Economics Discussion Papers
The Open-Access, Open-Assessment E-Journal

Discussion Paper 2007-16
April 11, 2007

An Idealized View of Financial Intermediation

*Carolyn Sissoko**

Occidental College, Economics Department, Los Angeles

Please cite the corresponding journal article:
http://www.economics-ejournal.org/economics/journalarticles/2007-5

Abstract:

Using the monetary model developed in Sissoko (2007), where the general equilibrium assumption that every agent buys and sells simultaneously is relaxed, we observe that in this environment fiat money can implement a Pareto optimum only if taxes are type-specific. We then consider intermediated money by assuming that financial intermediaries whose liabilities circulate as money have an important identifying characteristic: they are widely viewed as default-free. The paper demonstrates that default-free intermediaries who issue credit lines to consumers can resolve the monetary problem and make it possible for the economy to reach a Pareto optimum. We argue that our idealized concept of financial intermediation is a starting point for studying the monetary use of credit.

JEL: *E5, G2*

Keywords: *Fiat Money, Cash-in-advance, Financial Intermediation*

Correspondence:
Occidental College, Economics Department, 1600 Campus Road, Los Angeles, CA 90041
phone: 323-259-2932, fax: 323-259-2704, e-mail: *csissoko@oxy.edu*

*Neil Wallace, Chris Waller and an anonymous referee gave me invaluable comments while developing this paper. I also thank Jim Bullard, Gabriele Camera, Ricardo Cavalcanti, Lee Ohanian, Joe Ostroy, Shouyong Shi, Ross Starr, Ted Temzelides, Aaron Tornell, Randy Wright and seminar participants at the Office of the Comptroller of the Currency for helpful comments. All errors are, of course, my own.

When a private promise to pay in the future is generally accepted as a means of payment within an economy, we have a single financial asset that fits the definitions of both credit and money. The most common example of such an asset in the modern US economy is a merchant's credit account with a credit card company. The asset is both a privately issued liability and a liability that is almost universally accepted in payment. How can an asset have both of these attributes simultaneously? The essential link between them is this: the private issuer is widely viewed as almost default free. Very little time is spent by merchants worrying about what to do in case Visa, MasterCard or American Express fails to meet its obligations.

This paper uses the assumption that financial intermediaries are default-free to set up a perfect world where intermediation can effortlessly overcome the monetary problem created by the friction in our model. In fact, our perfect world is in many ways a replica of the competitive model – with one important distinction: the role of financial intermediaries and their most important characteristic have been defined. Just as the competitive model posits the existence of an ideal real world in order to articulate the nature of economic relations between agents, we hope that by positing the existence of an ideal financial world we can articulate the role that financial institutions play in the real economy.

In the paper the assumption that financial intermediaries are default-free means that their liabilities are accepted as a means of payment, and this is essential to the economy's ability to reach the first-best. This assumption is strong, but can be motivated by the work of Cavalcanti and Wallace (1999b), which demonstrates that bankers with public histories choose not to default in equilibrium. And, of course, we recognize that the ideal financial world we model is only a beginning and that a full understanding of the nature of financial intermediation will require a careful study of the effects of relaxing our assumptions – but that is future work.

Section 1 of the paper introduces a model of fiat money. The model is based on a standard infinite horizon general equilibrium endowment (SGE) economy[1] with one change: the general equilibrium assumption that every agent can buy and sell goods simultaneously is relaxed. In every period of our model each agent is randomly required to either sell first and then make purchases or to buy first and then sell his product. Introducing this simple friction into the SGE model means that each consumer will with probablity one half face an endogenous cash-in-advance constraint. We find that in this environment implementation of an efficient allocation using fiat

[1] We use an abbreviation from Wallace (1998).

money is possible only if the government can collect type-specific taxes. In section 2 of the paper we consider an alternative monetary regime based on default-free intermediaries and find that the first-best can be attained whenever agents are sufficiently patient and the credit line they are offered is sufficiently high.

1 The Model

There are n perishable goods indexed by $j \in \{1, ..., n\}$ and a continuum of infinitely lived consumers with unit mass. Each consumer is endowed with a quantity, y, of one good in every period where $y \in \{1, ..., k\}$. Let $i \in I \equiv \{1, ..., n\} \times \{1, ..., k\}$ index the different types of agents. Assume that each type of consumer, i, has mass $\frac{1}{nk}$. Let Y be the aggregate endowment of each good, or $Y = \frac{1}{nk} \sum_{y=1}^{k} y$.

All consumers have the same preferences and maximize their expected utility:

$$U^i = E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{n} u\left(c_{jt}^i\right) \forall i \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor and c_{jt}^i is the consumption of good j by an agent of type i at date t. The period utility function, $u(c)$, is increasing and strictly concave.

