Cheap Talk by Two Senders in the Presence of Network Externalities

Jeahan Jung
POSTECH

Jeong-Yoo Kim
Kyung Hee University

Abstract

We develop a model of cheap talk with two senders in the presence of network externalities so that their utility functions are inreasing in the network size. We first show that if there is no noise in private information that each sender receives, the full information is revealed by the harshest cross-checking strategies, that is, strategies to punish the senders unless their messages exactly coincide. Then, we show that with even a small noise cross-checking strategies cannot induce full revelation if utility functions of senders are linear in the network size, while full revelation is possible if utility functions are strictly concave. We find a sufficient condition for the existence of a fully revealing equilibrium which is supported by the cross-checking strategy with a positive confidence interval independent of each sender's private information.

Keywords: cheap talk; cross-checking strategy; fully revealing equilibrium; network externality; word-of-mouth communication

JEL Classification Code: C7, D8

1 Introduction

Is a recommendation letter of a professor credible? Is a car dealer selling used cars trustworthy? Is a lawyer's legal advice reliable? Since the seminal paper by Crawford and Sobel

(1982) – hereafter, abbreviated to CS – it is quite well known that partial (not full) information of the informed can be transmitted to the uninformed if their interests are similar enough.

In many cases, however, an uninformed party refers not to a single informed party but to multiple informed parties. Then, a natural question will be whether or not the uninformed can really elicit more accurate information by doing so. Why do universities require multiple letters of recommendation from applicants? Why do major academic journals make it a rule for multiple referees to review an unsolicited article? Why do wealthy people hire more than one attorney at a time? What made two independent opinions of Siskel and Ebert, two famous movie critics, appear side by side?

One obvious answer is to elicit more accurate information from the informed. It may be true if the informed always provide honest opinions. If the interests of the informed are not aligned with the interest of the uninformed, however, it can be only a partial answer, especially when university professors, article referees, attorneys, movie critics have some common interests with the students, article authors, the opposite legal party, movie producers/directors, because it does not take into account the effect on the incentive of the informed to misrepresent their information. Therefore, a more satisfactory answer should address how the presence of other speakers discipline the incentive of a speaker to distort information.

There are many articles on information transmission by multiple informed parties whose interests possibly differ with the interest of the uninformed.[1] Each of them provides different models basically addressing the question of whether or not the uninformed is enabled to get more accurate information by multiple informed parties than by a single informed party. All of the analyses are, however, based on the preference assumption that the uninformed's actions that are favorites to the informed and the uninformed are both increasing in private

[1]A short list, if not comprehensive, includes Gilligan and Krehbiel (1989), Austen-Smith (1993), Krishna and Morgan (2001), Battaglini (2002), Ambrus and Takahashi (2008), Li (2010), Galeotti *et al.* (2013), McGee and Yang (2013) and Li *et al.* (2016). In particular, Ambrus and Takahashi (2008) assume that the state space is multi-dimensional and bounded, both of which critically depart from our model that assumes a unidimensional and unbounded state space. Galeotti *et al.* (2013) consider multiple senders who send messages to some or all of the others. So, each player can be both a sender and a receiver. However, there is no network externality assumed in the paper. McGee and Yang (2013) is very close to our model, but again, no network externality is assumed.

information. That is, in all of the models, it is an essential ingredient for credible communication that the informed with different private information have different preferences over the actions by the uninformed.

In this paper, we will assert that such a condition on preferences is not necessary for credible communication in the presence of multiple informed parties. As a matter of fact, situations where such a condition is not satisfied abound. For instance, suppose there is an experience good whose quality is not learned before a consumer purchases one. An uninformed consumer who has to decide whether to buy one or not may refer to informed consumers for the quality. If the purchasing decision by this potential consumer does not affect the utility of existing consumers at all, that is, no consumer externalities are involved, existing consumers who are referred to will have no incentive to garble their own information about the quality. In this case, it would not be surprising that all references were truth-revealing.[2] However, if it does affect the utility of other existing consumers, that is, there are network externalities, they might have an incentive to exaggerate the quality of the good to boost the demand for it. In fact, it was often observed that old-time Mac-users alleged ultra-superioity of Macintosh even though they felt that inconvenience due to the limited network size exceeded the benefit from the relative quality advantage after the advent of the window system. In this situation, a consumer may wonder if word-of-mouth communication can be a reliable source of information regarding the quality of an experience good with network externalities.[3]

In this paper, we show that even in such a situation where there is no room for coordination between an informed party and an uninformed party, truthful revelation is possible if the uninformed party solicits references from multiple informed parties. The intuitive reason for this is that, in this case, the uninformed has a means of checking the truth of the message from one informed party probabilistically, which is the message from the other informed parties. Also, the situation is like a coordination game among informed parties. Even though coordination is in fact realized by the action of the uninformed, communication messages of the informed parties are a vehicle of implementing coordination, and more

[2] This argument will be more convincing if the reputational effect is taken into account.

