

Evaluating the New Keynesian Phillips Curve under VAR-based learning.

Reply to Referee 1 & Referee 2

2 July 2008

Some of the substantive points raised by the two referees, which I wish to thank for their careful reading of the paper and for the informed comments, are similar, and can be addressed jointly. Other comments are specific, and will be addressed separately. The comments of both referees have been very helpful for the revision of the paper.

Constant β .

Both referees are perplexed about the assumption (iii) stated in the Introduction of the paper, in which I deliberately assume that agents' learning rule involves only the parameters associated with the short run transient dynamics of the system, and not the cointegration parameters; the cointegration parameters are replaced in the empirical application with the estimates obtained from the entire sample. Of course, I fully understand the concern of the two referees; the relaxation of the assumption (iii), which in my view deserves a thorough treatment in future research, is in the research agenda. I understand, however, that the perplexity of the two referees is sound, and must be addressed.

While the Referee 1 argues that the analysis can be carried out by estimating β recursively and testing the cross-equation restrictions by the cointegrated VAR in Eq. (14) - rather than by means of the transformed system in Eq. (16) -, the Referee 2 points out that the assumption (iii), and the idea of replacing β with its full sample estimate $\hat{\beta}_{T^{\max}}$ in the empirical illustration, is acceptable to the extent that the cointegration parameter ϕ in β can be regarded as structurally constant over the chosen monitoring period (and this is a testable hypothesis). I have attempted to fulfil the requirements of both referees, with some qualifications, though.

As regards the point raised by the Referee 1, I agree that the cross-equation restrictions can be derived by focusing on the cointegrated VAR in Eq. (14); to see this, it is sufficient to consider the companion form representation of the cointegrated VAR

$$\tilde{Z}_t = \Upsilon \tilde{Z}_{t-1} + \tilde{\varepsilon}_t \tag{1}$$

where

$$\tilde{Z}_t = \begin{pmatrix} \Delta Z_t \\ \beta' Z_{t-1} \\ \Delta Z_{t-1} \\ \vdots \\ \Delta Z_{t-k+2} \end{pmatrix}, \quad \Upsilon = \begin{bmatrix} \Phi_1 + \alpha\beta' & \alpha & \Phi_2 & \cdots & \Phi_{k-1} \\ \beta' & I_p & 0 & \cdots & 0 \\ I_p & 0 & 0 & \cdots & 0 \\ 0 & I_p & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & I_p & 0 \end{bmatrix}$$

$$\tilde{\varepsilon}_t = \begin{pmatrix} \varepsilon_t \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix};$$

the forecasts in the Eqs. (20)-(21) (for simplicity I have omitted the superscript “ d ”) can be replaced with the quantities

$$\begin{aligned} \widehat{E}_{t-1}\Delta\pi_{t+1} &= g'_\pi \widehat{E}_{t-1} \tilde{Z}_{t+1} = g'_\pi \Upsilon^2 \tilde{Z}_{t-1} \\ \widehat{E}_{t-1}\Delta\pi_t &= g'_\pi \widehat{E}_{t-1} \tilde{Z}_t = g'_\pi \Upsilon \tilde{Z}_{t-1} \\ \widehat{E}_{t-1}(\beta' Z_t) &= g'_\beta \widehat{E}_{t-1} \tilde{Z}_{t+1} = g'_\beta \Upsilon^2 \tilde{Z}_{t-1} \end{aligned}$$

where g_π and g_β are selection matrices of suitable dimensions. In this case the cross-equation restrictions read as

$$g'_\pi \Upsilon (I_{pk} - \psi \Upsilon) - \omega g'_\beta \Upsilon^2 = 0_{1 \times pk}$$

and can be analyzed along the lines described in the paper. The Referee 1 argues that the representation (1) has the advantage that all parameters in Υ , including those in β , can be recursively updated as the information set increases over time. This is right, however, one may also define the W_t vector in Eq. (15) as

$$W_t = \begin{pmatrix} \widehat{\beta}'_t Z_t \\ v' \Delta Z_t \end{pmatrix} \equiv \begin{pmatrix} W_{1t} \\ W_{2t} \end{pmatrix} \quad \begin{matrix} r \times 1 \\ (p-r) \times 1 \end{matrix}$$

where $\widehat{\beta}_t$ is the recursive estimate of β based on the information set available up to time t , and is drawn from the recursive estimation of the cointegrated VAR. In other words, the analysis can be generalized to the case where

also the coefficients in β are updated recursively, irrespective of whether the statistical representation of the data is given in the form of Eq. (14), or in the equivalent form of Eq. (16), provided that W_t is opportunely defined.

Figure A reports the sequence of LR tests statistics (with corresponding critical values) obtained using the system $W_t = (Z_t' \hat{\beta}_t : \Delta Z_t' v)'$, with $\hat{\beta}_t$ updated recursively over the monitoring period, 1986:1-2006:4. The graph seems to confirm Referee 1's intuition that the relatively weaker evidence against the NKPC hypothesis under the chosen formulation of the adaptive learning hypothesis documented in the paper, might at least in part be ascribed to the assumption of a fixed cointegration relation. However, the graph in Figure A has been produced without testing the hypothesis of a stable cointegration relation over the chosen monitoring period (see below).

