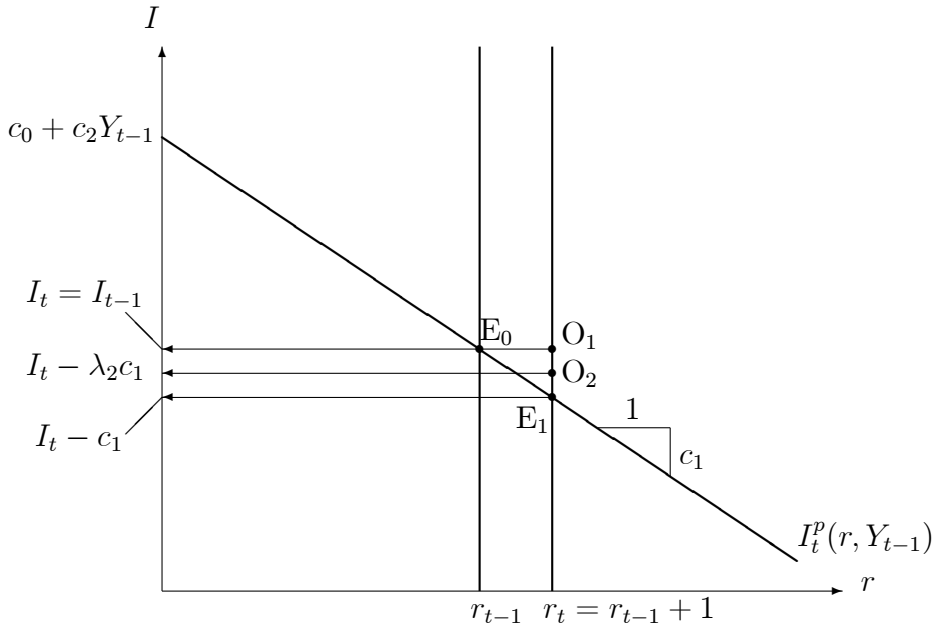


Figure 4: The investment demand schedule at time t , together with a shift in the interest rate. In the case of simple adaptive expectations, the movement is from E_0 to O_1 , in period t , to O_2 , in period $t + 1$, and to E_1 in period $t + 2$. When expectations are rational, the movement is from E_0 to O_1 , in period t , and then directly to E_1 , in period $t + 1$. Under perfect foresight the point moves directly to E_1 in period t .

are formed, the observed (I_t, r_t) deviates by some amount from the equilibrium.

In the diagram, the adjustment in the wake of an interest shock is also illustrated: Suppose that the economy has been in the equilibrium, E_0 , up to and including period $t - 1$. In period t there is a unit shock to the real interest rate, and no other shocks occur. When expectations are adaptive, the movement is from E_0 to O_1 , in period t , then to O_2 , in period $t + 1$, and from $t + 2$ to the new equilibrium E_1 . Once the shock has occurred it is known and the optimal level of investment in period $t + 1$, is $I_t - c_1$. When $\lambda_2 < 1$, the actual investment is higher.

4.3.2 Rational expectations

In this case, $r_t^e = E(r_t | x_{t-1}, \dots, x_0) = r_{t-1}$, and the matrices are as in (66), except that $\gamma = 0$ because $\lambda_2 = 1$. When expectations are rational, the movement is from E_0 to O_1 , in period t , and then directly to E_1 in period $t + 1$. All information is used rationally, and the equilibrium error in (69), becomes the white noise, $\beta' \varepsilon_t$. Thus, the response to shocks becomes faster than before. This way of formulating rational expectation resembles what is often found in macroeconomic text books (See e.g. Heijdra and van der Ploeg 2002).

Note that, for the CVAR(k),

$$\beta' x_t + s = (I_r + \beta' \alpha)(\beta' x_{t-1} + s) + \beta' \Gamma_1 \Delta x_{t-1} + \dots + \beta' \Gamma_{k-1} \Delta x_{t-(k-1)} + \beta' \varepsilon_t, \quad (70)$$

and hence, the hypothesis that $\beta' x_t + s$ is a white noise is,

$$I_r + \beta' \alpha = 0, \quad \beta' \Gamma_i = 0, \quad i = 1, \dots, k - 1. \quad (71)$$

For the static supply- and demand model in section 4.1, this would imply $I_r + \beta' \alpha = 0$, or equivalently,

$$\alpha = \begin{pmatrix} -\frac{b_1}{b_1+a_1} & -\frac{a_1}{b_1+a_1} \\ -\frac{1}{b_1+a_1} & \frac{1}{b_1+a_1} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}. \quad (72)$$

4.3.3 Perfect foresight

For illustrative purposes, distinguish between two cases: The pure case of perfect foresight, in which $Y_t^e = Y_t$, in addition to $r_t^e = r_t$, and the case, $Y_t^e = Y_{t-1}$, as previously.

