# "Dynamic Pricing for Inventories with Reference Price Effects" 

## Reply to Reviewer 1 comments

## Comment 0 .

"The paper studies the problem of dynamically pricing perishable items in the presence of reference effects under a general demand model. The methodology is a continuous time monopoly with deterministic demand and a finite time horizon. Demand intensities are time homogeneous and depend on a firm's current price as well as a reference price, which is based on the firm's past prices. Inventory holding costs and discounting is taken into account; inventory replenishment is excluded.

The introduction reads well and motivates the objective of the paper. The primary objective of the paper is to investigate the impact of reference price effects under a general demand model. Based on Pontryagin's maximum principle necessary optimality conditions for open-loop controls are derived in order to infer structural properties of optimal price paths.

In my assessment, the paper deals with an interesting and relevant topic: dynamic pricing with customer behavioral effects. Although the analysis remains only theoretical, I think there is some merit in the paper. However, I see two weaknesses: (i) due to the complexity of the model, general useful structural properties can hardly be derived, and (ii) the presented approach is not constructive, i.e., it remains unclear how to compute optimal prices.

Hence, the authors should add numerical results to their model in order to (i) verify that (under certain conditions) the interplay between prices and reference prices follows some rules and (ii) to show a way how to actually derive prices in practical applications.

I have the following critical comments/questions:"

## Reply 0:

We greatly thank the reviewer for his/her comments on our article. The reviewer has provided insightful remarks and suggestions, which we will account for in the revision of the article.

The main recommendation of the reviewer is to provide a numerical example, which would help 1) to provide stronger insights about the relationship between the selling and reference prices and 2) to show how to compute price in firm applications.

We thank the reviewer for this careful suggestion, and we'll revise the article by providing a numerical instance of price policy.

## Comment 1.

"(1) Could the authors provide a numerical example? Can the solution of the model be derived using approximation techniques? "

## Reply 1:

We thank the reviewer for this remark. Following reviewer's suggestion, a numerical example will be provided in the paper to illustrate the solution of the model.

## Comment 2.

"(2) Is there an incentive to start with extremely high prices (e.g., I million) to build a potential in order to generate demand by reducing prices afterwards? "

## Reply 2:

We thank the reviewer for the suggestion to better discuss the form of the pricing strategy. Our model shows that starting with an extremely high price can not be optimal in the presence of inventory, with inventory costs. Indeed, when inventory is costly, selling a lot at the beginning of the period allows to reduce firm's inventory costs all through the planning period. Following reviewer's comments, this point will be better discussed in the paper.

## Comment 3.

"(3) Can the model be generalized by time-dependent demand? "

## Reply 3:

We thank the reviewer for the remark. It is to be noted that, as acknowledged by the reviewer, the model is already complex. Therefore, introducing a time depence will complexify even more the model. We recognize though that such generalization of the model would make it more realistic. Therefore, the generalization of the model to timedependent demand will be discussed in the paper.

## Comment 4.

" (4) Proposition 1 and 2 are interesting. However, they hardly allow to infer how the optimal open-loop price path should look like as we have a complex interplay of (i) discounting, (ii) holding costs, (iii) time-to-go, (iv) price impact, and (v) reference price effect.
Is there a relation to corresponding optimality conditions in the literature (for models without reference effect), e.g.,

Stiglitz, J.,E. (1976) Monopoly and the rate of extraction of exhaustible resources. The American Economic Review 66(4), 655-660

Cao, P., Li,J., Yan,H. (2012). Optimal dynamic pricing of inventories with stochastic demand and discounted criterion, European Journal of Operational Research 217, 580588.

Further, can the necessary optimality conditions be used to actively construct optimal open-loop solutions as done in, cf. (13), Schlosser, R. (2015). Dynamic Pricing with Time-Dependent Elasticities, Journal of Revenue and Pricing Management 14 (5), 365383

Is it possible to analyze feedback controls by looking at optimality conditions for openloop controls for given states (time t, reference price r)? This way, it may be possible to derive structural properties regarding single state variables."

## Reply 4:

We thank the reviewer for the very interesting pointers in the literature. In the revised version of the paper, we'll discuss our approach according to such pointers. In particular, the references to Stiglitz (1976), Cao et al. (2012) and Schlosser (2015) will be included in the discussion of the paper. This will allow to better relate our results to optimality conditions in the literature (without reference effect), to discuss open-loop solutions and to derive structural properties for state variables, as suggested by the reviewer.

## Comment 5.

"(5) In the conclusion the authors discuss further extensions of the model, such as marketing expenses or promotion activities. There is related work that studies structural properties of pricing and advertising controls under general demand models, e.g., Schlosser, R. (2016). Joint Stochastic Dynamic Pricing and Advertising with TimeDependent Demand, Journal of Economic Dynamics and Control 73, 439-452.

In my opinion, the author's model with reference price effects is interesting. However, the current presentation of the results of the paper has to be improved. The authors should describe how optimized prices can be derived."

## Reply 5:

We thank the referee for the very interesting reference to Schlosser (2016) on structural properties of pricing and advertising under general demand models that could enrich our conclusion. The revised version of the paper will discuss this reference.

## Comment 6

"Minor comments:
page line comment
2-12 I think at the end of the sentence should be a "."
4-8 Is it ensured that $I(t)$ may not be negative?
77 Shouldn't be t finite, i.e., $0<t<T$ ?
8 -7 I think the reference "equation (2)" is wrong"

## Reply 6:

We thank the reviewer for the careful reading that allowed identifying several typos and errors. We'll correct all these typos/errors in the revised article.