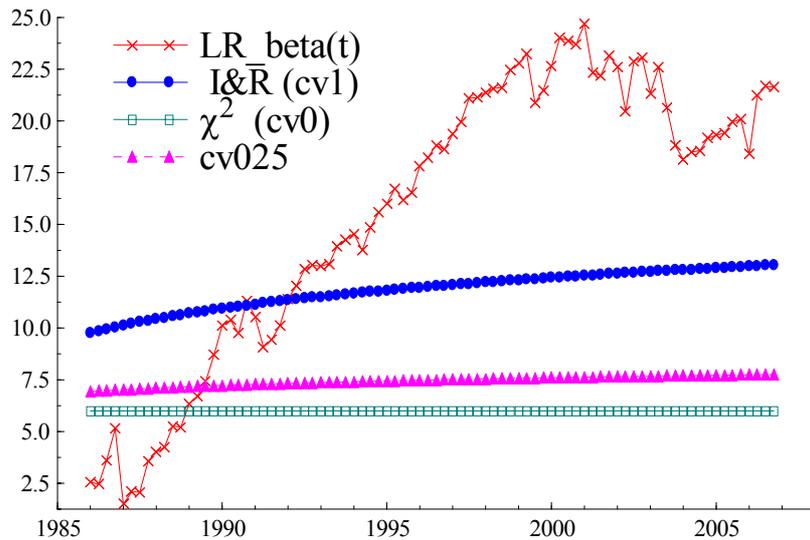


Figure A. Euro area data. Sequence of recursively computed likelihood ratio (LR) statistics obtained through the estimation of the system $W_t = (Z_t' \hat{\beta}_t : \Delta Z_t' v)'$, with $\hat{\beta}_t$ updated recursively over the monitoring period, 1986:1-2006:4; I&R is the critical value taken from Table 1 in Inoue and Rossi (2005); cv025 is the critical value calculated as described in Section 3.2 of the paper.

Coming to the comment by the Referee 2, I agree with the observation that the only reasonable way to motivate empirically the assumption (iii) of

the paper, is to show that the cointegration parameter ϕ in β is structurally constant; (incidentally, a structurally constant β allows me to face the argument of the Referee 1). The Figure B ¹ plots the recursively calculated test of the hypothesis $\tilde{\beta} \subseteq sp(\beta_t)$, $t = 1986:1, \dots, 2006:4$, where $\tilde{\beta} = \hat{\beta}_{T^{\max}} = (1, -\hat{\phi}_T, 0)'$ is estimated on the full sample 1981:2-2006:4 ($T^{\max} = 2006:4$), see e.g. Juselius (2006), Chap. 9. The test suggests that the hypothesis of a structurally constant cointegration parameter ϕ , although not overwhelming, can be taken as a reasonable approximation of the data from 1987 onwards (excluding the date 2005:4, where the rejection of the null is marginal).

Overall, I have revised the paper by observing that even accepting the hypothesis of structurally constant β , the evidence in favor of the NKPC under the chosen formulation of the ALH is not clear-cut (see also below).

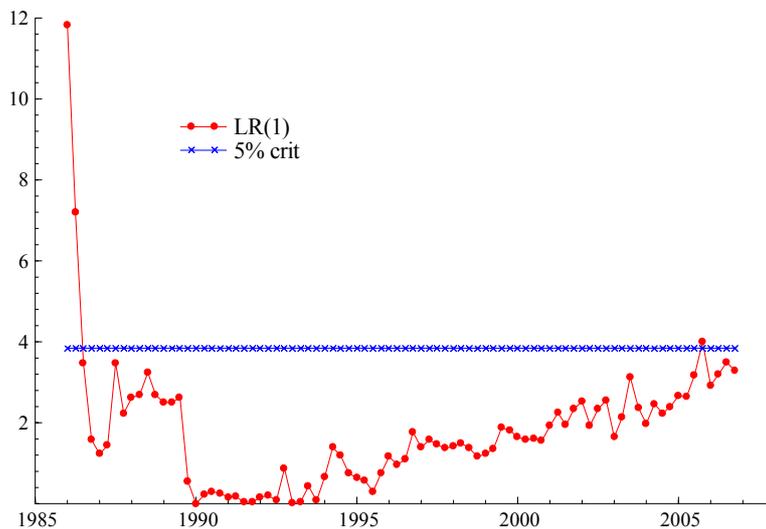


Figure B. Recursively calculated test of $\tilde{\beta} \subseteq sp(\beta_t)$, $t = 1986:1, \dots, 2006:4$, where $\tilde{\beta} = \hat{\beta}_{T^{\max}} = (1, -\hat{\phi}_T, 0)'$ is estimated on the full sample 1981:2-2006:4. The test is calculated in the so-called X-form, that means that both short-run and long-run, are re-estimated jointly at each t .