In the first case, the investment relation becomes static, and the model is,

$$\begin{aligned} I_t &= c_0 - c_1 r_t + c_2 Y_t + \varepsilon_{It}, \\ r_t &= r_{t-1} + \varepsilon_{rt}, \\ Y_t &= Y_{t-1} + \varepsilon_{Yt}, \end{aligned} \quad (73)$$

i.e. current effects are introduced. The reduced form parameters α and β are as in (66). The equilibrium error in (69) now becomes ε_{It} , which is intuitively clear, since the ability to foresee output and the interest rate implies that the realized point responds to the shocks in the same period, implying that the only error is ε_{It} . In Figure 4, the observed point moves directly to E_1 in period t in the case of the isolated interest rate shock.

In the second case when $Y_t^e = Y_{t-1}$, the reduced form parameters are the same as in (66). The only change is that the equilibrium error is now $\varepsilon_{It} - c_2 \varepsilon_{Yt}$, since Y is no longer perfectly foreseeable.

These simple examples have demonstrated how hypotheses about expectation can be related to the parameters of adjustment in the CVAR.

4.4 General equilibrium

4.4.1 A simple general equilibrium model

The framework established in section (4.1) can readily be generalized to consider the important distinction in economics, between *partial*- and *general* equilibrium. It is well-known that general equilibrium comparative static effects may be radically different from the corresponding effects based on partial equilibrium - quantitatively but also *qualitatively*. As a result, even though we are only interested in the supply- and demand elasticities in one market, we might have to model the markets for related goods as well.

The basic ideas can be illustrated with a model with two markets, and as the example we could extend the partial equilibrium model (1) - (3), by including the labour market thereby endogenizing the wage, W . Instead, another equally simple theory model is considered, which

illustrates exactly the same point.

Consider the markets for two related goods, chicken and beef, say, with quantities and prices denoted Q_1 , Q_2 , and P_1 , P_2 , respectively. The demand for Q_1 , is related negatively to P_1 , and positively to P_2 (Q_1 and Q_2 are substitutes). Supply depends positively on P_1 , and negatively on some input price, denoted P_I .

The partial equilibrium model for market 1 assumes that P_2 and P_I are exogenous (resembling the model in section 4.1), and is given by,

$$Q_1^d = \frac{d_0}{d_1} - \frac{1}{d_1}P_1 + \frac{d_2}{d_1}P_2, \quad (74)$$

$$Q_1^s = -\frac{e_0}{e_1} + \frac{1}{e_1}P_1 - \frac{e_2}{e_1}P_I, \quad (75)$$

$$Q_1^s = Q_1^d, \quad (76)$$

where, as before, all coefficients are positive. The chosen normalization on prices (divisions with d_1 and e_1) is purely notational, implying that the inverse demand expressions, which are the ones we draw, enter the cointegrating relations.

The assumption that the price, P_2 , is exogenous, in the partial equilibrium model, implies that when P_2 changes there is no feed back on it from P_1 , which seems unrealistic: An increase in P_2 shifts demand, Q_1^d , which will ignite an increase in P_1 , which, in turn, will raise demand for good 2, causing a higher price P_2 . This will further feed back positively on demand for good 1, so that the increase in P_1 would be reinforced, and so on. Hence, the results from the partial equilibrium analysis are invalidated, and to account for this, we also need to include the market for good 2, i.e. impose general equilibrium.

