The Macroeconomic Consequences of Artificial Intelligence: A Theoretical Framework

Xu Huang，Yan Hu and Zhiqiang Dong

Abstract: Artificial intelligence is different from previous technological changes. It can improve capital and labor productivity at the same time, and has a strong spillover effect on other industries. The authors construct a theoretical model to distinguish between complementary and alternative artificial intelligence. It is found that in the case of alternative artificial intelligence, if the artificial intelligence technology enhances the laborer's effect more than the capital, the labor share increases and the capital share decreases. Conversely, if the capital enhancement effect is greater than the labor force, the labor share decreases and the capital share rises. In the case of complementary artificial intelligence, if the reinforcing effect on the laborer is greater than the capital, the labor share is decreased and the capital share is increased; otherwise, the opposite is true. Further, the authors construct a two-sector model to examine the impact of the development of artificial intelligence technology on the transformation and upgrading of industrial structure. When the production products of the two sectors are replaced by each other, if the substitution elasticity of sector 1 is greater than that of sector 2, and the net capital enhancement effect of artificial intelligence technology on department 1 is greater than that of sector 2, the development of artificial intelligence technology leads to capital inflow into capital-intensive enterprises. The labor force has similar results.

JEL J23 J24

Keywords: artificial intelligence, automation, economic growth, the share of labor.

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The authors would like to acknowledge the financial support from Ministry of Education Humanities and Social Sciences Planning Fund Project (No. 14YJC790109); and National Natural Science Foundation Project (No. 71473089).

Citation Xu Huang, Yan Hu, and Zhiqiang Dong (2019). The macroeconomic consequences of artificial intelligence: a theoretical framework. Economics Discussion Papers, No 2019-48, Kiel Institute for the World Economy.

http://www.economics-ejournal.org/economics/discussionpapers/2019-48

1 Introduction

Artificial intelligence is manually designed system which possesses the ability of autonomous perception and decision making based on artificially designed algorithms and data. Artificial intelligence can help or even substitute humans to finish the work that required human intelligence previously. Artificial intelligence is different from previous technological changes. It can improve capital and labor productivity at the same time, and has a strong spillover effect on other industries. Automation is the process that machine or system achieve the desired goals set by humans through automatic detection, information processing, analysis and judgement, and control, with little human involvement. Therefore, automation is a kind of basic subjects, of which artificial intelligence is a branch. Indeed, few scholars distinguish between artificial intelligence and automation strictly, most studies regard them as substitutes for each other.

In terms of the relation with labor force, artificial intelligence can be classified into two categories: alternative and complementary. On the one hand, artificial intelligence has begun to replace many jobs and this trend is expected to continue (Frey and Osborne, 2017; Arntz et al., 2016; Acemoglu and Restrepo, 2017). For example, autonomous vehicles can speed transportation up, 3D printers can produce highly specialized products. On the other hand, artificial intelligence can assist humans in improving work efficiency. For example, Watson, a kind of AI product developed by IBM, incorporates more than 300 medical journals and various clinical guidelines, which can help doctors to make diagnoses and finish detailed reports.

There are two ways to study the theoretical effects of artificial intelligence on economic growth, and artificial intelligence is incorporated into the production function as a new factor in both ways. One is to improve the neoclassical production function (Hanson, 2001; Autor et al., 2013; Prettner 2017a, 2017b; Lankisch et al., 2019; Decanio, 2016), and the second is to extend the task-based model (Zeira, 1998; Acemoglu and Autor, 2011; Acemoglu and Restrepo, 2018a-f; Acemoglu and Restrepo, 2019).

Improve the neoclassical production function

The most important feature of the neoclassical production function is the incorporation of the elasticity of substitution between labor and capital. Hanson (2001) improves the Cobb-Douglas production function and finds that computer capital and labor complement each other while machine intelligence has a displacement effect on labor, and the wide use of smart machines strongly stimulates economic growth. However, this model does not consider the creation of new jobs, therefore underestimate the impact of smart machines on the economy. Aghion et al. (2017) improves the CES production function and analyzes the changes in the share of automated goods by exploring the changes in the elasticity of substitution. The model explains the Baumol's Cost Disease, which indicates that when the productivity of automation sector improves, its share in GDP declines. It is shown that economic growth is constrained to its weakness rather than advantages.

Some studies combine the Cobb-Douglas function with the CES function to explain labor market polarization, wage inequality and unemployment (Autor et al., 2013; Lankisch et al., 2019). Some others adopt the two-level nested CES production function to discuss the effects of economic growth on wages and labor structure (Decaio, 2016), which no longer focuses on the substitution of artificial intelligence capital for a production factor only, but also pays attention to the elasticity of substitution between artificial intelligence capital and two production factors.

Recent literatures embed the Overlapping Generational Model (OLG) in neoclassical production functions, thus the studies on artificial intelligence move into the dynamic era (Benzell et al., 2015; Gasteiger and Prettner, 2017; Prettner and Strulik, 2017; Sachs et al., 2015). The most important feature of the OLG model is that it is iterative indefinitely, so it can be adopted to explore the impact of artificial intelligence on the economy in the long run. A common conclusion of the above literatures is that artificial intelligence slows down economic growth and reduces social welfare in the long run. For one reason, those models generally ignore the impact of newly created tasks on the economy, only consider the displacement effect of artificial intelligence for low-skilled labor and the complementary effect for high-skilled one. For another, the displacement effect of artificial intelligence for labor reduces wages and thus savings. Consequently, corporate investment which in fact comes from savings drops, and finally economic slowdown appears. In addition, artificial intelligence decreases both the total wages and the the share of labor, however, it increases both the return on capital and the capital share of income, and the inequality is exacerbated.

In summary, improving the neoclassical production function relies on the introduction of the elasticity of substitution to connect technology with employment. Employing the Cobb-Douglas production function or the CES production function to improve the neoclassical production function is simple and convenient, however, such approach is not general enough and ignores the traditional capital. By contrast, adopting the two-level nested CES production function is more general but more complicated. Furthermore, this approach can only perform static analysis, because there is often no dynamic equilibrium solution without the help of numerical simulation. The simultaneous adoption of both the Cobb-Douglas and the CES production function can take many determinants into consideration. It is also convenient to obtain the equilibrium solution and analyze long-term economic growth.

Task-based Model

Task-based model shows that automation takes place of existing tasks and creates new tasks, so that the change in job nature is reflected. Zeira (1998) is a pioneer in developing the task-based model. He divides work into tasks and employ intervals to denote tasks. Recent studies extend Zeira’s model to explore the effect of artificial intelligence on economic growth (Acemoglu and Autor, 2011; Acemoglu and Restrepo, 2018a-f; Acemoglu and Restrepo, 2019). Although the models become more and more general, the main results stay unchanged. It is found that artificial intelligence has a displacement effect on labor, but it also brings four positive effects, which include productivity effect, capital accumulation, the deepening of automation deepening and the creation of new tasks. Among those four, the creation of new tasks has the most important effect, because humans have comparative advantages in performing new tasks. Automation reduces the labor share, but the new task increases it. Although the creation of new tasks can offset some negative effects of automation, the adjustment process in labor market is slow and painful. Because searching for new jobs takes time, and labor force need to develop new skills to adapt to new jobs. Moreover, the imperfection of labor market results in frictions and mismatches.

