Should We Trust the Empirical Evidence from Present Value Models of the Current Account?

Benoît Mercereau
Sinopia Asset Management, Paris

Jacques Miniane
International Monetary Fund, Washington

Please cite the corresponding journal article:
http://www.economics-ejournal.org/economics/journalarticles/2008-34

Abstract:
The present value model of the current account has been very popular, as it provides an optimal benchmark to which actual current account series have often been compared. We show why persistence in observed current account data makes the estimated optimal series very sensitive to small-sample estimation error, making it close to impossible to determine whether the paths of the two series truly bear any relation to each other. Moreover, the standard Wald test of the model will falsely accept or reject the model with substantial probability. Monte Carlo simulations and estimations using annual and quarterly data from five OECD countries strongly support our predictions. In particular, we conclude that two important consensus results in the literature – that the optimal series is highly correlated with the actual series, but substantially less volatile – are not statistically robust.


JEL: C11, C52, F32, F41
Keywords: Current account; present value model; model evaluation

Correspondence: Jacques Miniane, International Monetary Fund, 700 19th Street, Washington, N.W., D.C. 20431, U.S.A. Telephone: (202) 623-8791. Fax: (202) 623-4358. Email address: jminiane@imf.org.
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This Discussion Paper is based on "Challenging the Empirical Evidence from Present Value Models of the Current Account" (IMF WP/04/106) by Benoit Mercereau and Jacques Miniane.
I. INTRODUCTION

The intertemporal approach to the current account first popularized by Sachs (1981) views net accumulation of foreign assets as a way for domestic residents to smooth consumption intertemporally in the face of idiosyncratic income shocks. The intertemporal approach has been very popular over the last two decades. Under some simplifying assumptions and using a methodology developed by Campbell and Shiller (1987) in a different context, one can estimate the current account series that would have been optimal from a consumption smoothing perspective.

Starting with Sheffrin and Woo (1990), economists have been eager to compare actual current account data with this optimal benchmark. Several studies conducted at the International Monetary Fund have estimated this benchmark to assess the optimality of emerging economies’ external borrowing (see, e.g., Ostry, 1997 or Callen and Cashin, 2002). Numerous academic papers have looked at both emerging and industrial countries, and a consensus has emerged from this literature: while the model-predicted current account is positively correlated with the actual series, the latter is substantially more volatile, leading statistical tests to reject the model. Positive correlation has been interpreted as evidence that consumption-smoothing plays a role in the dynamics of the current account (Obstfeld and Rogoff, 1996). But given that the present value model assumes full capital mobility, the finding of excess current account volatility has been used as evidence against Feldstein and Horioka’s famous proposition of limited international capital mobility (see, e.g., Gosh, 1995). More recent papers have tried to “augment” the model in several directions to generate extra predicted volatility. Bergin and Sheffrin (2000) show that allowing for variable real exchange rates and interest rates improves the fit of the model for Australian, Canadian, and British data. Gruber (2004) generates extra volatility in the predicted series by way of habit formation and excess smoothness in consumption. Nason and Rogers (2006) test competing additions to the model and find that exogenous shocks to the world real interest rate best reconcile the extended model with Canadian data.

Our paper shows that none of the key results in the literature rests on robust statistical grounds. Specifically, we show that: (i) the dominant test in this literature - the Wald test of the cross-equation restrictions of the model - has very poor small-sample coverage, and hence inference based on this test can be very misleading; (ii) the model-predicted series is excessively sensitive to small-sample estimation error, making it close to impossible to conclude whether actual current accounts are highly correlated or not correlated at all with the model-predicted series, or more or less volatile.

1 The intertemporal approach to the current account is surveyed in Obstfeld and Rogoff (1995, 1996).


3 Exceptions include Ghosh and Ostry (1995) who find that the model fits many developing countries.
Small-sample problems of Wald tests of present value models have already been documented elsewhere, but not in studies of the current account. In particular, Bekaert and Hodrick (2001) show that Wald tests have the worst small-sample coverage of all the tests they consider for present value models of the term structure of interest rates.\(^4\) We replicate their findings for models of the current account using appropriate data-generating processes. Moreover, unlike Bekaert and Hodrick, we explain why Wald tests have such poor coverage in this literature. The tests rely on a linear approximation of a variance-covariance matrix, but this approximation is likely to be very imprecise in short samples when the observed current account is persistent, as it typically is.\(^5\) In our Monte Carlo simulations, Wald tests can erroneously reject the model at 95% confidence with a probability ranging between 11.7% and 28.3% depending on the data generating process assumed, implying an average rate of (false) rejection across DGPs three and half times the proper rate. When we take the tests to quarterly and annual data for five different countries, we find that in four out of the ten samples the Wald tests accepts (rejects) the model when other tests with good coverage – a well-known F-test and a simple linear Wald test – reject (accept) the model. Such results are not encouraging considering that the Wald test has been the dominant test in the literature.\(^6\)

As mentioned previously, the main selling point of the present-value model has been its ability to generate a simple, easy to interpret optimal series with which actual current accounts have often been compared compared. The second contribution of our paper is to show that persistence in actual current accounts will generate large uncertainty around the estimated optimal series, and make any comparison between actual and optimal series moot. For instance, our estimated confidence bands around the optimal current account are very wide in all our data samples, and easily encompass the observed current account.\(^7\) Empirical distributions of the variance ratio between actual and predicted current account that incorporate estimation uncertainty are strikingly dispersed in all our ten samples of data, and indicate substantial probability that the actual current account is either several times more or

\(^4\) Campbell and Shiller (1989) also use Monte-Carlo simulations to find large small sample bias in the predictions of the present value model of the dividend ratio, as well as poor coverage for the non-linear Wald test.

