Comments on “Level, Slope, Curvature: Characterising The Yield Curve in a Cointegrated VAR Model”

This paper studies the cointegration structure of the US term structure. In contrast to work undertaken by Hall, Anderson and Granger (1992) who looked at the short end of the yield curve and found a single common I(1) factor, this work looks at both the short and long ends of the yield curve, and finds evidence of two common I(1) factors. Although this two factor result is not consistent with standard versions of the expectations hypothesis of the term structure, the author is nevertheless is able to link her findings to widely accepted theory in the finance literature that posits level, slope and curvature factors. Further, she provides evidence that her first factor relates levels (i.e. the yields themselves), and her second factor relates slopes (yield spreads). She also relates her common trends (i.e. the factors) to macro variables, finding a long run relationship between her levels factor and inflation, and between her slope factor, inflation and total capacity utilisation.

Overall, this paper constitutes a nice piece of empirical work. The analysis is technically competent, and the author’s interpretation of this analysis provides a well written and interesting discussion of the relationships between the properties in the data and various economic theories that might explain these properties. I would like to author to include more discussion on the second factor and relate this back to the expectations hypothesis. My reasoning is that the second (slope) factor might be given a term (risk) premium interpretation via (4), and the finding that long term premia are related to inflation and total capacity utilisation is not only interesting (and quite plausible) in itself, but also the non-stationarity of inflation and total capacity utilisation can then be used to explain why long-term premia might be non-stationary, and hence why standard versions of the expectations hypothesis fail, when subjected to the Hall et al (1992) type tests. An interesting point here, is that short term interest rates don’t depend on the second factor, so that long term economic conditions are not relevant for short term investments.

I have the following minor comments about various aspects of the empirical analysis

1. It would be appropriate to provide test statistics and p-values related to setting the insignificant columns in the Γ matrices to zero (in Table 1).
2. Since each of the serial correlation tests reported in Table 2 have twenty five degrees of freedom, (rather than 25, 50, 75 and 100 for first to fourth order respectively), I am guessing that the author has separately tested for the individual significance of each lag. This is a little unusual and surprising, especially since she reverts to standard practice when testing for ARCH, (where the number of test regressors increases with lag length.
3. The author might also include $b_t^{48} - b_t^1$ and $\Delta(b_t^{48} - b_t^1)$ in Figure 3.
4. It would be useful to be specific about which restrictions were tested using the $\chi^2_{(8)}$ test reported under (12), and then be specific about whether all of these were imposed. I am guessing that the tested restrictions include weak exogeneity of b_t^{120}, unit vectors in α for b_t^1 and b_t^3, and stationarity of $b_t^3 - b_t^1$, but the author provides no clear statement of this, and then seems to proceed with a specification that imposes the first and third of these, but not the second (see the $\hat{\alpha}$ matrix presented at the bottom of P13, which does not seem to incorporate the unit vector restrictions). This is a little confusing. Following on from this, it is then unclear which of these restrictions have been imposed in the model

that delivers equation (14), what constitutes the complete set of restrictions that are included in the $\chi^2_{(11)}$ test reported on P15, and whether all of these restrictions were actually imposed.

5. It would also be useful to provide some tests of whether the medium and long ends of the term structure are characterised by stationary curvature. Using the formulae provided in footnote 5, the ratio of coefficients for $b_t^{18} - b_t^1$ to $b_t^{120} - b_t^1$ should be .857 to -.143 (rather than .759 to -.241), so that the normalised cointegrating vector should be (-857, 0, 1, 0, -.143). Further, the ratio of coefficients on $b_t^{48} - b_t^{18}$ to $b_t^{120} - b_t^{48}$ should be .706 to -.294 (rather than .467 to .533), so that the normalised cointegrating vector should be (0, 0, -702, 1, -.294).
6. Define C^* in equation (13).
7. It would be useful to be specific about which restrictions were tested using the $\chi^2_{(5)}$ test reported above (17). If I am not mistaken, the exogeneity restrictions involve just 4 degrees of freedom, so I am wondering what else has been tested here. Also, it might be useful to comment on why the second and third columns of the $\hat{C}$ matrix on P19 are zero.
8. Although the author has provided graphs of all variables in this analysis, readers might also like to see the results of formal unit root tests