Structural parameters and cointegration parameter.

¹This figure will become Figure 3 in Section 4 of the revised version of the paper.

As concerns the link between the structural parameters of the NKPC and the cointegration parameter, let me re-write the Eqs. (9)-(11) of the paper as

$$\begin{aligned}\gamma_f &= \frac{\psi}{1 + \psi - \omega} \\ \gamma_b &= \frac{1}{1 + \psi - \omega} \\ \lambda &= [1 - (\gamma_f + \gamma_b)]\phi.\end{aligned}\tag{2}$$

The structural parameter λ captures in the NKPC the pass-through from marginal costs to inflation; the equation above clearly shows that if the inflation rate and the wage share are cointegrated, λ is intimately related to cointegration parameter ϕ (other than to γ_f and γ_b), see Fanelli (2008) for details. The cointegration parameter serves as a kind of “anchor” for the pass-through parameter of the NKPC.

Thus, imagine that the estimator of ψ and ω is updated recursively with the increase of the information set (exactly as it happens in the paper using a grid search). Irrespective of whether ϕ in (2) is replaced with a full sample ($\widehat{\phi}_{T^{\max}}$) or a recursive ($\widehat{\phi}_t$) estimate, given $\widehat{\psi}_t$ and $\widehat{\omega}_t$, and hence $\widehat{\gamma}_{f,t}$ and $\widehat{\gamma}_{b,t}$, $\widehat{\lambda}_t$ is automatically determined, and is potentially allowed to vary over time, consistently with $\widehat{\gamma}_{f,t}$ and $\widehat{\gamma}_{b,t}$.

Robustness.

The Referee 1 suggests that robustness should be investigated by enlarging the information set (e.g. by adding real GDP in the system and keeping the short term nominal interest rate), whereas the Referee 2 argues, *inter alia*, that it suffices to consider a bi-variate VAR, including the inflation rate and the wage share alone. Of course, robustness can be explored in several ways and dimensions, depending on the objectives of the analysis. The idea of using also a VAR(3) in the empirical application, is motivated by the satisfactory forecast performance exhibited by both the VAR(2) and VAR(3) over the chosen monitoring period.

In principle, agents’ perceived law of motion (VAR) should include the minimum set of variables necessary to forecast inflation: the relevant economic theory should provide the relevant set of variables to include in Z_t (in addition to π_t). I have included in the system the wage share, and not the output gap, following Galí and Gertler (1999) and Galí et al. (2001), who

observe, and show on economic grounds, that the former represents a more reliable measure of firms' real marginal costs in the Calvo formulation of the NKPC. This consideration suggests that including real GDP in the system does not add too much.

A short term interest rate has been included in the vector of "additional" variables, a_t , for two reasons. First, the short term interest rate is supposed to capture the effects of monetary policy on firms' marginal costs through the cost channel (Chowdhury et al., 2005); of course, monetary policy typically exerts its effects over short/medium horizons, and I would not expect the nominal interest rate to cointegrate with any of the variables of the system (as it happens in practice). Second, the resulting tri-variate VAR, reads as the reduced form of a typical small scale system of monetary policy based on three equations (a demand - or wage - equation, the NKPC and the policy rule). In this set-up I have the possibility of testing the implications of the NKPC on such a VAR under the ALH, keeping the other equations of the system in reduced form. It is clear, however, that in future research the approach can be generalized to account for the restrictions implied by all three equations comprising the small scale DSGE model of monetary policy.

The revised version of the paper remarks these considerations.

Minor issues, Referee 1

1. Done.
2. Done.
3. I have quoted Mellander, Vredin and Warne (1992) in the revised version of the paper.
4. Typo fixed.
5. The VAR coefficients and the structural parameters of the NKPC are treated as fixed (not time-varying); the estimator of these parameters are updated recursively through a re-iterate application of maximum likelihood estimation (which corresponds to Recursive Least Squares if VAR disturbances are Gaussian). I consider the simplest formulation of the ALH, i.e. the one based on RLS.
6. I have observed in the revised version of the paper that aside from the discount factor, which is generally difficult to estimate from the data in this literature, the other estimated coefficients of the Calvo model seem reasonable.

7. Done.

Minor comments, Referee 2

1. I have revised the paper (abstract, Introduction and Conclusions) by observing that the evidence on the NKPC under the ALH is mixed.

2. Footnote 3 has been dropped.

3, 4, 5, 6. Done.

7. I have revised the sentence by observing that the evidence on the NKPC under the ALH is mixed, and depends on the chosen c.v., and the hypothesis of a structurally invariante cointegration parameter.

8. Done.

9. I have observed in the revised version of the paper that aside from the discount factor, which is generally difficult to estimate from the data in this literature, the other estimated coefficients of the Calvo model seem reasonable.

10. Typos fixed.

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