\(^5\) We note that when we talk about a “persistent” current account, we do not necessarily mean a non-stationary one. A stationary current account will be considered persistent if its process of mean reversion is slow. Clearly, if the current account is integrated of order one or higher then the model cannot be a correct representation of the data. What is crucial for our analysis is that the small sample problems we report in the paper can occur even if the current account is stationary but persistent, and there is no reason why a persistent but stationary current account cannot be model-consistent.

\(^6\) Out of the 15 papers that we cite which test the present value model of the current account, 10 report the (non-linear) Wald test only, 1 the F-test only, and 4 report both. None of these papers follow the simple linearization of the Wald test that we use in this paper, and which has the advantage of being sufficient while not relying on the Delta-method linear approximation.

\(^7\) As we will see, few papers in the literature report measures of estimation uncertainty such as confidence intervals around the estimated series, or the standard error of the correlation and variance ratio estimates between actual and model-predicted series. Moreover, we will show that some of the methods used to account for this uncertainty may be inappropriate when the current account is persistent.
several times less volatile than the optimal series. Distributions of the correlation coefficient between the actual and predicted series are also very dispersed, with often substantial probability that the correlation is negative. These findings occur regardless of whether the more robust tests have accepted or rejected the model.

Finally, it should be noted that our paper is not the first to warn against traditional inference from present value models of the current account. Kasa (2003) warns against plausible income processes for which the model-consistent current account may not reflect all relevant information to accurately forecast income changes, and hence to estimate optimal series. As the author acknowledges, however, such processes are plausible but very hard to distinguish empirically from processes for which the optimal series can be derived with no problem. Our criticism of the literature is more general since our case rests on conditions (notably, current account persistence) which are easy to verify and almost always met in the data. Moreover, unlike Kasa, we quantify the implications of these problems in practical applications, through simulations and actual estimation.

The paper is organized as follows. Section II explains how the present value model is tested and exposes the problems of the empirical methodology under current account persistence. Section III and Section IV present the simulations and the empirical results based on OECD data. Section V concludes.

II. ASSESSING THE PRESENT VALUE MODEL

A. The Present Value Model

In its simplest form, assuming quadratic utility, a constant real return on a single internationally traded bond, and a discount factor equal to the inverse of the (gross) return, the present value model predicts that the current account is given by:

\[ CA_{p,t} = -E_t \sum_{i=t+1}^{\infty} \left( \frac{1}{1+r} \right)^{i-t} \left[ Y_i - Y_{i-1} \right] \]

where \( r \) is the (constant) real interest rate and \( Y \) is income net of government spending and investment. Equation (1) characterizes the current account in an economy where a representative agent smooths her consumption by “saving for a rainy day.” That is, permanent shocks to income have no effect on the current account. Positive transitory shocks raise it on impact. Anticipated future income increases lower the current account.

B. Assessing the Present Value Model

A methodology developed by Campbell and Shiller (1987) is used to estimate the optimal current account benchmark given by equation (1). This methodology uses a VAR to estimate the expected income declines on the right hand of equation (1). The model implies that the current account should contain all the relevant information the agents have to form their expectations of future income declines. Therefore, the current account should be included in the VAR used to estimate these expected future income declines. Thus, we estimate the following \( l \) order VAR:
\[ X_t = CX_{t-1} + u_t \]

where \( X_t \) is the \( 2l \) vector containing actual data on income changes and current account \( (X_t = [\Delta Y_t, ..., \Delta Y_{t-l+1}, CA_t, ..., CA_{t-l+1}]') \); \( C \) is the companion matrix of the VAR; and \( u_t \) is a \( 2l \) column vector of zero-mean homoskedastic errors \([u_{\Delta Y}, 0, ..., 0, u_{CA}, 0, ...]'.\)

In this setup, the expected income change \( k \) periods ahead is given by:

\[ E_t \Delta Y_{t+k} = AC^k X_t \]

where \( A \) is the \( 2l \) row vector \([1 \ 0 \ 0 \ ... \ 0].\) Plugging these expressions into equation (1) yields the following expression for the current account predicted by the model:

\[ CA_{p,t} = KX_t \]

where \( K = -\frac{AC}{1+r} \left[ I - \frac{C}{(1+r)} \right]^{-1} \)

We then obtain the model’s estimated prediction, \( \hat{CA}_{p,t} \), by replacing \( C \) in (2) with its empirical estimate \( \hat{C} \). \( \hat{CA}_{p,t} \) is the estimation of the country’s optimal current account from a consumption-smoothing perspective. We can compare the actual current account to this benchmark. This can be done in several ways.

First, we can simply plot the actual and predicted current account paths. The literature has drawn inference by comparing these paths. For example, some authors use this approach to try to identify the shortcomings of a consumption-smoothing approach to the current account. Sheffrin and Woo (1990) note that the model underestimates UK current account deficits generated by the first oil shock. Ghosh (1995) draws similar conclusions for Japan following the second oil shock. Also, the aforementioned IMF studies use this estimated benchmark to assess if a country’s external borrowing is excessive.