1.1 A Competitive Baseline

In order to establish a baseline allocation, let us find the standard competitive equilibrium of this environment. We will introduce one-period privately issued bonds at each date t into our model to give our agents a means of transferrring wealth from one period to the next. b_t^i is the quantity of bonds an agent of type i holds from date $t-1$ to maturity at date t. Then the competitive problem for an agent of type $i = (j', y)$ is to choose consumption goods and bonds to maximize equation 1 subject to the budget constraint:

$$p_{j't} y + (1 + i_t) b_t^i \geq \sum_{j=1}^{n} p_{jt} c_{jt}^i + b_{t+1}^i \tag{2}$$

where p_{jt} is the price of good j at date t and i_t is the nominal interest rate paid on a bond held from $t-1$ to t.

Definition 1 *A **competitive equilibrium** is an allocation of goods $\{c_{jt}^i\}$ and of bonds $\{b_t^i\}$ and sequences of prices $\{p_{jt}\}$ and of interest rates, $\{i_t\}$, such that* *(i) given prices and interest rates, the competitive problem is solved for each type i, and*

(ii) markets clear

(a) $Y = \frac{1}{nk}\sum_{i\in I} c^i_{jt} \forall j, t$

(b) $0 = \frac{1}{nk}\sum_{i\in I} b^i_t \forall t$

The first order conditions for the competitive problem, using λ^i_t, as the Lagrangian multiplier on the budget contraint for a consumer of type i, are:

$$\beta^t u'\left(c^i_{jt}\right) \leq p_{jt}\lambda^i_t \tag{3}$$

$$(1 + i_{t+1})\lambda^i_{t+1} \leq \lambda^i_t \tag{4}$$

Because our utility function is concave and our constraint convex, there is a unique solution to these equations. The solution ensures that our equilibrium is better than any alternative allocation that differs only over a finite horizon. To ensure that over the infinite horizon our consumers do not choose to accumulate assets or liabilities that will not be repaid, we must also impose a transversality condition on the bond holdings of each type of agent:

$$\liminf_{t\to\infty} \lambda^i_t b^i_t = 0 \text{ for all } i$$

The unique competitive equilibrium is:

(i) an allocation of goods, $c^i_{jt} = \frac{y}{n}$ for all i, j, t

(ii) an allocation of bonds, $b^i_t = 0$ for all i, t

(iii) intratemporal prices that are the same for all goods, $p^*_t = p_{jt}$ for all j and

(iv) a real return on bonds that compensates bond holders for holding an asset over time: $(1 + i_{t+1})\frac{p_t}{p_{t+1}} = \frac{1}{\beta}$.

Observe that if we were to raise the welfare of one type of agent, we would have to reduce the consumption and therefore the welfare of another type of agent, so this allocation is a Pareto optimum. Furthermore this is the Pareto optimum that can be reached without transfers of endowments. Therefore the rest of the paper will investigate how fiat money and intermediated credit can be used to reach this particular allocation. Throughout the rest of the paper let $c^{i*}_t = \frac{y}{n}$ represent optimal consumption for a consumer of type i at date t and $C^* = \{c^{i*}_t\}$ represent the optimal allocation.

1.2 Fiat Money

Now that we have established a baseline allocation for our economy, let us explore how fiat money can be used to reach this Pareto optimum. This section of the paper makes an argument that runs parallel to an argument we make in a standard cash-in-advance economy in Sissoko (forthcoming). The important distinction between the two approaches is that here the monetary environment is endogenous.

To create a monetary environment we will have to relax the SGE assumption that every agent can buy and sell simultaneously. To do so, we assume that in each period, trade in goods takes place over the course of two sub-periods and the continuum of consumers is divided by a non-atomic probability distribution into first sub-period buyers and first sub-period sellers. The distribution is i.i.d. and ensures that in each period every consumer faces a 50% probability of being a first sub-period buyer and a 50% probability of being a first sub-period seller. In the second sub-period all of the agents who were buyers in the first sub-period become sellers and vice versa. In this market, every agent has to buy and sell at a different point in time. Thus in each sub-period the market receives goods only from sellers and distributes goods only to buyers.[2] Consumption takes place at the end of the second sub-period.

As in Sissoko (2007) we keep track of each agent's history of being a first sub-period buyer or first sub-period seller: $\chi_t \in \{B, S\}$ where B represents a first sub-period buyer and S a first sub-period seller. Let H_t be the set of possible histories at t and let an individual agent's history be represented by $h_t \in H_t$. $f_t(i, h_t)$ is the mass at date t of agents of type i with history, h_t. Clearly, $f_t(i, h_t) = \frac{1}{nk}\frac{1}{2}^{t+1}$.

To introduce fiat money into this environment, assume that each consumer of type i has m_0^i units of fiat money at date 0, and thus the aggregate date 0 money supply is $M_0 = \frac{1}{nk}\sum_i m_0^i$. The government controls the money supply by imposing a tax, τ_t^i, on each type i paid at the end of date $t \geq 0$. So, the aggregate money supply changes as follows: $M_{t+1} = M_t - \frac{1}{nk}\sum_i \tau_t^i$. The government burns the proceeds of the tax – or, if the tax is negative, costlessly prints fiat money to transfer to every consumer.

Definition 2 *A* ***government policy*** *is a series of initial money supplies,* m_0^i*, and a sequence of taxes,* $\{\tau_t^i\}$.