To keep the exposition as simple as possible while still illustrating the main points, we assume that the supply of good 2, Q_2^s , and P_I are exogenous. Hence, I retain the endogenous-exogenous dichotomy, but P_2 has become endogenous. The demand for good two is,

$$Q_2^d = \frac{f_0}{f_1} - \frac{1}{f_1}P_2 + \frac{f_2}{f_1}P_1, \quad (77)$$

and the equilibrium condition is,

$$Q_2^s = Q_2^d. \quad (78)$$

The general equilibrium model is described by (74) - (78), with P_I and Q_2^s as exogenous. Solving the model yields the general equilibrium,

$$\begin{aligned} Q_1^* &= \frac{d_0 + d_2(f_0 - f_1Q_2) + (d_2f_2 - 1)(e_0 - e_2P_I)}{D}, \\ P_1^* &= \frac{(e_0 - e_2P_I)d_1 + e_1(d_0 + d_2(f_0 - f_1Q_2))}{D}, \\ P_2^* &= \frac{(e_1 + d_1)(f_0 - f_1Q_2) + f_2((e_0 - e_2P_I)d_1 + e_1d_0)}{D}, \end{aligned} \quad (79)$$

where $D \equiv d_1 - e_1(d_2f_2 - 1)$ is the determinant of the coefficient matrix to the system. Thus,

the equilibrium exists, if and only if, $D \neq 0$, which is assumed. Again, the comparative static effects are readily computed as the partial derivatives with respect to P_I and Q_2 in (79).

The embedding of this theory model in the VAR can be done exactly as in section 4.1, introducing the mappings from latent plans to the observable variables. For simplicity however, I add the observation mapping for Q_{2t} ,

$$Q_{2t} = Q_{2t}^s, \quad (80)$$

and also introduce only one new adjustment coefficient, α_{33} . The equations for price- and quantity adjustment for the good 1 market are as before, so that, altogether, the implied CVAR(1) is,

$$\begin{aligned} \Delta Q_{1t} &= \alpha_{11}(P_1 - (d_0 - d_1Q_1 + d_2P_2))_{t-1} + \alpha_{12}(P_1 - (e_0 + e_1Q_1 + e_2P_I))_{t-1} + \varepsilon_{Q_{1t}}, \\ \Delta P_{1t} &= \alpha_{21}(P_1 - (d_0 - d_1Q_1 + d_2P_2))_{t-1} + \alpha_{22}(P_1 - (e_0 + e_1Q_1 + e_2P_I))_{t-1} + \varepsilon_{P_{1t}}, \\ \Delta P_{2t} &= \alpha_{33}(P_2 - (f_0 - f_1Q_2 + f_2P_1))_{t-1} + \varepsilon_{P_{2t}}, \\ \Delta P_{I t} &= \varepsilon_{P_{I t}}, \\ \Delta Q_{2t} &= \varepsilon_{Q_{2t}}. \end{aligned} \quad (81)$$

This corresponds to the matrices,

$$\alpha = \begin{pmatrix} \alpha_{11} & \alpha_{12} & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ 0 & 0 & \alpha_{33} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \beta = \begin{pmatrix} d_1 & -e_1 & 0 \\ 1 & 1 & -f_2 \\ -d_2 & 0 & 1 \\ 0 & -e_2 & 0 \\ 0 & 0 & f_1 \end{pmatrix} \text{ and } s = \begin{pmatrix} -d_0 \\ -e_0 \\ -f_0 \end{pmatrix}, \quad (82)$$

in equation (18), with orthogonal complements,

$$\alpha_{\perp} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \beta_{\perp} = \begin{pmatrix} \frac{(d_2f_2-1)e_2}{D} & \frac{-d_2f_1}{D} \\ \frac{d_1e_2}{D} & \frac{-e_1d_2f_1}{D} \\ \frac{d_1f_2e_2}{D} & \frac{-(d_1+e_1)f_1}{D} \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (83)$$

and common trends and loadings (as $|\alpha'_{\perp}\beta_{\perp}| = 1$),

$$CT_t = \begin{pmatrix} \sum_{i=1}^t \varepsilon_{P_{I i}} \\ \sum_{i=1}^t \varepsilon_{Q_{2 i}} \end{pmatrix}, L = \begin{pmatrix} \frac{(d_2f_2-1)e_2}{D} & \frac{-d_2f_1}{D} \\ \frac{d_1e_2}{D} & \frac{-e_1d_2f_1}{D} \\ \frac{d_1f_2e_2}{D} & \frac{-(d_1+e_1)f_1}{D} \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (84)$$