Second, we can informally assess the model by computing the correlation between the estimated and the actual current account, as well as the ratio of their (in sample) volatility. For example, the literature has often emphasized that actual current account series are typically more volatile than the model’s predictions (see among others Ghosh, 1995, Obstfeld and Rogoff, 1996, Nason and Rogers, 2006, and Gruber, 2004). In fact, Ghosh (1995) uses excess current account volatility as evidence against Feldstein and Horioka’s claim that international capital markets are not highly integrated. Also, the correlation between actual and predicted series tends to be quite high (see, e.g., Obstfeld and Rogoff, 1996 and Nason and Rogers, 2006), and some authors have taken this as evidence that consumption smoothing plays a role in the dynamics of the current account.
Third, we can run a formal test of the model. Testing whether $\hat{C}A_{p,t}$ and $CA_t$ are equal is akin to testing whether the vector $K$ in equation (2) equals the 2l vector $T$ whose elements are all zero except for the $(l+1)^{th}$ element, which equals one.\textsuperscript{8} This is the standard Wald test of the cross-equation restrictions of the model (we present this test in greater detail later).

The literature has sometimes used an alternative test. Let’s define $R_t = CA_t - (1 + r)CA_{t-1} - \Delta Y_t$ and $I_{t-1}$ as the information set containing all the values of $CA_{t-1}, \ldots, CA_{t-n}, \Delta Y_{t-1}, \ldots, \Delta Y_{t-\infty}$ as well as the $t-1$ and previous values of any other variable. Equation (1) implies that $E_t(R_t | I_{t-1}) = 0$. One can thus regress $R_t$ on the variables in $I_{t-1}$ with the appropriate number of lags and do a simple $F$-test on the joint nullity of the coefficients of all the regressors (this test is a necessary but not sufficient test of equation 1). While both tests have been used in the literature, the non-linear Wald test has been more popular, probably because it focuses on the same equation (equation 2) that allows constructing the optimal path predicted by the model.\textsuperscript{9}

In short, the Campbell-Shiller methodology has been used to assess whether consumption-smoothing determines the dynamics of the current account, as well as to draw inference on capital mobility and, occasionally, on the optimality of a country’s external borrowing. We show, however, that the methodology is problematic under near-singularity conditions commonly generated by current account data.

\section*{C. Problems Under Near-Singularity}

From equation (2), we can see that $CA_{p,t}$ is a linear function of the inverse of the matrix $M = \left[ I - \frac{C}{(1+r)} \right]$. When $M$ has at least one eigenvalue close to zero (i.e. when $C$ has at least one eigenvalue close to $1 + r$), a small error in the estimated VAR parameters translates into potentially very large deviations in the inverse of $M$ and hence on the model predicted current account (intuitively, one can think of the function $1/c$ as $c$ is close to zero: a small variation in $c$ translates into a large deviation in $1/c$). The estimated optimal current account is therefore very imprecise, as are its (in sample) volatility and its correlation with the actual current account. In such circumstances, to draw inference from comparing the actual current account to the estimated optimal path as described above is dubious—a point we will illustrate in subsequent sections.

The non-linear Wald test of the model also becomes problematic, leading to false rejection and false acceptance of the model. To perform this test, we need an estimate of the variance-covariance of $K$. Since $K$ is a non-linear function of the VAR parameters, researchers

\textsuperscript{8} More precisely, this is a joint test of the model and of the assumption that the data generating process of income changes and the current account is given by the unrestricted VAR.

\textsuperscript{9} Out of the 15 papers that we cite which test the present value model, 10 report the (non-linear) Wald test only, 1 the $F$-test only, and 4 report both.
approximate its variance-covariance by \( [JVJ] \), where \( V \) is the variance-covariance matrix of the VAR parameters and \( J \) is the Jacobian of \( K \). This is the Delta method linear approximation of the variance-covariance. Then, the statistic:

\[
W = (K - T) * [JVJ]^{-1} * (K - T)
\]  

(3)

has an asymptotic chi-square distribution with \( 2l \) degrees of freedom.

In the near-singularity region mentioned above, the Delta method greatly distorts this variance-covariance in short samples. To see the problem, assume for simplicity that the element in \( K \) we are trying to test for is proportional to \( \frac{1}{c} \), where \( c \) is some parameter to be estimated. Figure 2 shows what happens when the true value \( c_0 \) is close to zero. In this example, the small sample estimate \( \hat{c} \) falls a bit further from zero but its probability interval still contains the true value. However, the interval \([\hat{K}_{l,0} \hat{K}_{r,0}]\) computed by linear approximation will be “too small” given the steepness of the curve and may therefore not include the true value \( K_0 \). Clearly, the problem arises because the slope of \( \frac{1}{c} \) changes rapidly in the neighborhood of 0. Also of note, distortion is a short sample issue since the interval around \( \hat{K} \) shrinks as sample size increases. We see from equations (2) and (3) that the reasoning in Figure 2 can be extended to the Chi-square test of the present value model. Under near-singularity, the Delta method would produce a variance-covariance matrix which is “too small” and a \( W \) statistic which is “too large”, leading to a false rejection of the model.