[3] While we were revising this paper, an anonymous referee informed us of the existence of the literature on cheap talk with transparent motives. See Section 2 for more details.

fundamentally, correlation among them is the genuine source of their coordination. That is, in this situation, word-of-mouth communication is not a way of coordination between an informed party and an uninformed party, but a way of coordination among informed parties with correlated private information.

To support the truth-revealing outcome as an equilibrium, we will use a specific form of strategy of the uninformed – what will be called "cross-checking strategies". By a cross-checking strategy, we mean a strategy to reward senders if their messages are similar to each other and otherwise punish them.

We consider two cases: the noiseless information case and the noisy information case. If there is no noise in private information that each sender receives, the cross-checking strategy takes the following form; the uninformed believes that either one of the informed parties was fibbing as far as their messages are not exactly the same, and then takes a punishing action harsh to both of them, and otherwise the uninformed believes them literally and takes the optimal action given the updated posterior belief. If there is some noise in their information, the cross-checking strategy takes a rather complicated form. If the messages sent by the informed are observed not to be too far apart, more specifically, to be within a certain distance, the uninformed believes them thereby taking a rewarding action, i.e., choosing the maximum amount that the updated posterior belief allows, and otherwise punish them by choosing the minimum amount that the posterior belief allows.

We first show that if there is no noise in private information that each sender receives, the full information is revealed by the harshest cross-checking strategies, that is, strategies to punish the senders unless their messages exactly coincide. Then, we show that with introducing even a small noise, the cross-checking strategy cannot induce full revelation if the utility functions of senders are linear in the network size. The difficulty in this case arises mainly because even a small noise makes off-the-equilibrium messages vanish completely under the normal distribution of the noise. Even if a message is too high or too far from the other messages, it is a possible event, although the likelihood is very low. So, the uninformed receiver cannot believe that it is a consequence of a sender's lying. This makes it difficult to penalize a sender who sends a higher message than the true value strongly enough. However, we also show that if the utility functions are strictly concave, full revelation is possible with the cross-checking strategy. In this case, strict concavity of utility functions can make the penalty from inflating the message exceed the reward from it, so that it can discipline senders to be tempted to lie.

In Section 2, we briefly review related literature. In Section 3, we introduce the model. In Section 4, we analyze the noiseless case in which both senders receive exactly the same information. In Section 5, we analyze the noisy case. Section 6 contains concluding remarks and an avenue for future research.

2 Related Literature

It was earlier noticed by Seidmann (1990) and Gibbons (1989) that cheap talk could influence the receiver's equilibrium actions even if all the types of the sender share a common preference ordering over the actions of the receiver. Seidmann (1990) shows, in a setting with one sender, that if either the receiver is itself privately informed, or his action is multi-dimensional, the sender's types may disagree in their preferences over distributions of actions generated by the distribution of the receiver's types or over the pair of actions by the receiver due to their different trade-offs between the actions. Gibbons (1989) presents a model that is closest to ours. He analyzes a model of conventional arbitration in which the employer and the union simultaneously submit offers and then the arbitrator imposes a settlement. He also obtains the truth-revealing result that the parties' offers perfectly reveal their private information to the arbitrator. The crucial difference of his model from ours is that the parties observe the same noisy signal of the underlying state variable and that the arbitrator himself receives a direct correlated signal. This feature of correlation between senders' information and the receiver's information drives his result of perfect communication.

In a series of papers on legislative decisions (Gilligan and Krehbiel (1989), Austen-Smith (1993), Epstein (1998)), authors explore the informational role of the committee. Gilligan & Krehbiel and Epstein both consider models of legislative organization and two committee members with diverse preferences (presumably from different parties). Gilligan & Krehbiel assert that, if the committee preferences are symmetric about the floor's ideal point, floor members can get better information on the bill reported to the floor when two committee members with diverse preferences both agree to support the bill. Epstein shows that the argument by Gilligan and Krehbiel does not hold under asymmetric committee preferences.

Austen-Smith considers a model in which an informed House multiplely refers legislation to two committees with diverse preferences. He shows that more information can be communicated under multiple referral than under single referral.