resulting in the long-run matrix,

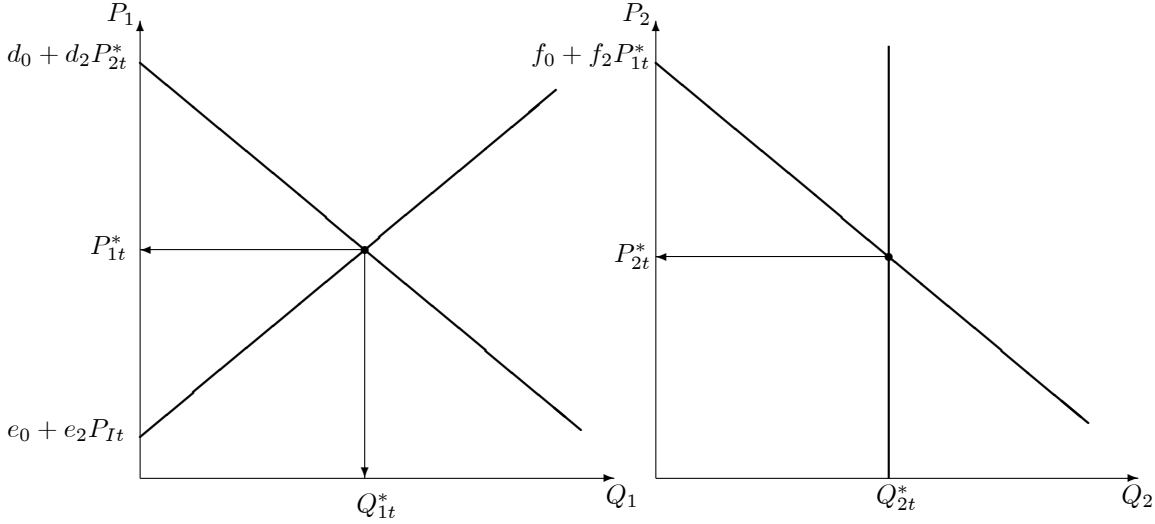


Figure 5: The economy in general equilibrium. Note how the demand curves are drawn for the equilibrium values of the price on the related market.

$$C = \begin{pmatrix} 0 & 0 & 0 & \frac{(d_2 f_2 - 1)e_2}{D} & \frac{-d_2 f_1}{D} \\ 0 & 0 & 0 & \frac{d_1 e_2}{D} & \frac{-e_1 d_2 f_1}{D} \\ 0 & 0 & 0 & \frac{d_1 f_2 e_2}{D} & \frac{-(d_1 + e_1)f_1}{D} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (85)$$

The general equilibrium at time t , for given values of the exogenous variables, P_I and Q_2 is illustrated in Figure 5.

Equations (82) - (85) show that the interpretations from section 4.1 generalize straightforwardly: The economically exogenous variables become the common trends, and L and C capture the comparative static effects. For example, the comparative static effect, $\frac{\partial Q_1^*}{\partial Q_2}$, corresponds to the fifth element of the first row of the C matrix.

In terms of Figure 5, the transition from one equilibrium to the next, in the comparative static experiment, now involves a *sequence* of shifts in the curves, since the markets interact, as opposed to the model in section 2. As before, this (static) theory model abstracts from this interaction altogether. It can be shown, that for the theory model to have a stable equilibrium, $D > 0$ is necessary, and gives reasonable comparative static effects, cf. Samuelson's Correspondence Principle (Samuelson 1941). This was also assumed in section 2, as $a_1 + b_1 > 0$. This principle is discussed further in section 5.

As opposed to the simple model in section 2, this model involves two notions of equilibrium: The *short-run* equilibrium, i.e. the *equilibrium at time t* , which is involved in the sequential interaction between the markets, and the *long-run* equilibrium, or "steady state". In section 2 these equilibrium concepts coincide. Furthermore, as before this "steady state" is a moving (stochastic) equilibrium, when the permanent shocks, $\varepsilon_{P_{1t}}$ and $\varepsilon_{Q_{2t}}$, are introduced.

The partial equilibrium model corresponds to the special case $f_2 = 0$, in which case there is an influence from P_2 on P_1 but not vice versa. As a result, the general equilibrium effects

on Q_1 and P_1 , from changes in P_I , are equivalent to the partial equilibrium effects. If this is the case, one would probably prefer the partial equilibrium model since it is easier to handle, analytically as well as econometrically.