Figure 3 shows the opposite problem. Here, the estimate \( \hat{c} \) falls a bit closer to zero and its interval excludes \( c_0 \). However, the interval around \( \hat{K} \) will be “too large” given the steepness of the curve and will include \( K_0 \). Translating this to the Chi-square test, the Delta method would produce a variance-covariance matrix which is “too small” and a \( W \) statistic which is “too small”, leading to a false acceptance of the model. Figures 2 and 3 also show that the level of distortion in the test is not solely related to the distance to singularity: the width of the confidence interval or, more generally, the level of estimation uncertainty also matter.

While the F-test discussed in the previous section avoids the distortions created by the Delta-method, it is a necessary but not sufficient test of the model. However, it is possible to obtain a sufficient test that does not rely on the Delta method by simply “linearizing” the non-linear Wald test. Strangely enough, linear Wald tests have not been used in this current account literature, but have been used to test present value models in finance (see for instance Campbell and Shiller, 1989). Rather than testing whether \( K = -\frac{AC}{1+r} \left[ I - \frac{C}{(1+r)} \right]^{-1} = T \) as in the standard test, one would simply postmultiply both sides by \( \left[ I - \frac{C}{(1+r)} \right] \) and test whether \( -\frac{AC}{1+r} = T \left[ I - \frac{C}{(1+r)} \right] \).
While the \( F \)-test and the linear Wald test of the model do not rely on the problematic linear approximation, it is not straightforward to claim that these tests are more robust than the non-linear Wald test. Indeed, the estimated parameters have only an asymptotic justification. Moreover, if the unit root case is approached with sample size held fixed, the usual OLS intervals become less and less precise in the sense of coverage probability. It is therefore not theoretically impossible that the distortion created by the Delta method luckily offsets (rather than worsens) the other statistical issues. It is reasonable to think that this is unlikely to happen in practice, and our simulation results will confirm that the \( F \) and linear Wald tests have indeed much better coverage in small samples.

The relevant question now is: how common is near-singularity in practice? Unfortunately, the answer is “very common.” Table 1 shows for each country sample one estimated eigenvalue of the VAR companion matrix. These eigenvalues are above 0.9 for all five countries in quarterly data, and in three out of five cases in annual data. In fact, the eigenvalues are often close to \( 1 + r \) which is the critical value for singularity. In our VAR estimations the coefficients on lagged income changes are often small (and insignificant.) It is current account persistence that generates near-singularity in our data.\(^{10}\)

We illustrate the problems discussed in this section in two steps. First, we do Monte-Carlo simulations assuming that the “observed” current account is fully consistent with the model. Then, we turn to actual rather than simulated data to show how problematic the methodology is in typical situations faced by practitioners.

### III. Simulations

#### A. Simulation Set Up

The idea behind our simulations is as follows. We assume a given process for net income and simulate \( x \) observations from this data-generating process. We then derive \( x \) observations of the current account assuming that they are given by the model, that is: (i) in each period, the current account is the present discounted value of all expected future income declines, and (ii) expectations of future income declines are rational and based on knowledge of the income process and all relevant data. Using these simulated income and current account series as our “observed” data, we estimate a bivariate VAR and obtain the sample estimate of the optimal current account using equation (2). We then compute the correlation coefficient between the “observed” and optimal series as well as their variance ratio. We also test the model using the Wald and \( F \) tests. We repeat this sequence 10,000 times.

Because the “observed” series is by construction model-consistent, the estimated optimal series should track it very closely if the Campbell-Shiller methodology worked as intended. In other words, correlation coefficients and variance ratios obtained from the simulations

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\(^{10}\) This does not necessarily mean that the current account is non-stationary. Note from our previous discussion that near-singularity can easily arise with a persistent but stationary current account.
should fall very close to one. Moreover, tests of the model should (falsely) reject the null that the model is a valid representation of the data with probability (1-\(\alpha\)), where \(\alpha\) is the confidence level of the test.

Choice of the data generating process for income is likely to be crucial for the results of the simulations. As we have seen, the data generating process for income that is assumed in this literature is an unrestricted bivariate VAR in income changes and the current account. To follow the literature closely, we generate observations on income changes and a second variable – call it \(z\) – using an unrestricted VAR, where the coefficients come from our estimations of a VAR in income and the current account using quarterly data from Belgium, Canada, Denmark, Sweden, and the UK.\(^{11}\) In a second step, we use this imposed VAR matrix and the observations of net income changes and \(z\) in equation (2), to generate our model-consistent current account.\(^{12}\) The reason we chose this two step procedure is to avoid imposing the null that the model is true when generating income changes. After all, the model specifies the current account for a given income process, but says nothing about what that income process should be.

The shocks to each bivariate VAR process are drawn from a bivariate normal distribution using the variance-covariance matrix of the empirical residuals. Finally, the simulated sample length is set at a 120 observations, or thirty years of quarterly data.\(^{13}\) This is approximately the average available sample length across the different countries and is what one typically finds in the literature.

**B. Simulation Results**

Figure 1 shows the distributions of the correlation coefficient and the variance ratio obtained in our five sets of simulations. It is worth emphasizing that the country names as used in this figure and this section identify the different simulations and not the true country data, which is the purpose of our next section.