[2] This is the same monetary friction that we use in a previous paper, Sissoko (2002).

A price vector, p_t, at which money can be exchanged for goods becomes public information at the start of each period. In the first sub-period the consumers of type B use their money holdings to buy goods and those of type S exchange the goods they choose to sell for cash. In the second sub-period type B agents can now sell their goods in exchange for cash while type S agents make their purchases of goods. After the second sub-period ends a financial market opens in which bonds pay returns and are traded.

The goods market friction creates a role for money within a SGE model by forcing every consumer to buy and sell at different points in time. However, because half of the continuum of agents buy and half sell in both the first and the second sub-periods, a first best allocation is feasible. On the other hand, individuals who are first sub-period buyers face an endogenous cash-in-advance constraint.

By developing the model with both money and bonds, the relationship between this model and standard cash-in-advance models is made clear. However, the fact that money is necessary within periods, while privately issued bonds can be traded from one period to the next implies that agents are anonymous in some transactions, but not in others. This is an issue, not just for this paper, but for the whole cash-in-advance literature. We explore this issue more thoroughly in Sissoko (2007) where we also derive the budget constraints below.

The choice variables in our environment depend both on the consumer's type, where $i = \{j', y\}$, and on the consumer's history of being a first sub-period buyer or seller. Thus the budget equations are as follows: consumers who are first sub-period sellers can sell their endowment before making purchases and face only the standard budget constraint.

$$p_{j't}y + m_t^i(h_{t-1}) - \tau_t^i + (1+i_t)b_t^i(h_{t-1}) \geq m_{t+1}^i(h_{t-1}, S) + b_{t+1}^i(h_{t-1}, S) + \sum_{j=1}^{n} p_{jt}c_{jt}^i(h_{t-1}, S) \quad (5)$$

On the other hand, first sub-period buyers do not have the opportunity to sell their endowment before they make their purchases and thus they face both a cash-in-advance constraint and the standard budget constraint:

$$m_t^i(h_{t-1}) \geq \sum_{j \neq j'} p_{jt}c_{jt}^i(h_{t-1}, B) \quad (6)$$

$$p_{j't}y + m_t^i(h_{t-1}) - \tau_t^i + (1+i_t)b_t^i(h_{t-1}) \geq m_{t+1}^i(h_{t-1}, B) + b_{t+1}^i(h_{t-1}, B) + \sum_{j=1}^{n} p_{jt}c_{jt}^i(h_{t-1}, B) \quad (7)$$

The friction that we introduce into the model forces trade in goods to take place in two sub-periods. For this reason, the goods market will have to clear in each sub-period. Let I' denote the set of agents who are endowed with good j', $I' = \{(j, y) : j = j'\}$ and I^- the set of agents who are not endowed with good j', $I^- = \{(j, y) : j \neq j'\}$. Then the market clearing conditions in the goods market are:

$$\sum_{i \in I'} \sum_{h_s \in H_{t-1}} (y - c^i_{j't}(h_s, S)) f_t(i, h_s, S) = \sum_{i \in I^-} \sum_{h_s \in H_{t-1}} c^i_{j't}(h_s, B) f_t(i, h_s, B) \text{ for all } j', t \quad (8)$$

$$\sum_{i \in I'} \sum_{h_s \in H_{t-1}} (y - c^i_{j't}(h_s, B)) f_t(i, h_s, B) = \sum_{i \in I^-} \sum_{h_s \in H_{t-1}} c^i_{j't}(h_s, S) f_t(i, h_s, S) \text{ for all } j', t \quad (9)$$

Because each agent chooses the current period's consumption and next period's monetary stock and bond holdings *after* learning whether he is a Buyer or Seller in the current period, at date t each consumer solves the following fiat money problem:

$$\max_{c_{jt}, b_{t+1}, m_{t+1}} \beta^t \sum_{j=1}^{n} u\left(c^i_{jt}(h_t)\right) + E_t \sum_{s=t+1}^{\infty} \beta^s \left(\frac{1}{2} \sum_{j=1}^{n} u\left(c^i_{js}(h_{s-1}, B)\right) + \frac{1}{2} \sum_{j=1}^{n} u\left(c^i_{js}(h_{s-1}, S)\right) \right) \quad (10)$$

subject to the budget constraints, equations 5, 6 and 7.

Definition 3 *A **fiat money equilibrium** is an allocation for each type and each possible history of goods $\{c^i_{jt}(h_t)\}$, of bonds, $\{b^i_t(h_{t-1})\}$, and of money $\{m^i_t(h_{t-1})\}$ and sequences of prices, $\{p_{jt}\}$, interest rates, $\{i_t\}$, and taxes, $\{\tau^i_t\}$, such that*

(i) given the government policy, prices and interest rates, the fiat money problem is solved for agents of all types and histories, and

(ii) markets clear

(a) in the goods market, equations 8 and 9,

(b) $M_t = \sum_{i \in I} \sum_{h_t \in H_t} m^i_t(h_{t-1}) f_t(i, h_t)$ for all t

(c) $0 = \sum_{i \in I} \sum_{h_t \in H_t} b^i_t(h_t) f_t(i, h_t)$ for all t.

