Level, Slope, Curvature: Characterising The Yield Curve in a Cointegrated VAR Model

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Reply to Referee 1

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Many thanks for your comments on the paper. They were very helpful.

Your point linking the term premium to Equation (4) and to macro variables is observant and interesting. I added a discussion at the end of Section 3.3 to this effect.

1. Test statistics for restricting coefficients of lagged differences. Test statistics and information criteria comparing the model with unrestricted Γ_i to that with certain lags set to zero, as detailed in Table 2, are now included in the paper.

2. Vector auto-correlation test. Auto-correlation test results in Table 3 have been updated to show lags 1-1, 1-2, 1-3 and 1-4, and not as erroneously reported 1-1, 2-2, 3-3, and 4-4.

3. Figure 3 has been updated to include $b_t^{48} - b_t^1$ and $\Delta(b_t^{48} - b_t^1)$.

4. Over-identifying restrictions in yield model. The $\chi^2(8)$ test includes weak exogeneity restrictions for the ten-year yield in α and over-identifying restrictions on β as detailed in (10), (11) and (12). This should be clearer in the text now. Since α and β are jointly determined, one cannot test all the restrictions discussed in the section jointly, and hence we focus on the long-run restrictions in β and weak-exogeneity restrictions in α.

However, the unit vector restrictions in α for the one- and three-month yields are implicitly taken up again in Section 3.2.5 where the spread between cumulated residuals of the four-year and eighteen-month yields is found to make up a common trend on its own. The other common trend is given by the cumulated residuals of the ten-year yield, and hence the one- and three-month yields do not feature in the common trends as was suggested by the tests for unit vectors in α.

5. Testing theoretical curvature restrictions. It is indeed an interesting exercise to test the theoretical curvature restrictions which given Footnote 5 should be $0.857(b^{18} - b^{1}) - 0.143(b^{120} - b^{18})$ and $0.706(b^{48} - b^{18}) - 0.294(b^{120} - b^{48})$ as you mention in your report. Testing these jointly with the short-term spread as in (10) and allowing for constants in all three β equations and for weak exogeneity of the ten-year yield in α leads to rejection. The test statistic is 43.635 which compared to a $\chi^2(9)$ gives a p-value of [0.000]. However, when testing the restrictions on each β equation separately (i.e. leaving other two unrestricted), the first set of restrictions, including the short, medium and long end of the curve, is only borderline rejected with a statistic of 10.427 ($\chi^2(4)$, p-value: 0.034). The second set of restrictions, including only the medium and long end, is still rejected, though ($\chi^2(4) = 20.148$, [0.000]).

It is not surprising that the theoretical restrictions are harder to find in the data for the long end of the yield curve. Due to problems of discounting time to maturity, it may be that at the long end of the yield curve perceptions of time are compressed. Hence we would not expect to find exactly these theoretical relations. The general notion of a weighted rather than exact difference between the spreads should be characteristic of the curvature of the yield curve, though, as we show in (10), (11) and (12).

6. $C^*(L)$ denotes coefficients of the stationary part of the processes (and this description has been added in the paper). Further detail is provided in Johansen (1996).

7. Over-identifying restrictions in macro-yield model. The $\chi^2(5)$ test includes restrictions on α regarding the weak exogeneity of both $CT1$ and tcu as well as the over-identifying restrictions on β as given by (17) and (18). The two zero columns in the $\hat{C}$-matrix on p. 21 are due to the weak exogeneity restrictions. Since the model has two weakly exogenous variables, each of these makes up one of the $p - r = 2$ common trends of the model. Hence

$$\alpha'_{\perp} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

and similarly for $\hat{C}$. The text should be clearer on this now.

8. Unit root tests. Unit root tests (performed in *PcGive*) show that the null hypothesis of a unit root is not rejected in any of the augmented Dickey-Fuller (ADF) tests conducted (up to 12 lags including and excluding a constant) for all the yields (one-month, three-month, eighteen-month, four-year and ten-year). The results are as follows (only results for significant differenced lags are shown):

Lag	Δb_t^1	Δb_t^3	Δb_t^{18}	Δb_t^{48}	Δb_t^{120}
0	-0.392	-0.499	-0.806	-1.024	-1.237
1		-0.572	-0.809	-0.980	
No constant	Critical values: 5% = -1.94, 1% =-2.58				
0	-1.581	-1.105	-1.368	-1.548	-1.299
1		-1.381	-1.734	-1.894	
Constant	Critical values: 5%=-2.88, 1%=-3.47				

I have not, however, included this table in the paper because unit root tests do not give us any additional information when using the cointegrated VAR framework. If we had stationary variables in the system, we should find that each stationary variable makes up a cointegrating relation of its own which is not the case in the present analysis.