One expects correlation coefficients to display the smallest dispersion away from a unit value. Correlations are not very demanding tests: all that is required for a high value is that the estimate of the optimal current account move in the same direction as the “actual” (model-consistent) series. Since the estimate of the optimal current account and the “actual”

\(^{11}\) In other words, our variable \(z\) is given by the same data-generating process assumed in the literature for the current account. But this is irrelevant for the simulations, since we are trying to replicate observed net income series, not current account series. The current account series are generated by the model given this process for net income.

\(^{12}\) In other words, we have two VARs in our simulations. One that we impose to generate observations of net income changes and model-consistent current account, and one that we estimate assuming that these are our “observed” series.

\(^{13}\) We actually generate 620 observations and discard the first 500 to eliminate the effect of initial values, all set to zero.
(model consistent) series are both linear combinations of the same variables, one should expect the correlations to be often high (this point is further discussed in the next section). Correlation distributions are nonetheless a bit dispersed for the case of Belgium and the United Kingdom, and very dispersed in the case of Canada. In the case of Belgium, there is a 9.1% probability that the correlation will be below 0.8. For the UK, this probability jumps to 19.6%, and for Canada to 78.4%. In fact, there is a 12.8% probability for Canada that the correlation will actually be negative.

In the case of the variance ratio the distributions are now very dispersed for Belgium, Canada, the UK and to a lesser degree Denmark. For Belgium, there is a 24.5% probability that the estimated series is less than half as volatile as the “observed” series, and over 10% probability that it is at least twice as volatile. For Canada, these figures are 10.6% and 32.7% respectively; for the UK, 11.2% and 13.6%. In all three cases, the probability that the variance ratio is between 0.75 and 1.25 is less than 1/3. At the other extreme, this probability is 100% for Sweden. Dispersion as we see for Canada, Belgium, and the UK is startling and makes standard claims in the literature about excess volatility of the actual current account series – or lack of – dubious at best.

We also computed the frequency of (false) model rejection at 95% confidence, using the $F$-test as well as the linear and non-linear Wald tests. Table 2 summarizes the results. For the $F$ and the linear Wald tests, this frequency is always very close to 5%, showing that these tests have good coverage in small samples even though they only have an asymptotic justification. But as predicted by the previous section, the performance of the non-linear Wald test is substantially worse. Rejection probabilities are 11.7% for Belgium, 28.3% for Canada, 15.1% for Denmark, 13% for Sweden, and 16.8% for the UK. Deviations from the 5% benchmark are not trivial if one considers that the “observed” current account was assumed perfectly model consistent.

To verify that these are small sample issues as argued in the previous section, we redid our simulations assuming five hundred years of quarterly observations instead. Test performance improves significantly indeed while correlation and variance ratio distributions tighten around one. For practical applications it is fair to ask if the problems would also “go away” with a sample size longer than thirty years but still realistically short. The answer is no. When sample size is set at sixty years of quarterly observations – more than can be expected in the near future for most countries - significant problems persist. Finally, when one assumes a sample size of forty observations instead – the typical size of annual data sets in the literature – all problems are greatly magnified.\(^\text{14}\)

\section*{IV. Empirical Results}

We now turn to actual data to show how problematic the methodology is in typical situations faced by practitioners. We use annual and quarterly data from the same five countries as

\footnotesize{\textsuperscript{14} We do not present detailed simulation results for all these alternative specifications to be concise. The results can be requested from the authors.}
above, which are the ones chosen by Obstfeld and Rogoff (1996) to survey the literature. The data appendix details the data sources, and discusses data construction and other estimation issues. As mentioned before, the eigenvalues of the variance-covariance matrix in Table 1 show that near-singularity is a real issue in most country samples considered. In this context, we study each aspect of the Campbell-Shiller methodology as used in this literature.

A. Tests of the Model

Table 1 gives the $p$-value of the Wald and $F$-tests. We see that the difference between the non-linear Wald and the $F$ and linear Wald tests can be large. The non-linear Wald statistic suggests significance levels that are up to eighty-eight percentage points different from the level given by the $F$ or linear Wald tests. Most strikingly perhaps, in four out of ten cases the non-linear Wald test leads to the inference opposite to that of the $F$- and linear Wald tests at the traditional 95 percent level of confidence (the $F$- and linear Wald tests lead to the same inference in all 10 cases). In particular, the non-linear Wald test always rejects the model with quarterly data while the $F$- and linear Wald tests accept it for two out of five countries (Belgium and Canada). It should also be noted that in three of the four papers cited that use both the $F$ and non-linear Wald tests (Sheffrin and Woo 1992, Cashin and McDermott 1998a, Gruber 2003) the two tests frequently lead to opposite conclusions regarding the validity of the model.

B. Graphical Analysis

We noted that the literature has often drawn inference by comparing the paths of the actual and predicted current account during economically significant periods. To evaluate the robustness of such inference, we construct the empirical distribution of the predicted current account. For each country sample, we generate 10,000 draws from the multivariate Normal distribution given by the estimated VAR parameters and their associated variance-covariance matrix. For each of these draws we compute the associated $K$ vector and the corresponding predicted current account.

Figures 5 and 7 plot the 2.5th and 97.5th percentiles of the empirical distribution of the predicted current account at each point in time. These confidence bands are typically very wide compared with the actual series, often dramatically so. This has important implications. Consider the case of Sheffrin and Woo (1990), who analyze the UK current account after the first oil shock and conclude that actual deficits in the UK did exceed the model predicted series following the first oil shock (see Figures 4 or 6). Yet particularly in

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15 We cannot reject the null of joint normality of the residuals at a 95% confidence level in any of the annual samples. In quarterly data, we can only reject it for Sweden. We do not detect serial correlation in any of the residual series. In the case of the UK quarterly data we generate 5,000 draws for computational reasons.