Using $\lambda^i_t(h_{t-1})$, $\mu^i_t(h_{t-1})$ and $\gamma^i_t(h_{t-1})$ as the Lagrangian multipliers on budget constraints 5, 6 and 7 respectively, we find the following first-order conditions for an agent of type $i = (j', y)$ who is solving the fiat money problem:

$$\frac{\beta^t u'\left(c^i_{jt}(h_{t-1}, S)\right)}{p_{jt}} \leq \lambda^i_t(h_{t-1}) \forall j, t \quad (11)$$

$$\frac{1}{2}\left(\lambda^i_{t+1}(h_{t-1}, S) + \mu^i_{t+1}(h_{t-1}, S) + \gamma^i_{t+1}(h_{t-1}, S)\right) \leq \lambda^i_t(h_{t-1}) \quad (12)$$

$$\frac{1 + i_{t+1}}{2}\left(\lambda^i_{t+1}(h_{t-1}, S) + \mu^i_{t+1}(h_{t-1}, S) + \gamma^i_{t+1}(h_{t-1}, S)\right) \leq \lambda^i_t(h_{t-1}) \quad (13)$$

$$\frac{\beta^t u'\left(c^i_{jt}(h_{t-1},B)\right)}{p_{jt}} \leq \mu^i_t(h_{t-1}) + \gamma^i_t(h_{t-1}) \forall j \neq j', t \tag{14}$$

$$\frac{\beta^t u'\left(c^i_{j't}(h_{t-1},B)\right)}{p_{j't}} \leq \gamma^i_t(h_{t-1}) \forall t \tag{15}$$

$$\frac{1}{2}\left(\lambda^i_{t+1}(h_{t-1},B) + \mu^i_{t+1}(h_{t-1},B) + \gamma^i_{t+1}(h_{t-1},B)\right) \leq \gamma^i_t(h_{t-1}) \tag{16}$$

$$\frac{1+i_{t+1}}{2}\left(\lambda^i_{t+1}(h_{t-1},B) + \mu^i_{t+1}(h_{t-1},B) + \gamma^i_{t+1}(h_{t-1},B)\right) \leq \gamma^i_t(h_{t-1}) \tag{17}$$

And, of course, we must also consider the transversality conditions to ensure infinite horizon optimality.[3]

$$\lim_{t\to\infty} \inf \lambda^i_t(h_{t-1}) m^i_t(h_{t-1}) = 0 \text{ for all } i, h_t$$

$$\lim_{t\to\infty} \inf \lambda^i_t(h_{t-1}) b^i_t(h_{t-1}) = 0 \text{ for all } i, h_t$$

Equations 12, 13, 16 and 17 demonstrate that we have an environment where each agent faces a cash-in-advance constraint with probability one half. Here we will investigate whether the Pareto optimum defined in section 1.1 is an equilibrium of the model. As one might expect our results mimic those of the standard cash-in-advance environment studied in Sissoko (forthcoming).

Given the Pareto optimum of section 1.1, it is no surprise that in our environment we find that the Friedman Rule holds and that equilibrium prices are deflationary. (Proofs are in the appendix.)

Proposition 1 *If consumption is Pareto optimal in a fiat money equilibrium, then nominal interest rates are zero at all dates t.*

Proposition 2 *In a Pareto optimal fiat money equilibrium, $p_{jt} = p_t$ for all j, t and $p_{t+1} = \beta p_t$ for all t.*

Let $p^*_t = p_{jt}$ for all j, t and $p^*_{t+1} = \beta p^*_t$ for all t. Then $\{p^*_t\}^\infty_{t=0}$ represents a sequence of prices that is in the set of prices consistent with a Pareto optimal allocation. We can now find the set of government policies that will implement the Pareto optimal allocation given Pareto optimal prices and interest rates. First we demonstrate in two lemmas (i) that government policy completely determines the path of money holdings for every consumer when the allocation is Pareto optimal, and (ii) that this fact will constrain tax policy in a Pareto optimal equilibrium.

[3]Cole and Kocherlakota (1998) demonstrate the sufficiency of these conditions in a standard cash-in-advance environment.

Lemma 1 *In a Pareto optimal equilibrium, money holdings for each type of consumer change only due to taxation.*

Proof. In a Pareto optimal equilibrium $\sum_{j=1}^{n} p_{jt} c_{jt}^{i}(h_t) = np_t^* \frac{y}{n} = p_t^* y$ and no agent will choose to buy or sell bonds, so by equations 5 and 7

$$m_{t+1}^{i}(h_t) = m_t^{i}(h_{t-1}) - \tau_t^{i} \text{ for all } i, t, h_t$$

■

Lemma 2 *In a Pareto optimal equilibrium, at every date t and for every type i money holdings must equal or exceed the expenditure a first sub-period buyer requires in order to purchase the Pareto optimal allocation.*

