

Calculations for Appendix

■ Performance Pay

■ The profit function (17) is

$$\begin{aligned} \text{In}[1] := \text{prof} &= p (\rho x + (1 - \rho) y) - \rho w - (1 - \rho) \frac{y}{x} w - c; \\ \rho &= q \left(1 + \mu \text{Log} \left[\frac{w}{W} \right] \right); \end{aligned}$$

■ Its derivative with respect to w is

$$\begin{aligned} \text{In}[3] := \text{Dprof} &= \text{D}[\text{prof}, w]; \\ W &= w; \\ \text{General}::spell1 : & \\ &\text{Possible spelling error: new symbol name "Dprof" is similar to existing symbol "prof". MORE...} \end{aligned}$$

$$\text{In}[5] := \text{solution} = \text{Solve}[\{\text{Dprof} == 0, \text{prof} == 0\}, \{w, p\}]$$

$$\text{Out}[5] = \left\{ \left\{ w \rightarrow -\frac{-c q x^2 \mu + c q x y \mu}{(q x + y - q y)^2}, p \rightarrow -\frac{-c q x - c y + c q y - c q x \mu + c q y \mu}{(q x + y - q y)^2} \right\} \right\}$$

■ Check that expression (18) is correct

$$\text{In}[6] := \text{Simplify} \left[-\frac{-c q x^2 \mu + c q x y \mu}{(q x + y - q y)^2} == c q x \mu \frac{x - y}{(q x + (1 - q) y)^2} \right]$$

$$\text{Out}[6] = \text{True}$$

$$\text{In}[22] := \text{Clear}[W]$$

■ Wage Compression

■ The Lagrangean is

$$\begin{aligned} \text{In}[23] := \\ L &= q \left(1 + \mu \text{Log} \left[\frac{w}{W} \right] \right) (p x - w) + \\ &(1 - q) \left(1 + \mu \text{Log} \left[\frac{v}{V} \right] \right) (p y - v) - c - \lambda \left(q \text{Log} \left[\frac{w}{W} \right] + (1 - q) \text{Log} \left[\frac{v}{V} \right] \right); \end{aligned}$$

■ The derivatives are

$$\text{In}[24] := \text{DLw} = \text{D}[L, w]$$

$$\text{Out}[24] = -\frac{q \lambda}{w} + \frac{q (-w + p x) \mu}{w} - q \left(1 + \mu \text{Log} \left[\frac{w}{W} \right] \right)$$

In[13]:= $\mathbf{DLv} = \mathbf{D}[\mathbf{L}, \mathbf{v}]$

Out[13]= $-\frac{(1-q)\lambda}{v} + \frac{(1-q)(-v+PY)\mu}{v} - (1-q)\left(1 + \mu \operatorname{Log}\left[\frac{v}{V}\right]\right)$