16 The width of the bands varies over the sample. This is a direct consequence of the definition of the estimated optimal current account: $CA_{p,t} = KX_t$. The smaller the elements of $X_t$ (in absolute value), the narrower the band at time $t$. Also, there is no direct link between the highest eigenvalue and the width of the bands, because the width of the bands also depends on the error on the estimated VAR coefficients.
quarterly data the confidence bands easily encompass those deficits, showing that the conclusion that the model underestimated the deficits is unwarranted. Also, the imprecision of the estimated optimal current account casts doubt on the conclusions of empirical studies that used this estimated current account as a benchmark to assess the optimality of some emerging countries’ external borrowing (Ostry 1997, Callen and Cashin, 2002).

A limited number of papers in the literature have constructed confidence bands around the predicted series (see Cashin and McDermott 1998a and Hall et al. 2001 among others). Their bands are typically much narrower than ours, sometimes an order of magnitude so. Because the main claim in our paper is that the model predicted series is extremely sensitive to small sample estimation error, it is important to understand why our bands are so large. As it turns out, confidence bands in the above cited papers are built using standard bootstrapping techniques, as proposed by Runkle (1987). As Killian (1998) has shown however, the distribution of bootstrap VAR coefficients is biased towards stationarity, possibly severely so. In such a case, the bootstrap distribution of the $M = \left[I - \frac{C}{(1+r)}\right]$ matrix can be biased away from the singularity region, resulting in less extreme values for the vector $K$ and bands that are too narrow. To see this, Figure 8 shows bootstrap bands for our annual data but adjusted for stationarity bias following the method proposed by Killian. These bands are similar to ours: remarkably close in the case of Canada, slightly narrower but still very wide for Sweden and the UK, and wider for Belgium and Denmark. In all cases, they are of similar order of magnitude. In other words, our bands are representative of the true underlying uncertainty around the model predicted series, while confidence bands using non-bias adjusted bootstrap techniques may severely underestimate this uncertainty.

The imprecision of the estimated optimal current account is in line with our theoretical discussion. In the presence of singularity created by persistence, the coefficients of the $K$ vector in equation (2) can be very imprecisely estimated, leading to an unstable estimated optimal current account path (by construction $CA_{pt} = AX_{pt}$). To further illustrate the source of instability of the estimated optimal current account, Figures 9 and 10 present the distribution of the $l + lth$ coefficient of $K$, which is supposed to be equal to one under the null. For all country samples, the variance of the coefficient is very large. Even when the model is consistent with the data as determined by the $F$-test, there is a high probability that the coefficient will be far from its theoretical value. The coefficient can easily be negative. Some papers start the analysis by informally comparing the estimated $K$ vector with its theoretical value (see Obstfeld and Rogoff 1995, 1996). Our discussion suggests, however, that the point estimate of the $K$ vector is unlikely to be very informative.

The large variance of the $K$ vector and the associated imprecision of the predicted current account also has strong implications for the variance of the predicted current account and its correlation with actual data, as we will now discuss.

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17 To see why a large and negative coefficient can occur, note in Figure 2 that the probability interval around $\hat{c}$ can encompass small and negative values and hence imply large and negative values of $K$. 
C. Variance Analysis

We saw that the literature has often emphasized that actual current account series are typically more volatile than the model’s predictions. The literature typically reports only the variance of the predicted current account, without any indication on how precise this estimate is.\textsuperscript{18} Figures 11 and 12 plot the distributions of the intertemporal variance ratio.\textsuperscript{19} Variance ratios are often very dispersed. Even when the model is consistent with the data the predicted current account can still be much more or much less volatile than the actual. For example, the $F$-test suggests that the model is strongly consistent with Belgian annual data, yet there is a 40 percent probability that the predicted current account is over four times as volatile as the actual. There is also a 20 percent probability that the predicted current account displays less than a fourth of the volatility of the actual series. Similar observations hold for the other data sets which failed to reject the model. Also, the data does not support claims that the current account is excessively volatile. In our samples, the probability that the predicted series is more volatile than the actual is often large. It averages 44 percent over our 10 samples, ranging from 11 percent for Belgian quarterly data to over 97 percent for Swedish quarterly data. Note that one cannot conclude that the current account is less volatile than the model predictions either. These results cast doubt on the literature’s finding that actual current accounts are more volatile than predicted by the model, and by extension on the interpretation that excess current account volatility is evidence against Feldstein and Horioka’s claim of limited international capital mobility.\textsuperscript{20}

D. Correlation Analysis

Despite the supposed failure of the model to match current account volatility, some authors have claimed that the model has explanatory value in that the correlation between actual and predicted series tends to be quite high (see, e.g., Obstfeld and Rogoff 1996). Figures 13 and 14 show the distributions of the in-sample correlation between actual and predicted series. These distributions are once again very wide reflecting dispersion in the $K$ vector, casting

\textsuperscript{18} Gosh (1995), Gosh and Ostry (1995), and Hall et al. (2001) are exceptions.