Proof. To reach the Pareto optimal allocation, it must be the case that the cash constraint, equation 6, does not bind on any consumer at any date t: $m_t^{i}(h_{t-1}) \geq p_t^* y \frac{n-1}{n}$ for all i, t, h_t. This must be true of every consumer, because there is no way of knowing *ex ante* whether the consumer will be a first sub-period buyer or seller in period t. ■

Together lemmas 1 and 2 establish the constraint that, if government policy is to implement the Pareto optimal allocation, then the path of taxation for each type of agent, $i = (j, y)$, must have the property that:

$$m_0^{i} - \sum_{s=0}^{t-1} \tau_s^{i} \geq p_t^* y \frac{n-1}{n} \text{ for all } i, t \tag{18}$$

Thus we find that those who have an endowment of k need more money in every period than those with any other endowment. If the government wishes to have the same tax policy for two agents with different endowment quantities, then the lower endowment agent will have extra cash in every period. But it stands to reason, that an optimizing agent with extra cash in every period will choose to spend some of that extra cash today and thus will not choose the Pareto optimal allocation. This is the logic behind proposition 3.[4]

Proposition 3 *The Pareto optimal allocation cannot be implemented in equilibrium if government policy treats consumers with differing quantities of endowments equally:*
Given a Pareto optimal price sequence, $\{p_t^*\}_{t=0}^{\infty}$, if $m_0^{i} = m_0^{i'}$ and $\tau_t^{i} = \tau_t^{i'}$ for all t and for types $i = \{j, y\}$ and $i' = \{j', y'\}$ where $y \neq y'$, then the equilibrium allocation is not C^*.

[4] Bhattacharya, Haslag and Martin (2005) also find that type-specific lump-sum taxes implement the first-best in several heterogeneous agent environments.

Thus in an environment with an endogenous cash-in-advance constraint, a government policy that implements C^* must treatment agents with low endowments differently from those with high endowments. Since the requisite differential treatment will give agents an incentive to misrepresent their endowments, there is reason to doubt that such a government policy would be successful. As in Sissoko (forthcoming) we use this fact to motivate the exploration of credit-based money. We go beyond the argument in the previous paper by proposing a specific institutional framework that we believe is a starting point for analyzing the monetary role of financial intermediaries.

2 Default Free Intermediaries

In this section of the paper, we consider a different form of money. Here claims drawn on private financial intermediaries take the place of fiat money. The exploration of the coexistence of fiat money and intermediated money will be left to future work.

Let us consider the monetary role of default-free intermediaries.[5] We will demonstrate in this paper that private organizations that are default-free can serve as the infrastructure of a financial/monetary system. The most obvious real world examples of this phenomenon are American Express, Visa and MasterCard – almost all merchants in the United States accept credits in accounts with these financial intermediaries as payment and almost none buy insurance to protect these accounts in case of default. And according to *The 2004 Federal Reserve Payments Study* the value of credit card transactions in the U.S. is currently more than three times the value of ATM withdrawals.

In our model we find that when these default-free intermediaries offer credit lines to the consumers in our economy and play a trigger strategy – withdrawing credit in case of consumer default – the intermediaries make it possible for the economy to reach the first-best, if:

(i) the credit lines are sufficiently large, and

(ii) the consumers in our economy are sufficiently patient.

Consider agents with credit cards. There are competitive financial intermediaries, i.e. Credit Card Companies (CCC), who are default-free and risk neutral.[6] Due to competition (and the fact

[5] Observe that the assumption that intermediaries are default-free is common to papers that study the circulation of private liabilities issued by intermediaries – see, for example, Williamson (1999, 2004) and Bullard and Smith (2003). To our knowledge, only Cavalcanti and Wallace (1999a,b) make the repayment of intermediated private circulating liabilities endogenous.

[6] We call the intermediaries credit card companies only to emphasize that the type of intermediary that we model

that there is no discounting within periods) the CCC allow agents to borrow short-term (i.e. from the first sub-period to the second sub-period of any given period) at an interest rate of zero. The CCC all have policies of offering credit cards with a credit limit of m (denominated in good 1) to agents who have not defaulted in the past, refusing credit cards to agents who have defaulted in the past and sharing costlessly verifiable information on defaulters. The CCC also allow agents to make payments on their cards even if no money is owing. Because at date 0 none of the consumers in the economy has a history, at date 0 all consumers have credit cards.

Since the CCC are default-free every seller in the economy is willing to accept a credit with a CCC as money. Thus, the cash constraint of the fiat money model becomes a credit limit constraint. Furthermore, because the consumers in our economy have been given credit lines, they now choose whether or not to have a negative balance at the end of the period. We use $a_t^i(h_t)$ to represent the credit card balance that a consumer of type i and history, h_t, chooses to hold after all trade in period t has taken place. Then the analog to the fiat money problem, which we will call the credit card problem, is the following:

$$\max_{c_{jt},b_{t+1},a_t} \beta^t \sum_{j=1}^{n} u\left(c_{jt}^i(h_t)\right) + E_t \sum_{s=t+1}^{\infty} \beta^s \left(\frac{1}{2} \sum_{j=1}^{n} u\left(c_{js}^i(h_{s-1},B)\right) + \frac{1}{2} \sum_{j=1}^{n} u\left(c_{js}^i(h_{s-1},S)\right) \right) \tag{19}$$

