

Partial Response to a Referee's Comments on Paper 2007/49

. The pages 6 and 7 of the original version is to be replaced with the following.

The Poisson-Dirichlet Distribution of Innovations

We now describe how innovations stochastically enter the model. The innovations follow the *two-parameter Poisson-Dirichlet distribution*. It is denoted by $PD(\alpha, \theta)$, while the one-parameter PD distribution is denoted by $PD(\theta)$.

The new parameter α arise as follows: Suppose that there are already n agents in the model, and that they are partitioned into k clusters. Suppose that the next agent, which is the $(n+1)$ st one to enter the model, is of a new type. It starts a new cluster, of initial size 1.

The probability of this event is given by

$$\Pr(K_{n+1} = k + 1 | K_1, K_2, \dots, K_n = k) = \frac{\theta + k\alpha}{n + \theta},$$

where α is the second parameter in the PD distribution.

The probability that the entering agent is one of the known types, and the entering agent joins one of the clusters which are already in existence, is

$$\Pr(K_{n+1} = k | K_1, K_2, \dots, K_n = k) = 1 - \frac{\theta + k\alpha}{n + \theta} = \frac{n - k\alpha}{n + \theta}.$$

Note that with $n = 1$, the above expression reduces to

$$\Pr(K_2 = 1 | K_1 = 1) = \frac{1 - \alpha}{1 + \theta} > 0,$$

that is, $\alpha < 1$.¹

It is important to note that the larger the sizes of clusters the greater is the probability that entering agents join these clusters.

(This section replaces page 6 including (8) and (9) in the original version, and the equations (10) and (11) are now (8) and (9). This page is part of the revised paper, extracted here for the convenience of the referee whose comments were primary reason for this revision.)

¹For the mathematical machinery used to describe the two-parameter Poisson-Dirichlet process, see Pitman (2002, Ch.4) for example. For a simpler example in a continuous-time framework is found in Feng-Hoppe (1998) in which a pure birth process $I(t)$ introduces the agents into the model with birth rate $r_k = \lim_{h \rightarrow 0} (1/h) \Pr(I(t+h) - I(t) | I(t) = k) = \alpha(k-1) + \beta = \alpha k + \theta$, where $\theta = \beta - \alpha$, that is $\alpha + \theta = \beta > 0$.