The theory developed by Acemoglu and Restrepo(2018a-f;2019) extends the research on economic growth theory and lays a foundation for later research on the impact of artificial intelligence on the economy. However, there is no strict definitions of “task” and no explanations on how new tasks are created. Additionally, the task-based model only considers the impact of automation capital on the economy, but ignores the traditional capital. Furthermore, the model focuses on the substitution between different tasks and simply assumes that capital substitutes labor perfectly. In fact, capital is not perfect substitute for labor in some given task, but if both the elasticity of substitution between different tasks and that between capital and labor are considered, the model will become too complicated. In order to overcome the above difficulties, some scholars introduce traditional capital into the task-based model, or combine the two types of models to study the impact of labor market polarization and taxing robots on the economy (Guerreiro et al., 2017).

By contrast, the neoclassical production function mainly focuses on the substitution between artificial intelligence capital and labor, while the task-based model emphasizes the substitution between tasks. The task-based model is more able to reflect the change in job nature, while the improved neoclassical production function can examine traditional capital and artificial intelligence capital simultaneously. The current trend is to combine these two types of models and develop new production function which is closer to reality.

It is worth noting that none of the above studies distinguishes between complementary artificial intelligence and alternative artificial intelligence, thus none discusses the different impacts on the economy. Based on the previous work (Acemoglu, 2018a-f, 2019; Aghion, 2017, Autor, 2013; Prettner, 2017; Lankisch et al., 2019), we for the first time introduce the elasticity of substitution, differentiate between complementary artificial intelligence and alternative artificial intelligence to study the impacts on the economy, and present a general framework in theory.

The authors construct a theoretical model that distinguishes between complementary and alternative artificial intelligence. It is found that in the case of alternative artificial intelligence, if the artificial intelligence technology enhances the laborer's effect more than the capital, the labor share increases and the capital share decreases. Conversely, if the artificial intelligence enhances the capital more than the labor, the labor share decreases, and the capital share has risen. In the case of complementary artificial intelligence, if the reinforcing effect on the laborer is greater than the capital, the labor share decreases and the capital share rises. Conversely, if the reinforcing effect on the capital is greater than the labor, the labor share increases and the capital share decreases. The wages and capital prices of workers are affected by three effects at the same time, and they need to meet certain conditions before they rise.

Further, the authors construct a two-sector model to examine the impact of the development of artificial intelligence technology on the transformation and upgrading of industrial structure. When the production products of the two sectors are replaced by each other, if the substitution elasticity of sector 1 is greater than that of sector 2, and the net capital enhancement effect of artificial intelligence technology on sector 1 is greater than that of sector 2, the development of artificial intelligence technology leads to capital inflow into capital-intensive enterprises. If the development of artificial intelligence technology has a greater net labor force enhancement effect on sector 1 than sector 2, the development of artificial intelligence technology leads to labor inflow into capital-intensive enterprises. In the case of complementary products in the two sectors, the above conclusions are reversed.

2 Single Sector Model

We consider a setting with only one enterprise in a closed economy. All workforce is homogeneous and fully employed. There are two types of artificial intelligence. One is complementary to labor, and the other is a substitute for labor.

The production function of the enterprise is

$$Y = [A_K f(A_I)K]^{\alpha}\{[(A_L g(A_I)L)^{\frac{\sigma-1}{\sigma}} + (A_P h(A_I)P)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}\}^{1-\alpha} \tag{1}$$

where K is traditional capital, L is labor, and P is artificial intelligence capital. σ is the elasticity of substitution between labor and artificial intelligence capital, and $\sigma \geq 0$ .When $\sigma = 0$, $Y = [A_K f(A_I)K]^{\alpha} \min\{A_L g(A_I)L, A_P h(A_I)P\}$, when $0 < \sigma < 1$, artificial intelligence capital is complementary to labor. when $\sigma = 1$, $Y = [A_K f(A_I)K]^{\alpha}[A_L g(A_I)L]^{\beta}[A_P h(A_I)P]^{1-\alpha-\beta}$, when $\sigma > 1$, labor and artificial intelligence capital substitute each other, when $\sigma \to +\infty$, $Y = [A_K f(A_I)K]^{\alpha}[A_L g(A_I)L + A_P h(A_I)P]^{1-\alpha}$. A_K is the efficiency of traditional capital production. A_L is the productivity of workers, or artificial intelligence labor-augmenting technology. A_P is the production efficiency of artificial intelligence capital, or artificial intelligence capital-augmenting technology. A_I is artificial intelligence technology. $f(A_I) = e^{m_1 A_I}$ represents the spillover effect of artificial intelligence technology on traditional production methods. $g(A_I) = e^{n_1 A_I}$ indicates the enhanced effect of artificial intelligence technology on labor productivity. $h(A_I) = e^{q_1 A_I}$ indicates the enhanced effect of artificial intelligence technology on capital production efficiency, At the same time, $f(A_I), g(A_I), h(A_I)$ indicate that artificial intelligence technology has improved its production efficiency through self-learning. $f(A_I), g(A_I), h(A_I)$ take the form of exponential function to show that artificial intelligence technology can greatly improve the productivity of laborers and capital, and has a strong spillover effect, which is the difference between artificial intelligence technology and previous technological changes. Define $Z_{A_I} = (A_L g(A_I)L)^{\frac{\sigma-1}{\sigma}} + (A_P h(A_I)P)^{\frac{\sigma-1}{\sigma}}$.

Suppose the enterprise makes profits π, the worker's wage is ω, the rent of traditional capital is R_K , and the rent of artificial intelligence capital is R_P , thus

$$\pi = Y - \omega L - R_K K - R_P P \ .$$

The enterprise maximizes profits, so
the worker's wage is

(2) $$\omega = \frac{\partial Y}{\partial L} = (1-\alpha)(A_K e^{m_1 A_I} K)^{\alpha} (A_L e^{n_1 A_I})^{\frac{\sigma-1}{\sigma}} L^{-\frac{1}{\sigma}} Z_{A_I}^{\frac{1-\sigma\alpha}{\sigma-1}},$$

the rent of traditional capital is

(3) $$R_K = \frac{\partial Y}{\partial K} = \alpha (A_K e^{m_1 A_I})^{\alpha} K^{\alpha-1} Z_{A_I}^{\frac{\sigma(1-\alpha)}{\sigma-1}},$$

the rent of artificial intelligence capital is

(4) $$R_P = \frac{\partial Y}{\partial P} = (1-\alpha)(A_K e^{m_1 A_I} K)^{\alpha} (A_P e^{q_1 A_I})^{\frac{\sigma-1}{\sigma}} P^{-\frac{1}{\sigma}} Z_{A_I}^{\frac{1-\sigma\alpha}{\sigma-1}}.$$

The no arbitrage condition is $R_K = R_P$, hence

(5) $$K = \frac{\alpha}{1-\alpha} (A_P e^{q_1 A_I})^{\frac{\sigma-1}{\sigma}} P^{\frac{1}{\sigma}} Z_{A_I}.$$