subject to

$$m \geq \sum_{j \neq j'} \frac{p_{jt}}{p_{1t}} c_{jt}^i(h_{t-1},B) \tag{20}$$

$$\frac{p_{j't}}{p_{1t}} y + (1+i_t) b_t^i(h_{t-1}) + a_t^i(h_t) \geq b_{t+1}^i(h_t) + \sum_{j=1}^{n} \frac{p_{jt}}{p_{1t}} c_{jt}^i(h_t) \tag{21}$$

$$m \geq a_t^i(h_t) \tag{22}$$

While the objective function remains the same, we now have a credit limit constraint on purchases, a budget constraint that includes the possibility of going into debt and a credit limit constraint on the level of second sub-period debt. We find the same first-order conditions for consumption and bonds in the credit-card problem as we found in section one of the paper, equations 11, 13, 14, 15 and 17. Thus, we find that:

(i) $\mu_t^i(h_{t-1}) = 0$ for all i, t, h_t (or the credit limit constraint never binds) when consumption is Pareto optimal

is well established in the real world. Other intermediaries that offer unsecured credit lines to customers will also be consistent with our model.

(ii) $p_{jt} = p_t$ for all t in a Pareto optimal equilibrium, and

(iii) the gross real interest rate in a pareto optimal equilibrium compensates for holding an asset over time, $(1 + i_t)\frac{p_t}{p_{t+1}} = \frac{1}{\beta}$

Definition 4 *A **pareto optimal price system** is a sequence of prices and a sequence of interest rates such that criteria (ii) and (iii) above are met.*

The formulation of the problem above has a defect, however. It implies that, because there is no cost to maximizing $a_t^i(h_t)$, the optimal behavior for an agent is to choose $a_t^i(h_t) = m$ for all i, t, h_t. The problem leaves out the effects of defaulting on short-term debt. We know that whenever $a_T > 0$, the agent has defaulted on his debt at date T and that the discontinuous effects of default must also be incorporated into the problem. Since the intermediaries tell each other about defaults and play a trigger strategy that precludes a defaulter from borrowing in the future, an agent in default must consume his endowment for all $t > T$. Thus if $a_T^i(h_T) > 0$, the one-period utility for an agent of type $i = (j, y)$ at any date $t > T$ is $u^{id} = u(y) + (n-1)u(0)$. We incorporate the consequences of default into our concept of equilibrium as follows:

Definition 5 *An **intermediated equilibrium** is an allocation of goods, $\{c_{jt}^i(h_t)\}$, of credit balances, $\{a_t^i(h_t)\}$, and of bonds, $\{b_t^i(h_{t-1})\}$, and sequences of prices, $\{p_{jt}\}$, and of interest rates, $\{i_t\}$ such that*

(i) given prices and interest rates, the consumer's credit card problem is maximized for consumers of every type i and history, h_t

(ii) markets clear

(a) in the goods market, equations 8 and 9,

(b) $0 = \sum_{i \in I} \sum_{h_t \in H_t} b_t^i(h_t) f_t(i, h_t)$ for all t

(iii) if $a_T^i(h_T) > 0$ for some T, then the agent's one-period utility for all $t > T$ is u^{id}.

The last constraint represents the role played by the trigger strategy of our default-free intermediaries in changing the consumers' incentives to default. Let V^{id} be the continuation value of default for an agent of type i. This is just the utility an agent gets from consuming his endowment forever:

$$V^{id} = \frac{1}{1-\beta} u^{id}$$

Let V^{ic} be the continuation value of consuming the Pareto optimal allocation forever for an agent of type i:

$$V^{ic} = \frac{1}{1-\beta} nu(c^{i*})$$

Now we can show that intermediated money can implement the first-best without type specific policies.

Proposition 4 *If the credit limit constraint does not bind and β is sufficiently high, the pareto optimal allocation is an intermediated equilibrium.*

Proof. Consider the problem solved by an individual agent who faces a pareto optimal price system. An agent who chooses to default at time T, will borrow as much as possible at T and due to the concavity of the utility function will consume equal amounts of all goods at date T. Thus the value of default to an agent of type $i = (j, y)$ is: $nu(\frac{m+y}{n}) + \beta V^{id}$. Since the agent faces pareto optimal prices, his utility when he does not default is: $nu(c^{i*}) + \beta V^{ic}$. Let β^{i*} be the β at which $nu(c^{i*}) + \beta V^{ic} - nu(\frac{m+y}{n}) - \beta V^{id} \geq 0$ holds with equality. Since the left hand side of this equation is strictly increasing in β, for all $\beta \geq \beta^{i*}$ no agent of type i will choose to default in a pareto optimal equilibrium. Let $\beta^* = \max_{i \in I} \beta^{i*}$. Then for all $\beta \geq \beta^*$, no agent will choose to default.