There is recent literature on cheap talk with transparent motives. (See, for example, Chakraborty & Harbaugh [2010] and Lipnowski & Ravid [2018].) By transparent motives, they mean that the informed sender does not care about the state but only about the receiver's action. In that sense, senders in our model with network externalities also have transparent motives. The authors of both papers show that cheap talk can be informative even if the sender has a transparent motive. One important difference from our model is that their models are about cheap talk with one sender, not about cheap talk with multiple senders. In Charkraborty & Harbaugh (2010), informativeness of cheap talk relies on the multi-dimensionality of the state variable which implies that the receiver cares about multiple issues rather than one issue, unlike in our model. Also, while we assume that the state space and the receiver's action space are unbounded, Lipnowski & Ravid (2018) uses a different assumption that the state space is compact (and an implicit assumption that the action space is compact). Moreover, they does not consider the noisy information case which is central to our analysis.

Farrell and Saloner (1985) explore the role of communication in an industry with network externalities. They consider a situation in which potential users with independent private information on valuations of alternative technologies can engage in cheap talk to each other about which technology to adopt. They find that communication eliminates excess inertia where the preferences of the users coincide, while it increases inertia where their preferences differ. Their model assumes that all potential users are informed of their valuations on technologies before they purchase one without any explicit explanation of how they obtained the information. In their model, the role of cheap talk by potential consumers is to announce their intentions of which technology to purchase, while, in our model, it is made by existing consumers in order to inform the potential consumer of their valuation on the product.

Word-of-mouth communication has been modelled by several authors. In Ellison and Fudenberg (1995), decision-making agents ask several other individuals randomly chosen from the population about their current choice and payoff, to make their own choices between two alternatives, based on their reports, assuming that they are truth-telling. Since they assumed that each player's payoff is not influenced by the actions chosen by others, it seems natural, in their model, not to pay heed to the incentives of the informed consumers to be honest. Satterthwaite (1979) addresses the question how information about sellers flows among consumers. However, his analysis is also based on the assumption of naive speakers, who always speak honestly, and naive listeners, who always take messages for serious.

3 Model

We develop a model of cheap talk with two senders. There are two senders or speakers S_i, $i \in N \equiv \{1, 2\}$ and one receiver or listener R. The state of nature θ is a random variable with probability distribution function, $F(\theta)$, and density function, $f(\theta)$, supported on $\Theta \equiv \mathbb{R}$. For example, senders are consumers using the same computer of quality θ. R can be interpreted as a large organization such as a university or a company which is going to decide to buy a number of same computers.

For simplicity, we assume that θ is uniformly distributed over $\Theta = \mathbb{R}$,[4] i.e., senders have no information about θ or no bias *a priori*. Only S_i's observe a noisy signal on the state of nature $v_i \in V = \mathbb{R}$ where $v_i = \theta + \epsilon_i$, ϵ_i is stochastically independent with θ, and ϵ_i's are i.i.d. We assume that ϵ_i follows a normal distribution with its mean zero and the variance σ^2.[5]

The game proceeds as follows. S_i's send a payoff-irrelevant signal (cheap talk) $m_i \in M = \mathbb{R}$ to R simultaneously.[6] Then, receiving a vector of signals $\mathbf{m} = (m_1, m_2) \in M^2$, R updates his posterior belief about v_1 and v_2, $b_1(\mathbf{m})$ and $b_2(\mathbf{m})$, and then forms his belief about θ $b(\mathbf{m})$ by using $b_1(\mathbf{m})$ and $b_2(\mathbf{m})$, based on which he chooses an action $a \in A(= \mathbb{R})$ that is a network size.[7] A strategy of the receiver determines senders' payoffs as well as his own

[4] Note that we are assuming an improper prior distribution.

[5] Alternatively, we can assume that S_i observes the true value of θ with probability $1 - \epsilon_i$ and observes something else with ϵ_i. For this alternative modelling of introducing noise, see Battaglini (2002).

[6] Since the cheap talk messages of senders, m_i, are payoff-irrelevant by the definition of cheap talk, the payoffs of the players (U^{S_i} and U^R) which are described below should not depend on m_i. Kartik (2009) considers messages with lying costs. Sine a lying message of a sender affects the payoff of the sender, it is not cheap talk. In our model, cheap talk affects the payoffs of players not directly, but only through the belief of the receiver.

[7] By a network size, we mean the number of products which are same as, or at least compatible to the

payoff.