Substitute the Eq. (5) into the Eq. (2), (3) and (4), we have

(6) $$\omega = aL^{-\frac{1}{\sigma}} P^{\frac{\alpha}{\sigma}} (A_K e^{m_1 A_I})^{\alpha} (A_L e^{n_1 A_I})^{\frac{\sigma-1}{\sigma}} (A_P e^{q_1 A_I})^{\frac{(1-\sigma)\alpha}{\sigma}} Z_{A_I}^{\frac{1-\alpha}{\sigma-1}},$$

where $a = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. Thus,

(7) $$R = R_K = R_P = aP^{\frac{\alpha-1}{\sigma}} (A_K e^{m_1 A_I})^{\alpha} (A_P e^{q_1 A_I})^{\frac{(\sigma-1)(1-\alpha)}{\sigma}} Z_{A_I}^{\frac{1-\alpha}{\sigma-1}}.$$

the share of labor is

(8) $$\alpha_L = \frac{\omega L}{Y} = (1-\alpha) \frac{(A_L e^{n_1 A_I} L)^{\frac{\sigma-1}{\sigma}}}{(A_L e^{n_1 A_I} L)^{\frac{\sigma-1}{\sigma}} + (A_P e^{q_1 A_I} P)^{\frac{\sigma-1}{\sigma}}}.$$

the share of artificial intelligence capital is

(9) $$\alpha_P = \frac{RP}{Y} = (1-\alpha) \frac{(A_P e^{q_1 A_I} P)^{\frac{\sigma-1}{\sigma}}}{(A_L e^{n_1 A_I} L)^{\frac{\sigma-1}{\sigma}} + (A_P e^{q_1 A_I} P)^{\frac{\sigma-1}{\sigma}}}.$$

Proposition 1 (i) When $0 < \sigma < 1$, that is, artificial intelligence capital is complementary to labor, if the condition $\alpha_L > 1-\sigma$ is satisfied, artificial intelligence technology increases wages. When $\sigma > 1$, that is, artificial intelligence capital and labor substitute each other, if the condition $0 < \alpha_L < 1-\alpha\sigma$ is satisfied, artificial intelligence capital-augmenting technology increases wages.

Proposition 1 (ii) When $\sigma > 1$, that is, artificial intelligence capital and labor substitute each other, artificial intelligence technology raises the price of capital. When $0 < \sigma < 1$, that is, artificial intelligence capital is complementary to labor, if the condition $\alpha_P > (1-\sigma)(1-\alpha)$ or

$\alpha_L < \sigma(1-\alpha)$ is satisfied, artificial intelligence technology raises the price of capital .

Proposition 1 (iii)

(10) $$\frac{\partial \ln \alpha_L}{\partial A_I} > 0 \Leftrightarrow (\sigma-1)(n_1-q_1) > 0, \frac{\partial \ln \alpha_L}{\partial A_I} < 0 \Leftrightarrow (\sigma-1)(n_1-q_1) < 0$$

(11) $$\frac{\partial \ln \alpha_P}{\partial A_I} > 0 \Leftrightarrow (\sigma-1)(n_1-q_1) < 0, \frac{\partial \ln \alpha_L}{\partial A_I} < 0 \Leftrightarrow (\sigma-1)(n_1-q_1) > 0$$

Proof of Proposition 1:

(i) by (6)

$$\ln \omega = \alpha m_1 A_I + \frac{\sigma-1}{\sigma} n_1 A_I + \frac{(1-\sigma)\alpha}{\sigma} q_1 A_I + \frac{1-\alpha}{\sigma-1} \ln Z_{A_I} + \ln(aL^{-\frac{1}{\sigma}})$$

$$+\frac{\alpha}{\sigma}\ln P + \alpha \ln A_K + \frac{\sigma-1}{\sigma}\ln A_L + \frac{(1-\sigma)\alpha}{\sigma}\ln A_P,$$

thereby

(12) $$\frac{\partial \ln \omega}{\partial A_I} = \alpha m_1 + \frac{\sigma-1+\alpha_L}{\sigma} n_1 + \frac{(1-\sigma)\alpha+\alpha_P}{\sigma} q_1,$$

Similarly

(13) $$\frac{\partial \ln R}{\partial A_I} = \alpha m_1 + \frac{\alpha_L}{\sigma} n_1 + \frac{(\sigma-1)(1-\alpha)+\alpha_P}{\sigma} q_1,$$

By(12), When $0<\sigma<1$, $\alpha m_1 > 0,$ $\frac{(1-\sigma)\alpha+\alpha_P}{\sigma} q_1 > 0$,if $\frac{\sigma-1+\alpha_L}{\sigma} n_1 > 0$,ie $\alpha_L > 1-\sigma$,then $\frac{\partial \ln \omega}{\partial A_I} > 0$. When $\sigma > 1$, $\alpha m_1 > 0,$ $\frac{\sigma-1+\alpha_L}{\sigma} n_1 > 0$,if $\frac{(1-\sigma)\alpha+\alpha_P}{\sigma} q_1 > 0$,ie $\alpha_P > \alpha(\sigma-1)$,then $\frac{\partial \ln \omega}{\partial A_I} > 0$ .Because $\alpha_L + \alpha_P = 1-\alpha$,thus $0 < \alpha_L < 1-\alpha\sigma$.

（ii） By(12), When $0<\sigma<1$, $\alpha m_1 > 0,$ $\frac{\alpha_L}{\sigma} n_1 > 0$,if $\frac{(\sigma-1)(1-\alpha)+\alpha_P}{\sigma} q_1 > 0$,ie $\alpha_P > (1-\sigma)(1-\alpha)$,then $\frac{\partial \ln R}{\partial A_I} > 0$. When $\sigma > 1$, $\frac{\partial \ln \omega}{\partial A_I} > 0$.

(iii)By (8), (9),

$$\frac{\partial \ln \alpha_L}{\partial A_I} = -\frac{\partial \ln \alpha_P}{\partial A_I} = \frac{(\sigma-1)}{\sigma} \bullet \frac{(n_1-q_1)(A_P e^{q_1 A_I} P)^{\frac{\sigma-1}{\sigma}}}{(A_L e^{n_1 A_I} L)^{\frac{\sigma-1}{\sigma}} + (A_P e^{q_1 A_I} P)^{\frac{\sigma-1}{\sigma}}}.$$

The economic intuition of Proposition One is that the share of labor is related to the elasticity of substitution and the effect of reinforcement. When $\sigma > 1$, that is, in the alternative artificial intelligence, the artificial intelligence enhances the labor force more than the capital, the labor share will increase，the capital share will decrease. Conversely, when the artificial intelligence enhances the capital more than the labor, the labor share will decrease, the capital share will increase. When $0 < \sigma < 1$, that is, in the complementary artificial intelligence, the artificial intelligence enhances the labor force more than the capital, the labor share will decrease, the capital share will increase. Conversely, when the artificial intelligence enhances the capital more than the labor, the labor share will increase, the capital share will decrease.

From (12) and (13), wages and capital prices are simultaneously affected by three enhancement effects. Wages and capital prices do not necessarily increase with the improvement of artificial intelligence technology, and certain conditions must be met. When artificial intelligence is complementary, artificial intelligence technology will increase wages when the labor share is greater than $1\text{-}\sigma$. When the labor share is less than $\sigma(1-\alpha)$, artificial intelligence technology will increase the capital price. When artificial intelligence is alternative, artificial intelligence technology will always increase the price of capital. When the labor share is less than $1-\alpha\sigma$, artificial intelligence technology will increase wages.