If no agent defaults, the optimal value of $a_t^i(h_t) = 0$ for all i, t, h_t. Then, given a pareto optimal price sequence and a credit limit that is high enough that it never binds, the equilibrium will be pareto optimal. ■

Because the CCC communicate with each other and can force a defaulter into autarky forever, we find that as long as the consumers in this economy are sufficiently patient, they will not choose to default. While the penalty to default that we impose may seem excessive, it is important to note that the results above do not depend on the specific form of the penalty, but only on the existence of a one-period penalty, $u^{id} < u^{i*}$, that is repeated forever. Thus there is no reason to believe that the addition of fiat money to the economy will change the character of the results.

The intermediated environment that we propose in this paper has two important properties: First, unlike the fiat money economy, a Pareto optimum can be reached in the intermediated money environment without type-specific policies – the elastic nature of credit makes it a better solution to our monetary problem.[7] Second, when the credit limit is sufficiently high and our agents are

[7] Williamson (1999, 2004) also finds that the benefits of private money derive from its elasticity. Bullard and

sufficiently patient, the cash constraint and the default penalty are off the equilibrium path and are irrelevant to the equilibrium allocation. Thus when m and β are high enough, the problem becomes identical to the competitive problem without a cash constraint. In short, we model intermediaries as agents who use the fact that they are perceived to be default-free to resolve the liquidity problem in the economy and thereby make it possible for the economy to reach a first-best allocation.

Because credit in our environment solves a liquidity problem, we find that debt can be sustained by nothing more than the threat of losing the right to borrow in the future. This result stands in stark contrast to results of the existing literature on self-enforcing debt contracts – see for example Bulow and Rogoff (1989) and Kehoe and Levine (1993). The different results derive from different assumptions. The existing literature assumes that spot markets work perfectly in the absence of financial intermediation, whereas we assume that liquidity constraints can affect market outcomes and that financial intermediation is needed to make markets work. Because withdrawing credit in our model is equivalent to imposing a liquidity constraint on an agent, it is a severe penalty that is sufficient to support an equilibrium with debt.

That intermediaries are default free serves to emphasize the importance of confidence to a credit-based monetary system. The elastic nature of credit, however, gives it advantages over fiat money that are likely to be robust even when there is some default. The effects of relaxing the assumption that intermediaries are default-free is area for future research that will help us understand the role of confidence in the financial system.

The view of financial intermediation we present in this paper is idealized. The intermediated monetary system functions because the private intermediaries are universally viewed as default free, and thus the debt of intermediaries is universally accepted as a means of exchange. Consumers who make credit card purchases know that in case of default they will face a life-time penalty, and in our idealized world, this is sufficient to induce consumers to pay their debt. This very simple structure of intermediation allows our ideal economy to function as though there were no liquidity constraints. In future work, this first-best financial solution can serve as a baseline to which a more complex view of financial intermediaries can be compared.

Smith (2003) arrive at a similar result, but use the terminology of the real-bills doctrine. Sissoko (2006) also makes this point.

References

[1] Bhattacharya, Joydeep, Joseph Haslag and Antoine Martin (2005). "Heterogeneity, Redistribution and the Friedman Rule," *International Economic Review*, 46(2), pp. 437-54. *Further information in IDEAS/RePEc*

[2] Brock, William (1975). "A Simple Perfect Foresight Monetary Model," *Journal of Monetary Economics*, 1, pp. 133-50. *Further information in IDEAS/RePEc*

[3] Bullard, James and Bruce Smith (2003). "The Value of Inside and Outside Money," *Journal of Monetary Economics*, 50, pp. 398-417. *Further information in IDEAS/RePEc*

[4] Bulow, Jeremy and Kenneth Rogoff (1989). "Sovereign Debt: Is to Forgive to Forget?," *American Economic Review*, 79, pp. 43-50. *Further information in IDEAS/RePEc*

[5] Cavalcanti, Ricardo and Neil Wallace (1999a). "Inside and Outside Money as Alternative Media of Exchange," *Journal of Money, Credit and Banking*, 31(3), part 2, pp. 443-57. *Further information in IDEAS/RePEc*

[6] Cavalcanti, Ricardo and Neil Wallace (1999b). "A Model of Private Bank-Note Issue," *Review of Economic Dynamics*, 2(1), pp. 104-36. *Further information in IDEAS/RePEc*

[7] Cole, Harold and Narayana Kocherlakota (1998). "Zero Nominal Interest Rates," *Federal Reserve Bank of Minneapolis Quarterly Review*, 22(2), pp. 2-10. *Further information in IDEAS/RePEc*

[8] Federal Reserve System (2004). *The 2004 Federal Reserve Payments Study.* *Further information*

[9] Kehoe, Timothy and David Levine (1993). "Debt-Constrained Asset Markets," *Review of Economic Studies*, 60, pp. 865-888. *Further information in IDEAS/RePEc*

[10] Ljungqvist, Lars and Thomas Sargent (2000). *Recursive Macroeconomic Theory*, Cambridge, MA, MIT Press. *Further information*

[11] Sissoko, Carolyn (2002). "Short-Term Credit: A Monetary Channel Linking Finance to Growth," Occidental College Working Paper. *Further information in IDEAS/RePEc*

[12] Sissoko, Carolyn (2007). "A Model of Money," Occidental College Working Paper.