The C matrix in (85) demonstrates the central point in economics that general equilibrium comparative statics might be qualitatively different from those in partial equilibrium. For example, consider the effect of a supply shock, ε_{P_I} , on Q_1 . The partial equilibrium effect, is $-\frac{e_2}{d_1+e_1}$, setting $f_2 = 0$ in the first row, fourth column in (85). This is unambiguously negative. In contrast, in general equilibrium, the effect is $\frac{(d_2f_2-1)e_2}{D}$, which is negative only if $d_2f_2 < 1$. In terms of the graphs, the partial equilibrium model shows the initial upward shift in the supply curve for good 1, and then the story ends. In general equilibrium, the resulting rise in P_1 , spills over to the market for good 2, and shifts the demand curve on this market upwards, which feeds back and shifts demand for good 1 upwards etc.. Hence, in the wake of the shift in the supply curve in market 1, there is a sequence of upward shifts in the demand curve as well. If $d_2f_2 > 1$, the sum of these demand shifts is greater than the initial supply shift, and Q_1^* will therefore rise.

Thus, by formulating the general equilibrium, we can test whether the partial model is valid, by the parameter restriction, $f_2 = 0$. Alternatively, one can start with the small system (i.e. without Q_2), and include the variables of the partial equilibrium model. Then, one can test the exogeneity of P_2 . If accepted, one can stick to the partial analysis. If general equilibrium interaction effects are important, this is likely to show up in the small model. For example, one could imagine in practice, the test for weak exogeneity of P_2 would be rejected.

In practice, the latter approach, i.e. starting in the partial equilibrium system, is an example of the gradual model building approach advocated for in Juselius (1992) and Juselius (2006). That approach exploits the invariance property of cointegration with respect to adding variables, so that one can start with a smaller, and hence more manageable system, and then gradually extend it by one variable at the time, or, alternatively combine it with another small system.

The example in this section illustrates how theory information, i.e. the partial-, general equilibrium distinction may facilitate such a gradual model building approach in practice: For instance, finding a "strange" borderline stationary relation, it can be investigated whether this becomes stationary, and interpretable, when including the *ceteris paribus* variable(s) modelled in general equilibrium models (other prices).

4.4.2 The relation to Dynamic Stochastic General Equilibrium models

By now we have considered both *dynamic* theory models and *general equilibrium* in the context of the *stochastic* CVAR, and it is thus natural to relate to DSGE models.

A difficulty with the DSGE approach is that it often involves linearizations around a well-defined constant (growth-adjusted) steady state, which may seem difficult to reconcile with the prevalent persistence of macroeconomic data (Juselius and Franchi 2007)⁶. Hence, there seems

⁶See for example Smets and Wouters (2003) for a popular application, and Campbell (1994) for the log-linearization method.

to be a need for some econometric framework within which one can analyze (modified) DSGE models on a statistically sound basis, *when data are persistent*. Though simple, the above exposition may provide the skeleton of such a framework. To state it loosely: If the general equilibrium model in section 4.4.1, were instead derived from *expectations formation*, and *dynamic optimizing* agents faced with budget constraints, it would essentially be a DSGE model, implying cointegration: The expectations and dynamic optimization introduce dynamics, the shocks are stochastic, and the model obviously involves a general equilibrium.

I have not considered the theoretical problems that such derivation of a "DSGE-CVAR model" may introduce, and quite likely, some methodological problems and questions will appear. However, it is my belief that the exposition here has provided a simple framework that hopefully at least will facilitate the communication between CVAR modellers, and more "structurally orientated" DSGE econometricians.

5 Discussion and further generalizations

So far, many practical econometric and theoretical issues have been disregarded, in order to obtain an accessible and explicit exposition. A thorough discussion of all these issues is beyond the present scope, and instead a few remarks about the presented framework are given.

First, the approximation of persistence by I(1) can be generalized to I(d), $d > 1$. As an example, consider the simple static model in section 4.2, in which more lags were added. For example, it could happen that W_t and Z_t were even more persistent, and hence, better approximated as I(2) than I(1). As a result, $\gamma_{ii} = 1$, and the MA representation in (50) would be invalid. Instead, the MA representation for I(2) processes would apply (Theorem 4.6, Johansen 1996). As before, the I(2) property would merely be an assumption about the statistical-, and not the theoretical parameters of interest, and as a result the I(2) property is readily reconciled with a simple static demand- and supply model.