Special case

Next, we want to focus on the impact of substitution elasticity on wages, capital prices, labor share, and capital share, so assume $m_1 = m_2 = m_3 = 0$. The production function of the enterprise is

(14) $$Y = (A_K K)^{\alpha}\{[(A_L L)^{\frac{\sigma-1}{\sigma}} + (A_P P)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}\}^{1-\alpha} .$$

This assumption is equivalent to artificial intelligence technology that only increases the productivity of artificial intelligence capital.

(i)The production function of the enterprise is

$$Y = K^{\alpha}(L^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma(1-\alpha)}{\sigma-1}} ,$$

Define $Z = L^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}$.

Similarly, apply the no arbitrage condition, we have

(15) $$K = \frac{\alpha}{1-\alpha} ZP^{\frac{1}{\sigma}} .$$

Substitute the Eq. (5) into the Eq. (2), (3) and (4), we have

(16) $$\omega = aL^{-\frac{1}{\sigma}} Z^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha}{\sigma}} ,$$

Thus,

(17) $$R = R_K = R_P = aZ^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha-1}{\sigma}} .$$

Proposition 2 (i) The accumulation of artificial intelligence capital increases wages and lowers capital rent.

Proposition 2 (ii) The share of traditional capital stays unchanged at α .When $0<\sigma<1$, artificial intelligence capital is complementary to labor, it increases the share of labor but reduces the share of artificial intelligence capital. When $\sigma>1$, labor and artificial intelligence capital substitute each other, artificial intelligence capital lowers the the share of labor but promotes the share of artificial intelligence capital. When $\sigma=1$, both the share of labor and the share of artificial intelligence capital remain fixed.

All proofs are contained in the appendix.

The economic intuition of Proposition 2 is that if unemployment is ignored, artificial intelligence capital lowers the price of capital, so workers are relatively scarcer and wages are raised. When artificial intelligence capital is complementary to labor, automation assists in human working and improves productivity. Therefore, artificial intelligence capital is beneficial to labor by improving the the share of labor. When artificial intelligence capital and labor substitute each other, competition is formed. Artificial intelligence capital reduces the demand for labor, and lowers the the share of labor, thus is to the disadvantages of labor. In fact, technological progress does not always bring benefits. Take the first industrial revolution as an example, the British worker suffered and even started to smash machines during the Luddite movement. This indicated that in order to eliminate the negative effect of technological advancement, countries can choose to support the develop of complementary artificial intelligence, providing protection for labor. It is worth noting that the above conclusions are obtained without considering unemployment. If some industries have large demand for labor, especially low-skilled labor, it is advantageous to develop alternative artificial intelligence, which can make up the shortfall in labor supply.

(ii)When capital is used to improve the level of artificial intelligence capital-augmenting technology

Suppose that capital is invested to improve the level of artificial intelligence and the production function is

(18) $$Y=K^{\alpha}[L^{\frac{\sigma-1}{\sigma}}+(A_P P)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma(1-\alpha)}{\sigma-1}},$$

where A_P is artificial intelligence capital-augmenting technology. Define $Z_P=L^{\frac{\sigma-1}{\sigma}}+(A_P P)^{\frac{\sigma-1}{\sigma}}$.

Similarly, apply the no arbitrage condition, we have

(19) $$K=\frac{\alpha}{1-\alpha}A_P^{\frac{1-\sigma}{\sigma}}Z_P P^{\frac{1}{\sigma}},$$

(20) $$\omega=aA_P^{\frac{(1-\sigma)\alpha}{\sigma}}L^{-\frac{1}{\sigma}}Z_P^{\frac{1-\alpha}{\sigma-1}}P^{\frac{\alpha}{\sigma}},$$

(21) $$R=R_K=R_P=aA_P^{\frac{(1-\sigma)(\alpha-1)}{\sigma}}Z_P^{\frac{1-\alpha}{\sigma-1}}P^{\frac{\alpha-1}{\sigma}}.$$

Proposition 3(i) When $0<\sigma<1$, that is, artificial intelligence capital is complementary to labor, artificial intelligence capital-augmenting technology increases wages. When $\sigma > \frac{1}{\alpha} > 1$, that is, artificial intelligence capital and labor substitute each other, artificial intelligence capital-augmenting technology lowers wages.

Proposition 3 (ii) When $\sigma > 1$, that is, artificial intelligence capital and labor substitute each other, artificial intelligence capital-augmenting technology raises the price of capital. When $0 < \sigma < 1$, that is, artificial intelligence capital is complementary to labor, the relation between artificial intelligence capital-augmenting technology and the price of capital is uncertain.

Proposition 3 (iii) When $0 < \sigma < 1$, that is, artificial intelligence capital is complementary to labor, artificial intelligence capital-augmenting technology improves the labor share but reduces the share of artificial intelligence capital, and the share of traditional capital remains unchanged. When $\sigma > 1$, that is, artificial intelligence capital and labor substitute each other, artificial intelligence capital-augmenting technology reduces the labor share but improves the share of artificial intelligence capital, and the share of traditional capital remains unchanged. When $\sigma = 1$, both the labor share and the share of artificial intelligence capital are fixed.

All proofs are contained in the appendix.

The economic intuition of Proposition 3 is that while ignoring unemployment and assuming full employment of labor, the impacts of artificial intelligence capital-augmenting technology on wages and labor share depend on whether it is complementary or alternative. If artificial intelligence capital is complementary to labor, it is good for wages and labor share. If artificial intelligence capital and labor substitute each other, artificial intelligence capital-augmenting technology worsens the competitive situation of labor, as more workers are replaced, and both wages and labor share drop.

Facing the challenge of machines, it is more favorable for human to choose collaboration than competition, because artificial intelligence has extremely high productivity. Therefore, making artificial intelligence helpers for humans is a good idea. Humans may engage in the jobs which need creativity, decision-making and abstract thinking, leaving the dull and repetitive work to machines. Humans get rid of boring work and focus on what they are good at. Meanwhile, artificial intelligence improves productivity largely and promotes economic growth. In this way, labor force spends less time on work, enjoys more leisure but earns higher wages, thus the social welfare is improved.

(iii)When capital is used to improve labor-augmenting technology

When capital is invested to improve labor-augmenting technology, the production function is

(22) $$Y = K^{\alpha}[(A_L L)^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma(1-\alpha)}{\sigma-1}},$$

where A_L is the labor-augmenting technology. Define $Z_L = (A_L L)^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}$.

Similarly, apply the no arbitrage condition,

(23) $$K = \frac{\alpha}{1-\alpha} Z_L P^{\frac{1}{\sigma}}$$

(24) $$\omega = aA_L^{\frac{\sigma-1}{\sigma}} L^{-\frac{1}{\sigma}} Z^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha}{\sigma}}$$

(25) $$R = R_K = R_P = aZ_L^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha-1}{\sigma}}$$

Proposition 4(i) Labor-augmenting technology raises the price of capital. When $0<\sigma<\alpha$, that is, artificial intelligence capital is complementary to labor, labor-augmenting technology lowers wages. When $\sigma>1$, that is, artificial intelligence capital and labor substitute each other, labor-augmenting technology increases wages.