\textsuperscript{19} For each draw $i$ from the multivariate Normal, we construct the predicted current account $CA_{p,t}^i = K^tX_t$ for all $t$ in our time range, where $X_t$ is the time $t$ vector of data. For each draw $i$ the variance ratio is calculated as the intertemporal variance of $CA_{p,t}^i$ over the $t$ range divided by the intertemporal variance of the actual series.

\textsuperscript{20} Gosh (1995) and Gosh and Ostry (1995) often reject the null that the variance ratio equals one in favor of the alternative hypothesis that the variance of the predicted series is higher. The fact that they can reject the null implies that their estimated variance of the variance ratio is much smaller than ours. The authors do not specify how they compute this variance, and hence we cannot account for the discrepancy. Two things should be noted however: (i) the numerical expression of the variance of the variance ratio is a function of the variance of the cross-equation coefficients, hence if the latter is approximated by the Delta method then the results for the former will not be reliable; (ii) if the variance of the variance ratio is obtained by standard bootstrap simulations, the result may be strongly biased downwards, as discussed previously in the context of confidence intervals.
doubt on the above claim. Also, correlation values are in no way indicative of the model's statistical validity. Most strikingly in the case of Belgian and Danish annual data the $F$-test accepts the model, yet there is over 45 percent probability that the correlation lies between $-1$ and $-0.9$. Conversely, for Swedish annual data there is a 37 percent probability that the correlation will exceed 0.95 even though the test has rejected the model. Finally, the distributions often cluster around one and minus one which can also be explained by the behavior of $K$ under near-singularity. To see why, consider the case of one lag in estimation. Then:

$$CA_{p,t} = \hat{k}_1 \Delta Y_t + \hat{k}_2 CA_t$$

and

$$corr(\hat{CA}_p, CA) = \frac{\text{cov}(\hat{k}_1 \Delta Y + \hat{k}_2 CA, CA)}{\sqrt{\text{Var}(\hat{k}_1 \Delta Y + \hat{k}_2 CA) \ast \text{Var}(CA)}}$$

Intuitively, if $\hat{k}_2$ is positive (negative) and very large relative to $\hat{k}_1$, then $\hat{CA}_p$ is mostly driven by $\hat{k}_2 CA$ and the correlation will tend to one (minus one).

V. CONCLUDING REMARKS

Our discussion challenges the results found in the large empirical literature on present-value models of the current account. Our discussion also suggests ways to revisit this large empirical literature. First, priority should be given to the $F$- and linear Wald tests rather than the non-linear Wald test when assessing the relevance of a consumption-smoothing model in explaining current account fluctuations. Second, confidence bands around the predicted optimal current account that account for the true underlying estimation uncertainty are needed, as are empirical distributions of the correlation coefficient and the variance ratio. Methods to account for estimation uncertainty should avoid biasing results away from the singularity region, or rely on Delta-method approximations. However, uncertainty surrounding estimates of future changes in output is likely to be such as to compromise present value estimates. If future research confirms this, one of the workhorse models of the literature would have proved of little empirical use. It would then be time to devote more attention to other models of the current account.
REFERENCES


Nason J, Rogers JH. 2006. The present value model of the current account has been rejected: round up the usual suspects. Journal of International Economics 68: 159-187.


DATA APPENDIX

All our data are from the International Financial Statistics of the International Monetary Fund. The periods covered are indicated in the table below, noting that for each country we use the longest available sample in IFS.\(^{21}\)

Net output and current account are defined as: \(Y_t = GDP_t - G_t - I_t\) and \(CA_t = GNP_t - C_t - I_t - G_t\) and are expressed in real, per capita terms. Corresponding IFS series are as follows: GNP: gross national income (line 99a); G: government consumption (line 91f); I: sum of private gross fixed capital formation (line 93e) and increase/decrease in stocks (line 93i); C: household consumption (line 96f); and GDP: gross domestic product (line 99b). For conversion into real, per capita terms we use GDP volume in 1995 or 1996 terms (line 99b) and population (line 99z).

<table>
<thead>
<tr>
<th>Country</th>
<th>Annual Data</th>
<th>Quarterly Data</th>
</tr>
</thead>
</table>

As has been standard practice in the literature (see, e.g., Campbell, 1987, or Sheffrin and Woo, 1990), we remove the means from the current account and from the first difference in net output (we only test the dynamic restrictions of the theory). We set annual and quarterly real interest rates to 4 percent and 1 percent respectively. For the VAR we use the number of lags selected by the Akaike information criterion. Our results are robust to changes in the number of lags or in the value of the real interest rate.

Finally, some authors assume that the discount factor is not equal to the inverse of the gross real interest rate (see Ghosh, 1995). In such a case, the current account equation includes a consumption-tilting parameter which needs to be estimated. We followed this procedure as a robustness check. The estimated consumption-tilting parameters are usually close to one (their value when the discount factor is equal to the inverse of the gross real interest rate). The resulting series display similar properties as before, and our results remain robust to this specification.

