

## **Modeling the Effects of Financial Constraints on Firm's Investment**

Reply to invited reader comment

The main concern in the invited reader comment regards the contribution made in the paper in comparison to existing literature. In order to make the paper contribution clearer, in the present version the main model results are discussed in the context of a broader review of both the theoretical and the empirical literature. The paper reviews the most recent theoretical literature on the investment function, concerned with the implications of non convexities in the adjustment cost function, and provides some foundations for the model assumptions using the literature on asymmetric information, agency and investment. In order to clarify the purpose of the work and emphasize its findings a more extensive review of the empirical literature on the relation between finance and investment is also provided.

The findings in the paper could probably usefully be further developed with an empirical application, at this stage however the analysis has been limited to the task of applying a rigorous methodology in order to analyze some of the main unresolved questions that arise from empirical research in the field of finance and investment. In the paper it is also mentioned that some of this findings were also in the background of the more traditional empirical literature, although they have not yet found a proper definition in the economic research regarding the investment function. Further work of both theoretical and empirical nature is therefore certainly required.

Focusing on more specific comments:

1. In order to overcome the possible questions regarding the firm's cash-flow policies I relaxed the assumption that the firm's debts stock must be positive. The firm's debt stock in the present version is allowed to be negative and the firm in this way is allowed to transfer cash-flow across periods of time. The amount of debt stock is however bounded above by the constraints on the firm's new debt issue and cash-flow. With this modification the model is indeed more in line with the analogous models that arise in the literature on the consumption function. Since the focus of the paper is the investment function, further analysis of the firm's policies regarding these funds is not pursued in the paper although it could in principle be performed.

2. The appendix contains now a clearer expositions of the firm's optimization problem. In the paper the model solution is obtained through the Bellman equation and therefore the Hamiltonian function is not considered

explicitly. This could be done following for instance Malliaris and Brock (1982) but the Bellman equation approach does not require to write the Hamiltonian.

In the present version of the paper the firm's optimization problem and the Lagrange multipliers associated to each financing constraint are defined explicitly. The first order conditions and the complementary slackness conditions that must hold at an optimal are stated precisely and it is also explicitly mentioned that given the structure of the firm's optimization problem these are both necessary and sufficient for an optimum. The analysis of the firm's optimization problem in the appendix is used to derive equations (3.3) and (3.4) in the text.

In the main text it is also clarified that the existence of a solution for the firm's dynamic optimization problem is in general ensured by the assumptions that are made regarding the shape of the profit and costs of adjustment functions and of the financing constraints. Some additional references to the mathematical literature have been provided.

3. The assumptions concerning the firm's borrowing constraints have been clarified in the present version of the paper. In particular, the paper describes the correspondence between the upper bound on the firm's net borrowing and the upper bound on the firm's debt stock. In the literature usually borrowing constraints are made to depend on firm's characteristics such as net worth or assets. The assumptions in the paper do not rule out these type of dependence and they are made only for simplicity of notation. The main arguments in the paper are not affected by this notational simplification. That this simplification is made only for notational convenience is explicitly stated in the present version of the paper.

4. Section 5 and 6 have been revised in order to show how the firm optimal strategy depends on the fundamentals. In particular, the external finance premium terms that enter in the equation for the shadow price of capital and for the shadow price of firm's debt have been solved explicitly. This allows for a better characterization of the results of the paper with regard to the effects of future constraints on the firm's marginal  $q$  and on the shadow price of firm's debt. This characterization of the results is related to some recent contributions to the theory of credit constraints giving a better emphasis to the originality of the contents of the paper, also in comparison to the more traditional literature.

5. The meaning of firm's marginal  $q$  should be clarified by the fourth sentence of the abstract, although this may be subject to revisions.

6. The interpretation of conditions like (5.1) and (6.1) as no arbitrage conditions is quite standard in economics. In the present case, although we refer to  $q_t^B$  as the shadow price of capital we know that by definition it is the marginal value of an additional unit of debt at time  $t$  for the firm. Therefore  $q_t^B$  is defined in terms of market values, like the firm's marginal  $q$ . A better term for this quantity is currently not available, it might be noted however that it is defined in a similar manner as the firm's marginal  $q$ , which is sometimes referred to as the shadow price of capital.