Proposition 4 (ii) When $0<\sigma<1$, that is, artificial intelligence capital is complementary to labor, labor-augmenting technology lowers labor share but increases the share of artificial intelligence capital, and the share of traditional capital remains unchanged. When $\sigma>1$, that is, artificial intelligence capital and labor substitute each other, labor-augmenting technology raises labor share but reduces the share of artificial intelligence capital, and the share of traditional capital remains unchanged. When $\sigma=1$, both labor share and the share of artificial intelligence capital are fixed.

All proofs are contained in the appendix.

Proposition 4 has essentially the same logic as Proposition 2. When artificial intelligence capital is complementary to labor, the increase in labor-augmenting technology indeed helps humans to complete more tasks in less time, which saves labor and raises wages. Conversely, if labor-augmenting technology improves, humans are actually doing more for machines. Therefore, machines rather than humans are benefited, both wages and the share of labor. This is called Baumol's Cost Disease, which means that the share of one sector declines when its productivity improves.

When artificial intelligence capital and labor substitute each other, there exists competition between them. Higher productivity helps labor to win competitive advantages and earn higher wages. If artificial intelligence takes the place of a large quantity of jobs, labor will be required to improve productivity.

Combining Proposition 2 with Proposition 4, when artificial intelligence capital is complementary to labor, higher artificial intelligence capital leads to higher labor share, but higher labor-augmenting technology results in lower labor share. When artificial intelligence capital and labor substitute each other, labor-augmenting technology has positive effect on labor share while the effect of artificial intelligence capital is negative. Given that the productivity of artificial intelligence is much higher than humans, it is not wise for humans to compete with artificial intelligence by improving productivity.

The following conclusions can be obtained from Propositions 2, 3, and 4, as shown in Table 1, Table 2, and Table 3.

Table 1: Impact of artificial intelligence on workers' wages

	P	A_L	A_P
Complementary artificial intelligence	+	–	+
Alternative artificial intelligence	+	+	–

Note: "+" means that the wages of workers will increase, "—" means that the wages of workers will decrease.

Table 2: The impact of artificial intelligence on labor share

	P	A_L	A_P
Complementary artificial intelligence	+	_	+
Alternative artificial intelligence	_	+	_

Note: “+” means the labor share will increase, “—” means the labor share will decrease.

Table 3: Impact of artificial intelligence on capital share

	P	A_L	A_P
Complementary artificial intelligence	—	+	—
Alternative artificial intelligence	+	—	+

Note: “+” means the capital share will increase, “—” means the capital share will decrease.

（iv） The develop of artificial intelligence and the creation of new tasks

One of the shortcomings of the neoclassical model is that it ignores the creation of new jobs by technological progress. Following the task-based approach developed by Zeira (1998) and Acemoglu (2018a-f), we propose a more general model. Assuming that the interval of task i is [N-1, N]. A higher N means that the development of automation creates new demand for employment and thus new tasks, and the higher the N is, the more complicated the task is. N-1 gets larger as N becomes larger, indicating the disappearance of old tasks. The economic tuition is that the development of automation creates new jobs while eliminates some others. Define the production function of enterprise as follow,

$$(26)\qquad Y=[A_K f(A_I)K]^{\alpha}[\int_{N-1}^{I}(A_p(i)h(A_I)p(i))^{\frac{\sigma-1}{\sigma}}di+\int_{I}^{N}(A_l(i)g(A_I)l(i))^{\frac{\sigma-1}{\sigma}}di]^{\frac{\sigma(1-\alpha)}{\sigma-1}},$$

where, $p(i), l(i)$ are the capital and labor invested in the task i, A_p, A_l are capital-augmenting and the labor-augmenting technology respectively, and I indicates the frontier of artificial intelligence possibilities, a higher I means more advanced artificial intelligence. It is assumed that artificial intelligence capital works in a smaller interval, while workers work in a large one, because the low-skilled workers are more easily to be replaced by artificial intelligence capital, and finally workers mainly engage in more complex tasks, such as creative ones.

We use the production function (26) mainly to connect the neoclassical production function with the Task-based Model, which simplifies the complex operations of the Task-based Model. This production function combines the advantages of both models. It is worth mentioning that we can use this form of production function to analyze some specific problems, such as the impact of artificial intelligence on income inequality and population aging. In addition, from (26), you can get Propositions 1-4.

3 Two-sector model

This section builds a two-sector model based on the previous one. Production consists of a final product production department and two intermediate product production units.

3.1 Production, technology, factor market

Drawing on Acemoglu (2006), the final product production function is

(27) $$Y = [\gamma Y_1^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)Y_2^{\frac{\varepsilon-1}{\varepsilon}}].$$

Where Y is an intermediate product, Y_1 and Y_2 are two intermediate products, and $\varepsilon \in [0,\infty)$ is an elasticity of substitution between the intermediate products. γ is a distribution parameter indicating the importance of the two intermediate products.

The production functions of the two intermediate products are

(28) $$Y_1 = [A_{K_1} f_1(A_I)K_1]^{\alpha_1}\{[(A_{L_1} g_1(A_I)L_1)^{\frac{\sigma_1-1}{\sigma_1}} + (A_{P_1} h_1(A_I)P_1)^{\frac{\sigma_1-1}{\sigma_1}}]^{\frac{\sigma_1}{\sigma_1-1}}\}^{1-\alpha_1},$$

(29) $$Y_2 = [A_{K_2} f_2(A_I)K_2]^{\alpha_2}\{[(A_{L_2} g_2(A_I)L_2)^{\frac{\sigma_2-1}{\sigma_2}} + (A_{P_2} h_2(A_I)P_2)^{\frac{\sigma_2-1}{\sigma_2}}]^{\frac{\sigma_2}{\sigma_2-1}}\}^{1-\alpha_2},$$

Where $f_i(A_I) = e^{m_i A_I}$ represents the spillover effect of artificial intelligence technology on traditional production methods. $g_i(A_I) = e^{n_i A_I}$ indicates the enhanced effect of artificial intelligence technology on labor productivity. $h_i(A_I) = e^{q_i A_I}$ indicates the enhanced effect of artificial intelligence technology on capital production efficiency. $\sigma_i \in [0,\infty)$ is an elasticity of substitution between labor and artificial intelligence capital. We assume that department 1 is a capital-intensive sector and department 2 is a labor-intensive sector. Notes $T_{K_i} = A_{K_i} e^{m_i A_I}$, $T_{L_i} = A_{L_i} e^{n_i A_I}$, $T_{P_i} = A_{P_i} e^{q_i A_I}$, $Q_i = [(A_{L_i} h_i(A_I)L_i)^{\frac{\sigma_i-1}{\sigma_i}} + (A_{P_i} g_i(A_I)P_i)^{\frac{\sigma_i-1}{\sigma_i}}]^{\frac{\sigma_i}{\sigma_i-1}}$, $i = 1,2$.