\(^{21}\) For Belgium we cut the sample in 1998 as there is a break in the data following the adoption of the euro in 1999.
Table 1. Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>Number of Lags</th>
<th>Eigenvalue</th>
<th>F-test (p-value)</th>
<th>Linear Wald test (p-value)</th>
<th>Non-linear Wald test (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Annual data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgium 1953–1998</td>
<td>1</td>
<td>1.01</td>
<td>36.7%*</td>
<td>36.4%*</td>
<td>96.9%</td>
</tr>
<tr>
<td>Canada 1948–2002</td>
<td>1</td>
<td>0.61±0.2i</td>
<td>0.6%</td>
<td>0.9%</td>
<td>27.8%</td>
</tr>
<tr>
<td>Denmark 1966–2002</td>
<td>1</td>
<td>0.96</td>
<td>20%*</td>
<td>22.5%*</td>
<td>94.9%</td>
</tr>
<tr>
<td>Sweden 1950–2002</td>
<td>1</td>
<td>0.93</td>
<td>0.8%</td>
<td>0.7%</td>
<td>89.4%</td>
</tr>
<tr>
<td>United Kingdom 1948–2002</td>
<td>2</td>
<td>0.67</td>
<td>0.1%</td>
<td>0%</td>
<td>2.8%</td>
</tr>
<tr>
<td><strong>Quarterly data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgium 1980–1998</td>
<td>3</td>
<td>0.98</td>
<td>43.7%*</td>
<td>10.7%*</td>
<td>0.7%</td>
</tr>
<tr>
<td>Canada 1948–2002</td>
<td>8</td>
<td>0.94</td>
<td>38.1%*</td>
<td>31%*</td>
<td>2%</td>
</tr>
<tr>
<td>Denmark 1988–2002</td>
<td>4</td>
<td>0.93</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Sweden 1990–2002</td>
<td>4</td>
<td>0.97</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>United Kingdom 1955–2002</td>
<td>4</td>
<td>0.94</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Notes: A star denotes model acceptance by the $F$-test at the 95 percent confidence level. For each sample the number of lags was selected using the Akaike criterion.

Table 2. Simulations: Probability of Model Rejection at 95% Confidence (in percent)

<table>
<thead>
<tr>
<th></th>
<th>$F$-test</th>
<th>Linear Wald Test</th>
<th>Non-linear Wald Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>5.3</td>
<td>6.2</td>
<td>11.7</td>
</tr>
<tr>
<td>Canada</td>
<td>5.1</td>
<td>7.2</td>
<td>28.3</td>
</tr>
<tr>
<td>Denmark</td>
<td>5.3</td>
<td>6.3</td>
<td>15.1</td>
</tr>
<tr>
<td>Sweden</td>
<td>5.0</td>
<td>6.3</td>
<td>13.0</td>
</tr>
<tr>
<td>UK</td>
<td>4.8</td>
<td>6.3</td>
<td>16.8</td>
</tr>
</tbody>
</table>
Figure 1. Simulated Distributions

Note: The variance ratio is expressed as the variance of the predicted series over that of the actual.
Figure 2. False Rejection in the Singularity Region

Note: \( c_0 \) is the true value of the parameter, \( \hat{c} \) the empirical estimate, and \( [\hat{c}_L, \hat{c}_R] \) the probability interval around \( \hat{c} \). The Delta method yields the probability interval \( [\hat{K}_{L,\delta}, \hat{K}_{R,\delta}] \) around \( \hat{K} \) rather than the correct \( [\hat{K}_L, \hat{K}_R] \).
Figure 3. False Acceptance in the Singularity Region

Note: $c_0$ is the true value of the parameter, $\hat{c}$ the empirical estimate, and $[\hat{c}_L, \hat{c}_R]$ the probability interval around $\hat{c}$. The Delta method yields the probability interval $[\hat{K}_L, \hat{K}_R]$ around $\hat{K}$ which includes $K_0$ even though $[\hat{c}_L, \hat{c}_R]$ excludes $c_0$. 
Figure 4. Actual (-) and Predicted (--.) Series: Annual Data

Note: A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 5. Actual (-), Predicted (--), and Confidence Bands (Bold): Annual Data

Note: Bold lines correspond to the 2.5th and 97.5th percentiles of the distribution of the predicted series at each point in time. A star denotes model acceptance by the F- and linear Wald tests at the 95 percent confidence level.
Figure 6. Actual (-) and Predicted (--) Series: Quarterly Data

Note: A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Notes: Bold lines correspond to the 2.5th and 97.5th percentiles of the distribution of the predicted series at each point in time. A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 8. Actual (-), Predicted (--), and Killian-Adjusted Bands (Bold): Annual Data

Note: A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 9. Distributions of the \((l + 1)^{st}\) Coefficient of the \(K\) Vector: Annual Data

Note: A star denotes model acceptance by the \(F\)- and linear Wald tests at the 95 percent confidence level.
Figure 10. Distributions of the \((l + 1)^{th}\) Coefficient of the \(K\) Vector: Quarterly Data

Note: A star denotes model acceptance by the \(F\)- and linear Wald tests at the 95 percent confidence level.
Notes: The variance ratio is expressed as the variance of the predicted series over that of the actual. A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 12. Distributions of the Variance Ratio: Quarterly Data

Notes: The variance ratio is expressed as the variance of the predicted series over that of the actual. A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 13. Distributions of the Correlation Coefficient: Annual Data

Note: A star denotes model acceptance by the $F$- and linear Wald tests at the 95 percent confidence level.
Figure 14. Distributions of the Correlation Coefficient: Quarterly Data

Note: A star denotes model acceptance by the F- and linear Wald tests at the 95 percent confidence level.