[13] Sissoko, Carolyn (forthcoming). "Why Inside Money Matters," *Journal of Money, Credit and Banking*.

[14] Wallace, Neil (1998). “A Dictum for Monetary Theory,” *Federal Reserve Bank of Minneapolis Quarterly Review*, 22(1), pp. 20-26. *Further information in IDEAS/RePEc*

[15] Williamson, Stephen (1999). "Private Money," *Journal of Money, Credit and Banking*, 31(3), pp. 469-491. *Further information in IDEAS/RePEc*

[16] Williamson, Stephen (2004). "Limited Participation, Private Money and Credit in a Spatial Model of Money," *Economic Theory*, 24(4), pp. 857-75. *Further information in IDEAS/RePEc*

Appendices

A Proof of Proposition 1

Proposition 6 *If consumption is Pareto optimal in a fiat money equilibrium, then nominal interest rates are zero at all dates t.*

Proof. Assume $c^i_{jt}(h_t) = \frac{y}{n}$ for all i, j, t, h_t. Then equations 11, 14 and 15 hold with equality, $p_{jt} > 0$ for all j, t, and:

$$\lambda^i_t(h_{t-1}) = \frac{\beta^t u'(c^{i*})}{p_{jt}} = \mu^i_t(h_{t-1}) + \gamma^i_t(h_{t-1}) = \gamma^i_t(h_{t-1}) \text{ for all } i, t, h_t$$

Therefore: $\mu^i_t(h_{t-1}) = 0$ for all i, t, h_t.

Observe that by equation 6 Pareto optimal consumption is only possible if $m^i_{t+1}(h_t) > 0$ for all i, t, h_t. Then equations 12 and 16 also hold with equality. When we combine equations 12 and 13 we find that $1 + i_{t+1} \leq 1$ for all t. However, as bonds are denominated in money, a negative interest rate would create an arbitrage opportunity, and in any equilibrium, $i_t \geq 0$ for all t. Conclusion: $i_t = 0$ for all t. ■

B Proof of Proposition 2

Proposition 7 *In a Pareto optimal fiat money equilibrium, $p_{jt} = p_t$ for all j, t and $p_{t+1} = \beta p_t$ for all t.*

Proof. Assume that $p_{jt} > p_{j't}$. Then by equation 11 (which holds with equality) we know that the consumption of first sub-period sellers is skewed towards good j': $c^i_{j't}(h_{t-1}, S) > c^i_{jt}(h_{t-1}, S)$ for all i, t, h_{t-1}. So the consumption allocation is not Pareto optimal. Conclusion: in a Pareto optimal equilibrium, $p_{jt} = p_t$ for all j, t.

Dividing equation 11 by a lagged version of the same equation, we find:

$$\frac{p_{t+1}}{\beta p_t} = \frac{\lambda^i_t(h_{t-1})}{\lambda^i_{t+1}(h_t)} \text{ for all } i, t, h_t$$

But, given a Pareto optimal allocation, equations 11, 14, 15 and 16 hold with equality and:

$$\lambda^i_t(h_{t-1}) = \mu^i_t(h_{t-1}) + \gamma^i_t(h_{t-1}) = \gamma^i_t(h_{t-1}) = \lambda^i_{t+1}(h_t) \text{ for all } i, t, h_t$$

Conclusion: $p_{t+1} = \beta p_t$ for all t. ■

C Proof of Proposition 3

Proposition 8 *The Pareto optimal allocation cannot be implemented in equilibrium if government policy treats consumers with differing quantities of endowments equally:*
Given a Pareto optimal price sequence, $\{p_t^*\}_{t=0}^{\infty}$*, if* $m_0^i = m_0^{i'}$ *and* $\tau_t^i = \tau_t^{i'}$ *for all* t *and for types* $i = \{j, y\}$ *and* $i' = \{j', y'\}$ *where* $y \neq y'$*, then the equilibrium allocation is not* C^**.*

Proof. Without loss of generality assume $y > y'$. By lemmas 1 and 2 we know that m_0^i and $\{\tau_t^i\}_{t=0}^{\infty}$ must be chosen such that equation 18 holds for all t.

But now consider the behavior of an agent of type i'. This agent knows that at every date t, $m_0^{i'} - \sum_{s=0}^{t-1} \tau_s^{i'} \geq p_t^* y \frac{n-1}{n} > p_t^* y' \frac{n-1}{n}$. Thus this agent knows that if he chooses to carry $m_{t+1} < m_0^{i'} - \sum_{s=0}^{t-1} \tau_s^{i'}$ into the next period, he will still be able to consume c_s^{i*} for all $s > t$. So there is some ε such that at date t he can spend some of his excess money and consume $c_{jt}^i(h_t) = \frac{y+\varepsilon}{n}$ for all j and c_s^{i*} for all $s > t$. Since this allocation is both affordable and preferred by an agent of type i', C^* can not be an equilibrium allocation. ■