The final product is used as a valuation, and the price is standardized to 1. Intermediate product production is in complete competition, the price is C_i , $i = 1,2$,thus

(30) $$\frac{C_1}{C_2} = \frac{\gamma}{1-\gamma}\left(\frac{Y_1}{Y_2}\right)^{-\frac{1}{\varepsilon}}$$

Solve the problem of maximizing profit in the intermediate production department,

(31) $$R_{K_i} = \alpha_i C_i T_{K_i}^{\alpha_i} K_i^{\alpha_i-1} Q_i^{1-\alpha_i}$$

(32) $$\omega_i = (1-\alpha_i)C_i Y_i Q_i^{\frac{1-\sigma_i}{\sigma_i}} T_{L_i}^{\frac{\sigma_i-1}{\sigma_i}} L_i^{-\frac{1}{\sigma_i}}$$

(33) $$r_i = (1-\alpha_i)C_i Y_i Q_i^{\frac{1-\sigma_i}{\sigma_i}} T_{P_i}^{\frac{\sigma_i-1}{\sigma_i}} P_i^{-\frac{1}{\sigma_i}}$$

Where R_{K_i} is the price of traditional material capital, ω_i is the wage of workers, r_i is the artificial intelligence capital price.

All factor markets are completely competitive, and the market clearing conditions are

(34) $$K = K_1 + K_2$$

(35) $$L = L_1 + L_2$$

(36) $$P = P_1 + P_2$$

3.2 Static equilibrium

Next, let us analyze the impact of changes in state variables K, P, L, AI on factor prices and factor shares, focusing on the effects of artificial intelligence spillover effects and the elasticity of substitution.

Denote the fraction of traditional material capital, labor and artificial intelligence capital in the capital-intensive are $\lambda_k = K_1 / K, \lambda_l = L_1 / L, \lambda_p = P_1 / P$. The changes of variables λ_k, λ_l and λ_p respectively measure the reconfiguration process of production factors in the two production sectors, which reflects the industrial structure transformation and upgrading process. Without loss of generality, suppose $\lambda_p > \lambda_l$, $\alpha_1 > \alpha_2$ that is, the production sector 1 is a capital-intensive sector and the production sector 2 is a labor-intensive sector.

Combining equations (27)-(36), we obtain:

(37) $$\frac{\alpha_1}{\alpha_2} \cdot \frac{\gamma}{1-\gamma} \cdot (\frac{Y_1}{Y_2})^{\frac{\varepsilon-1}{\varepsilon}} \cdot \frac{1-\lambda_k}{\lambda_k} = 1$$

(38) $$(\frac{T_{P_1}}{T_{L_1}})^{\frac{\sigma_1-1}{\sigma_1}} \cdot \frac{(1-\lambda_p)^{\frac{1}{\sigma_2}}}{\lambda_p^{\frac{1}{\sigma_1}}} \cdot P^{\frac{1}{\sigma_2}-\frac{1}{\sigma_1}} = (\frac{T_{P_2}}{T_{L_2}})^{\frac{\sigma_2-1}{\sigma_2}} \cdot \frac{(1-\lambda_l)^{\frac{1}{\sigma_2}}}{\lambda_l^{\frac{1}{\sigma_1}}} \cdot L^{\frac{1}{\sigma_2}-\frac{1}{\sigma_1}}$$

(39) $$\frac{\alpha_1}{1-\alpha_1} \cdot \frac{\lambda_p^{\frac{1}{\sigma_1}}}{(1-\lambda_p)^{\frac{1}{\sigma_2}}} \cdot \frac{Q_1^{\frac{\sigma_1-1}{\sigma_1}}}{Q_2^{\frac{\sigma_2-1}{\sigma_2}}} \cdot P^{\frac{1}{\sigma_2}-\frac{1}{\sigma_1}} = \frac{\alpha_2}{1-\alpha_2} \cdot \frac{\lambda_k}{1-\lambda_k} \cdot \frac{T_{P_1}^{\frac{\sigma_1-1}{\sigma_1}}}{T_{P_2}^{\frac{\sigma_2-1}{\sigma_2}}}$$

(37)-(39) jointly determine the equilibrium.

By(32)、（33），the ratio of artificial intelligence capital income to labor income is

(40) $$d\ln\frac{rP}{\omega L} = \frac{\sigma_1-1}{\sigma_1} \cdot \frac{\sigma_1\lambda_l}{\sigma_1\lambda_l + \sigma_2(1-\lambda_l)} d\ln\frac{T_{P_1}}{T_{L_1}} + \frac{\sigma_2-1}{\sigma_2} \cdot \frac{\sigma_2(1-\lambda_l)}{\sigma_1\lambda_l + \sigma_2(1-\lambda_l)} d\ln\frac{T_{P_2}}{T_{L_2}}$$
$$+ \frac{(\sigma_1-1)\lambda_l + (\sigma_2-1)(1-\lambda_l)}{\sigma_1\lambda_l + \sigma_2(1-\lambda_l)} d\ln\frac{P}{L} + \frac{\lambda_p - \lambda_l}{\sigma_1\lambda_l + \sigma_2(1-\lambda_l)} d\ln\frac{\lambda_p}{1-\lambda_p}$$

The added value elasticity of capital and labor is

$$\eta_i^p = \frac{(A_{P_i} g_i(A_I)P_i)^{\frac{\sigma_i - 1}{\sigma_i}}}{(A_{L_i} h_i(A_I)L_i)^{\frac{\sigma_i - 1}{\sigma_i}} + (A_{P_i} g_i(A_I)P_i)^{\frac{\sigma_i - 1}{\sigma_i}}}$$

$$\eta_i^l = \frac{(A_{L_i} h_i(A_I)L_i)^{\frac{\sigma_i - 1}{\sigma_i}}}{(A_{L_i} h_i(A_I)L_i)^{\frac{\sigma_i - 1}{\sigma_i}} + (A_{P_i} g_i(A_I)P_i)^{\frac{\sigma_i - 1}{\sigma_i}}}$$

By (37)-(40),we obtain

Proposition 5 (i) $\frac{d\ln\lambda_P}{d\ln P} > 0 \Leftrightarrow (\alpha_1 - \alpha_2)(\varepsilon - 1)(\sigma_1 - \sigma_2)(q_1 - q_2) > 0$

$$\frac{d\ln\lambda_l}{d\ln P} > 0 \Leftrightarrow (\alpha_1 - \alpha_2)(\varepsilon - 1)(\sigma_1 - \sigma_2)(n_1 - n_2) > 0$$

Proposition 5 (ii) $\frac{d\ln\lambda_P}{d\ln A_I} > 0 \Leftrightarrow (\varepsilon - 1)(\sigma_1 - \sigma_2)[\eta_1^p(q_1 - n_1) - \eta_2^p(q_2 - n_2)] > 0$

$$\frac{d\ln\lambda_l}{d\ln A_I} > 0 \Leftrightarrow (\varepsilon - 1)(\sigma_1 - \sigma_2)[\eta_1^l(n_1 - q_1) - \eta_2^l(n_2 - q_2)] > 0$$

Proposition 5 (iii) $\frac{d\ln(rP/\omega L)}{d\ln P} > 0 \Leftrightarrow (\lambda_p - \lambda_l)\frac{d\ln\lambda_l}{d\ln P} > 0$

$$\frac{d\ln(rP/\omega L)}{d\ln A_I} > 0 \Leftrightarrow (\lambda_p - \lambda_l)\frac{d\ln\lambda_l}{d\ln A_I} > 0$$

All proofs are contained in the appendix.