Second, one should note that in the case when the endogenous-exogenous dichotomy is rejected the common trends no longer have the simple form, i.e. $\alpha'_\perp = (0, I_{p-r})$, as in (36) say. The common trends in CT_t , will now involve linear *combinations* of different cumulated shocks. This is normally how cointegration is understood.

Third, as discussed in section 4.1, the Taylor approximation (29) and (30), rests on stationarity of the equilibrium error, (43). However, given stationarity, the approximations may work better in some case than others. In general, it depends on the degree of non-linearity of the mappings, which probably depends on whether variables are in logarithms or not, whether transaction costs are negligible or not, etc. Moreover, the continuity assumption of $g()$ and $h()$, should also be viewed as a rough approximation, as transaction costs are likely to introduce *discontinuous* adjustment to disequilibria.

Fourth, Samuelson's Correspondence Principle may be related to the CVAR analysis above (Samuelson 1941). This simple but useful principle states that *stability* of the equilibrium implies comparative statics with "reasonable" signs. For example, above the equilibrium price, it

is often assumed that supply exceeds demand, so that the price level falls, implying stability. This assumption, thus involves a restriction on the slopes of the demand-, and supply curves, which make comparative statics have reasonable signs (See Samuelson 1941). This correspondence is not needed for the CVAR models in sections 4.1 and 4.4, as stability depends on the α in addition to β , of which it is the latter that the principle concerns. As a result we can have stable equilibria with "strange" comparative statics.

Finally, in the general equilibrium model in section 4.4, the α coefficients, α_{13} , α_{23} , α_{31} and α_{32} , were all set to zero to simplify (See equation 82). Relaxing this assumption in a corresponding model, introduces a flexible framework for modelling "sophisticated agents" gathering information, and acting simultaneously in several markets. In general, the framework provided, suggests that hypotheses about information, expectations, adjustment costs, etc. should be stated as restrictions on α and Γ_i .

6 Summary and Conclusion

In an attempt to bridge the gap between economic theory models and the cointegrated VAR, this paper has focused on facilitating the formulation and understanding of economic theory models as restrictions on the CVAR. As most economic models build on the same fundamental concepts, simple static- and dynamic theory models were considered to keep the exposition clear.

The point of departure was a well-specified VAR as the statistical model, with some of the estimated roots close to unity, corresponding to persistence of the series. Under the endogenous-exogenous dichotomy of the theory model, this persistence originates from the generation of the exogenous variables which is outside the theory model. Hence, roots at unity, do not contradict the theory, and should be imposed, as an approximative assumption about the DGP to obtain reliable inference from short samples of persistent series.

Approximating the exogenous variables as I(1) unit root processes, static models and simple dynamic models were thus analyzed as restrictions on a CVAR. This established an explicit correspondence between the basic concepts of theory models and the econometric concepts of the CVAR.

This correspondence shows that: The theoretical relations, i.e. demand-, and supply relations, correspond to the cointegrating vectors. The concept of exogeneity in economics is stronger than the econometric concept of strong (and thus weak-) exogeneity for β . The existence of the economic equilibrium implies the existence of the attractor set, and the economic equilibrium correspond to the so-called long-run value. The comparative statics are captured by the long-run impact matrix, C . The common trends, which determine the long-run movement of the system variables, correspond to the exogenous variables in the economic model. The loadings matrix can be interpreted to describe how the slopes of the demand-, and supply curves determine the impact on the endogenous variables, from shifts in the curves (i.e. in the exogenous variables). The matrix, $I_r + \beta'\alpha$, and, in particular, its largest eigenvalue

relates to the concept of market clearing, and interesting adjustment hypotheses (e.g. nominal rigidities etc.) can be related to this matrix. The example of the dynamic theory model, also demonstrates how hypotheses about expectations are related to the adjustment parameters of the CVAR, α and Γ_i .

As a generalization of the basic framework the distinction between general-, and partial equilibrium was also related to the CVAR: It was shown how to investigate whether comparative statics in general equilibrium differ from those in partial equilibrium, and how the empirical validity of the partial equilibrium model can be tested in the general equilibrium model.

As alluded to, given explicit hypotheses derived from detailed microeconomic assumptions about optimization, information, expectation etc., this paper should, to some extent, facilitate the formulation of such hypotheses as restrictions on the CVAR.

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