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The Delimitation of Giffenity for The Wold-Juréen (1953) Utility Function Using Relative Prices: A Note

Summary. The Giffen good issue comes up in microeconomic theory courses, and it has been addressed by several economic theorists over the years. That is, assuming that the good is not ostentatious, contrary to received wisdom, the demand for some goods increases when their price increase. This phenomenon has piqued the interest of economist since its introduction around 1884, and the instant paper under review is no exception. In that sense, the research is relevant.

Major points

1. Pg. 5, **Lemma 1**, Proof. The paper is silent on the divisibility of x_1 and x_2. Thus, the strict inequalities imposed on x_1 and x_2 implies that each good MUST be purchased in amounts that admit a very small increment ϵ near the boundaries. This raises an issue for the inequality $m < p_1 + 2p_2$. The latter implies that our DM cannot afford to buy 1-unit of Good 1 at price p_1 and 2-units of Good 2 at price p_2. Presumably, for other price combinations we have $x_1p_1 + x_2p_2 \leq m$ in order to satisfy the DM's budget constraint? If so, then the strict inequality has the following implications for small increments ϵ.

a) Near 0, for $x_2 = \epsilon$ and $x_1 > 1$ we have $x_1p_1 + \epsilon p_2 \leq m$. This implies
$$1 < x_1 \leq \frac{m - \epsilon p_2}{p_1}$$

b) Near 1, for $x_1 = 1 + \epsilon$ and $x_2 \leq (m - (1+\epsilon)p_1)/p_2$, the latter inequality implies that for $0 < x_2 < 2$ we have
$$\epsilon < x_2 < \min\left\{\frac{m - (1+\epsilon)p_1}{p_2}, 2\right\}$$

2. Page 6, **Proposition 3**. The above inequalities, in a), brings us to the following TE relationship:

$$\frac{\partial x_1}{\partial p_1} = -\left(\frac{m - \epsilon p_2}{p_1^2}\right) \qquad \text{(A)}$$

Evidently, $\frac{\partial x_1}{\partial p_1} < 0$ in (A) because $m - \epsilon p_2 > 0$ for small ϵ. However, (A) also implies that $\frac{\partial x_1}{\partial p_1} \geq 0$ for $\epsilon \geq \frac{m}{p_2}$. This is a requirement for Giffen goods (Silberberg & Walker, 1984; Jehle and Reny, 2011). The latter inequality requires that the price of Good 2 must bee much greater than the DM's income, i.e., $p_2 \gg m$. This implies that if x_2 is not divisible our DM cannot afford it at all and Giffenity cannot hold. If it is divisible, then it leads to an awkward situation in which the price of the good is much larger than the DMs income in order for Giffenity to hold. By virtue of admitting differential calculus we are implicitly assuming that the goods are divisible. All that to say, the sign of TE is not ambiguous but rather for Giffenity to hold in the Wold-Juréen utility function we have to accept conditions like $p_2 \gg m$ as absurd as it may be.

Conclusion

The author(s) underlying proposition that in order for Giffenity to hold the price of Good 1 must be greater than the price of Good 2. In other words, $p_1 > p_2 \gg m$ in the context of the major points above. This implies that the price of each good must be much greater than the DM's income for Giffenity to hold. It is not clear whether the author9s) contemplated this conundrum which is fundamentally different from the notion that the sign of the TE factor is ambiguous. Comparative statics in the major points above show that it is not ambiguous.

References

Jehle, G. A., & Reny, P. J. (2011). Advanced Microeconomic Theory. 3rd ed. Pearson Education, Ltd

Silberberg, E., & Walker, D. A. (1984). A modern analysis of Giffen's paradox. International Economic Review, 687-694.