We mainly analyze $\varepsilon > 1$($0 < \varepsilon < 1$ is the opposite),at this time the two-sector products are alternative, We also assume $\alpha_1 > \alpha_2$. We find the flow of capital and labor, and are influenced by the elasticity of substitution between capital and labor and the spillover effects of artificial intelligence. For(i), If the substitution elasticity of capital and labor in sector 1 is greater than the substitution elasticity of capital and labor in sector 2, and the artificial intelligence technology has greater capital enhancement effect on sector 1, capital will flow into the sector 1. If the artificial intelligence technology has a greater labor-enhancing effect on sector 1, the labor force will also flow into the sector 1.

For(ii), If you consider the reinforcing effect of capital on capital and labor, the situation will be much more complicated. We can use $q_i - n_i$ as a net enhancement effect of capital on artificial intelligence technology(Let $n_1 = n_2 = 0$, the result is more obvious). $\eta_i^p(q_i - n_i)$ can be seen as the net enhancement effect of artificial intelligence technology on the capital of sector i. If the substitution elasticity of capital and labor in sector 1 is greater than the substitution elasticity of capital and labor in sector 2, and the net enhancement effect of artificial intelligence technology on the capital of sector 1 is greater than sector 2, then the development of artificial intelligence technology will lead to the growth of capital-intensive sectors. The development of artificial intelligence technology is also the same for the change of the labor structure. If the substitution elasticity of capital and labor in sector 1 is greater than the substitution elasticity of capital and labor in sector 2, and the net enhancement effect of artificial intelligence technology on the labor of sector 1 is greater than sector 2, then the development of artificial intelligence technology will lead to the growth of capital-intensive sectors.

For(iii), the ratio of capital income to labor income, in addition to the above factors, is also affected by the proportion of capital and labor. As the capital stock increases, if $\lambda_p > \lambda_l, \sigma_1 > \sigma_2$, and $q_1 > q_2$ are satisfied at the same time, the ratio of capital income to labor income will increase, that is, the capital share will increase. If $\lambda_p > \lambda_l, \sigma_1 > \sigma_2$,and the net enhancement effect of artificial intelligence technology on the capital of sector 1 is greater than sector 2, The development of artificial intelligence technology will lead to an increase in capital share.

4 Conclusion and prospects

On the basis of previous research, we propose a general framework of the macroeconomic consequences of artificial intelligence. We assume that all workers are homogenous and fully employed, and focus on the effects of two kinds of artificial intelligence on wages, capital prices, labor share, capital share and economic growth. We also consider the enhancement effect of artificial intelligence on capital and labor, and point out that artificial intelligence has a strong spillover effect on other industries.

We construct a theoretical model that distinguishes between complementary and alternative artificial intelligence. It is found that in the case of alternative artificial intelligence, if the artificial intelligence technology enhances the laborer's effect more than the capital, the labor share increases and the capital share decreases. Conversely, if the artificial intelligence enhances the capital more than the labor, the labor share decreases, and the capital share has risen. In the case of complementary artificial intelligence, if the reinforcing effect on the laborer is greater than the capital, the labor share decreases and the capital share rises. Conversely, if the reinforcing effect on the capital is greater than the labor, the labor share increases and the capital share decreases. The wages and capital prices of workers are affected by three effects at the same time, and they need to meet certain conditions before they rise.

Further, we construct a two-sector model to examine the impact of the development of artificial intelligence technology on the transformation and upgrading of industrial structure. When the production products of the two sectors are replaced by each other, if the substitution elasticity of department 1 is greater than that of sector 2, and the net capital enhancement effect of artificial intelligence technology on sector 1 is greater than that of sector 2, the development of artificial intelligence technology leads to capital inflow into capital-intensive enterprises. If the development of artificial intelligence technology has a greater net labor force enhancement effect on department 1 than sector 2, the development of artificial intelligence technology leads to labor inflow into capital-intensive enterprises. In the case of complementary products in the two sectors, the above conclusions are reversed.

In fact, further analysis can be made by extending our general model.

(1) Unemployment. If most of the jobs are replaced by alternative artificial intelligence, what kind of actions the government can take to address the problem of mass unemployment. One solution is to tax robots and subsidize the unemployed. However, taxation will certainly result in the loss of efficiency and foreign transfer of artificial intelligence capital. Moreover, over subsidizing may cause more people stay at home while they can go out for work. How to make reasonable and effective policy is still a worthy concern.

(2) Labor market polarization. If technological progress decreases the share of medium-skilled workers and increases the shares of both high- and low- skilled ones, the polarization of labor market occurs. It is possible to discuss the possibility of artificial intelligence replacing medium-skilled workers and the changes in their wages and the share of labor.

(3) Aging of population. The aging problem is becoming more and more serious in most developed countries and some developing countries. Both the decreasing birth rate and increasing average life span exacerbate the labor shortage. Whether the development of artificial intelligence can solve this problem to keep economic growth and social welfare is another interesting topic.

(4) Endogenous artificial intelligence. We assume that the technology of artificial intelligence is exogenous in this model. Therefore, another question is what the effects of artificial intelligence on economic variables are if this assumption is relaxed.

(5) Education. Most countries in the world are facing a dilemma that a lot of college graduates cannot find a job while some emerging industries are starved for talents such as data analysts and AI developers. How to reform the education system to cope with the shock from artificial intelligence deserves attention.

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Appendix:

Proof of Proposition 2:

(i)

$$\ln\omega = \ln(aL^{-\frac{1}{\sigma}}) + \frac{1-\alpha}{\sigma-1}\ln Z + \frac{\alpha}{\sigma}\ln P$$

thereby

$$\frac{\partial(\ln\omega)}{\partial P} = \frac{1}{\sigma ZP}[P^{\frac{\sigma-1}{\sigma}} + \alpha L^{\frac{\sigma-1}{\sigma}}] > 0$$

Similarly

$$\frac{\partial(\ln R)}{\partial P} = -\frac{1-\alpha}{\sigma ZP}L^{\frac{\sigma-1}{\sigma}} < 0$$

（ii）The traditional capital share is

$$\alpha_K = \frac{RK}{Y} = \alpha$$

The labor share is

$$\alpha_L = \frac{\omega L}{Y} = (1-\alpha)\frac{L^{\frac{\sigma-1}{\sigma}}}{L^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}}$$

$$\alpha_P = \frac{RP}{Y} = (1-\alpha)\frac{P^{\frac{\sigma-1}{\sigma}}}{L^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}}$$

thereby

When $0<\sigma<1$，$\frac{\partial(\alpha_L)}{\partial P} > 0, \frac{\partial(\alpha_P)}{\partial P} < 0$；when $\sigma>1$，$\frac{\partial(\alpha_L)}{\partial P} < 0, \frac{\partial(\alpha_P)}{\partial P} > 0$.

When $\sigma=1$，α_L、α_P、α_K are all fixed values.

Proof of Proposition 3:

Proof: (i)

$$\frac{\partial(\ln\omega)}{\partial A_P} = \frac{L^{\frac{\sigma-1}{\sigma}}}{\sigma Z_P A_P}[(1-\sigma)\alpha + (1-\sigma\alpha)(\frac{A_P P}{L})^{\frac{\sigma-1}{\sigma}}]$$

thereby

when $0<\sigma<1$，$\frac{\partial(\ln\omega)}{\partial A_P} > 0$；when $\sigma > \frac{1}{\alpha} > 1$，$\frac{\partial(\ln\omega)}{\partial A_P} < 0$。

(ii)

$$\frac{\partial(\ln R)}{\partial A_P} = \frac{(1-\alpha)L^{\frac{\sigma-1}{\sigma}}}{\sigma Z_P A_P}[\sigma(\frac{A_P P}{L})^{\frac{\sigma-1}{\sigma}} + (\sigma-1)]$$

When $\sigma > 1$， $\frac{\partial(\ln R)}{\partial A_P} > 0$.

(iii)

$$\alpha_L = \frac{\omega L}{Y} = (1-\alpha)\frac{L^{\frac{\sigma-1}{\sigma}}}{L^{\frac{\sigma-1}{\sigma}} + (A_P P)^{\frac{\sigma-1}{\sigma}}}, \alpha_P = \frac{RP}{Y} = (1-\alpha)\frac{(A_P P)^{\frac{\sigma-1}{\sigma}}}{L^{\frac{\sigma-1}{\sigma}} + (A_P P)^{\frac{\sigma-1}{\sigma}}}$$

When $0 < \sigma < 1$， $\frac{\partial(\alpha_L)}{\partial A_P} > 0, \frac{\partial(\alpha_P)}{\partial A_P} < 0$;when $\sigma > 1$， $\frac{\partial(\alpha_L)}{\partial A_P} < 0, \frac{\partial(\alpha_P)}{\partial A_P} > 0$.

Proof of Proposition 4:

$$\frac{\partial(\ln \omega)}{\partial A_L} = \frac{1}{\sigma Z_L A_L}[(\sigma-\alpha)(A_L L)^{\frac{\sigma-1}{\sigma}} + (\sigma-1)P^{\frac{\sigma-1}{\sigma}}]$$

$$\frac{\partial(\ln R)}{\partial A_L} = \frac{1-\alpha}{\sigma Z_L} A_L^{-\frac{1}{\sigma}} L^{\frac{\sigma-1}{\sigma}} > 0$$

thereby

when $0 < \sigma < \alpha$， $\frac{\partial(\ln \omega)}{\partial A_L} < 0$； when $\sigma > 1$， $\frac{\partial(\ln \omega)}{\partial A_L} > 0$.

(ii)

$$\alpha_L = \frac{\omega L}{Y} = (1-\alpha)\frac{(A_L L)^{\frac{\sigma-1}{\sigma}}}{(A_L L)^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}}$$

$$\alpha_P = \frac{RP}{Y} = (1-\alpha)\frac{P^{\frac{\sigma-1}{\sigma}}}{(A_L L)^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}}$$

When $0 < \sigma < 1$， $\frac{\partial(\alpha_L)}{\partial A_L} < 0, \frac{\partial(\alpha_P)}{\partial A_L} > 0$;when $\sigma > 1$， $\frac{\partial(\alpha_L)}{\partial A_L} > 0, \frac{\partial(\alpha_P)}{\partial A_L} < 0$.

Proof of Proposition 5:

We will use the following results， $\lambda_p > \lambda_l \Leftrightarrow \eta_1^k > \eta_2^k$， $d\ln(1-z) = -\frac{z}{1-z}d\ln z$

By (37),

$$\ln\frac{\alpha_1}{\alpha_2} + \ln\frac{\gamma}{1-\gamma} + \frac{\varepsilon-1}{\varepsilon}(\ln Y_1 - \ln Y_2) + \ln(1-\lambda_k) - \ln\lambda_k = 0$$

Simultaneous differentiation on both sides,

(41) $$\frac{\varepsilon-1}{\varepsilon}(d\ln Y_1 - d\ln Y_2) - \frac{1}{1-\lambda_k} d\ln\lambda_k = 0$$

Because $Y_1 = (T_{K_1}K_1)^{\alpha_1}Q_1^{1-\alpha_1}, Y_2 = (T_{K_2}K_2)^{\alpha_2}Q_2^{1-\alpha_2}, Q_1 = [(T_{L_1}L_1)^{\frac{\sigma_1-1}{\sigma_1}} + (T_{P_1}P_1)^{\frac{\sigma_1-1}{\sigma_1}}]^{\frac{\sigma_1}{\sigma_1-1}}$,

$Q_2 = [(T_{L_2}L_2)^{\frac{\sigma_2-1}{\sigma_2}} + (T_{P_2}P_2)^{\frac{\sigma_2-1}{\sigma_2}}]^{\frac{\sigma_2}{\sigma_2-1}}$, thus

(42) $$d\ln Y_1 = \alpha_1(d\ln T_{K_1} + d\ln\lambda_k + d\ln K) + (1-\alpha_1)(d\ln Q_1)$$

(43) $$d\ln Y_2 = \alpha_2(d\ln T_{K_2} + d\ln(1-\lambda_k) + d\ln K) + (1-\alpha_2)(d\ln Q_2)$$

(44) $$d\ln Q_1 = \eta_1^l(d\ln L_1 + d\ln T_{L_1}) + \eta_1^p(d\ln P_1 + d\ln T_{P_1})$$

(45) $$d\ln Q_2 = \eta_2^l(d\ln L_2 + d\ln T_{L_2}) + \eta_2^p(d\ln P_2 + d\ln T_{P_2})$$

By (38),

(46) $$\frac{\sigma_1-1}{\sigma_1}(d\ln T_{P_1} - d\ln T_{L_1}) - [\frac{1}{\sigma_1} + \frac{\lambda_p}{\sigma_2(1-\lambda_p)}]d\ln\lambda_p + (\frac{1}{\sigma_2} - \frac{1}{\sigma_1})d\ln P$$

$$= \frac{\sigma_2-1}{\sigma_2}(d\ln T_{P_2} - d\ln T_{L_2}) - [\frac{1}{\sigma_1} + \frac{\lambda_l}{\sigma_2(1-\lambda_l)}]d\ln\lambda_l + (\frac{1}{\sigma_2} - \frac{1}{\sigma_1})d\ln L$$

By (39),

(47) $$[\frac{1}{\sigma_1} + \frac{\lambda_p}{\sigma_2(1-\lambda_p)}]d\ln\lambda_p + \frac{\sigma_1-1}{\sigma_1}d\ln Q_1 - \frac{\sigma_2-1}{\sigma_2}d\ln Q_2 + (\frac{1}{\sigma_1} - \frac{1}{\sigma_2})d\ln P$$

$$= \frac{1}{1-\lambda_k}d\ln\lambda_k + \frac{\sigma_1-1}{\sigma_1}d\ln T_{P_1} - \frac{\sigma_2-1}{\sigma_2}d\ln T_{P_2}$$

From (41)-(47), you can prove proposition 5.