The payoff to S_i is given by a continuously differentiable function $U^{S_i} : A \to \mathbb{R}$ for all i and the payoff to R is given by twice continuously differentiable function $U^R : A \times \Theta \to \mathbb{R}$. Throughout the paper, we will assume that (1) $U^{S_i}(a) = u(a)$ where $u' > 0$, $u'' \leq 0$, i.e., increasing in a, and (2) $U^R(a, \theta) = -(a - \theta)^2$. The receiver's utility function implies that it has a unique maximum in a for all θ and the maximizer of U^R, denoted by $a^R(\theta)$, is strictly increasing in θ. Utility functions of senders which are increasing in a mean that the decision of R involves a positive network externality. The monotonic increase of $a^R(\theta)$ in θ means that the receiver will want to buy more units of high θ which can be interpreted as quality. Asymmetry between the utility function of senders and the receiver comes from the feature that only the receiver (consumer) pays the price. That is, while the uninformed consumer wants to purchase more units as the quality is higher, the informed consumers who already purchased one want the uninformed consumer to buy as many as possible regardless of the quality, because of the positive network externality.

A strategy for S_i specifies a signaling rule given by a measurable function $s_i : V \to M$. A strategy for R is an action rule given by a function $\alpha : M^2 \to A$.

The equilibrium concept that we will employ is that of weak Perfect Bayesian equilibrium (wPBE). An equilibrium of this game consists of a vector of a signaling rule for S_i, an action rule of R and a system of beliefs $((s_i^*(v_i))_{i=1}^2, \alpha^*(\mathbf{m}), (b_i(\mathbf{m}))_{i=1}^2, b(\mathbf{m}))$ such that

(2-I) $s_i^*(v_i) \in \arg\max_{m_i} \int_{-\infty}^{\infty} U^{S_i}(\alpha^*(m_i, s_j^*(v_j)))h(v_j \mid v_i)dv_j$, where $h(v_j \mid v_i)$ is the conditional density function of v_j given v_i, for $j \neq i$

(2-II) $\alpha^*(\mathbf{m}) \in \arg\max_a U^R(a, b(\mathbf{m}))$.

(2-III) R's posterior belief $b_i(\mathbf{m})$ is consistent with the Bayes' rule on the equilibrium path and $b(\mathbf{m})$ is an unbiased estimator of $b_1(\mathbf{m})$ and $b_2(\mathbf{m})$.[8]

product that the senders are using.

[8]Since R is not informed of three values, v_1, v_2 and θ, he must form all of the three beliefs. However, the beliefs that he can infer from the weak consistency requirement of wPBE are only about v_1 and v_2, not about θ because the value of θ is not known to senders, either. Instead, R can obtain an estimator for θ from the two observations or the beliefs $b_1(m_1)$ and $b_2(m_2)$. But since θ is *not a type* of senders (because senders do not know the value), the definition of wPBE does not impose any requirement for the estimator. It could be any weighted average of the messages, $\lambda m_1 + (1 - \lambda)m_2$ where $\lambda \in [0, 1]$, in particular, m_1 or m_2

Before we characterize equilibria, we will adapt some standard definitions often used in literature.

Definition 1 *An equilibrium is communicative iff there exist two different vectors of observations* $\mathbf{v}$, $\mathbf{v}'$ *such that* $\mathbf{s}^*(\mathbf{v}) \neq \mathbf{s}^*(\mathbf{v}')$ *and* $\alpha^*(\mathbf{m}) \neq \alpha^*(\mathbf{m}')$ *where* $\mathbf{m} = \mathbf{s}^*(\mathbf{v})$, $\mathbf{m}' = \mathbf{s}^*(\mathbf{v}')$. *An equilibrium is uncommunicative (or babbling) otherwise.*

Definition 2 *A communicative equilibrium is fully-revealing iff* $\mathbf{s}^*(\mathbf{v}) \neq \mathbf{s}^*(\mathbf{v}')$ *for any* $\mathbf{v}, \mathbf{v}'$ *such that* $\mathbf{v} \neq \mathbf{v}'$. *In particular, if* $\mathbf{s}^*(\mathbf{v}) = \mathbf{v}$, *a fully-revealing equilibrium is a truth-revealing equilibrium.*[9]

Definition 3 *A message vector* $\mathbf{m}$ *induces an action* a *iff* $a = \alpha^*(\mathbf{m})$.

In this paper, we will restrict our attention to symmetric equilibria such that $s_i^*(v_i) = s_j^*(v_j)$ if $v_i = v_j$, for all $j \neq i$. Let the symmetric equilibrium strategy be denoted by $s^*(\cdot)$. Then, the definition of the fully-revealing equilibrium is reduced to $s^*(v_i) \neq s^*(v_i')$ for any v_i, v_i' such that $v_i \neq v_i'$.

Observe that, in this model, unlike the CS model, if only one informed party can engage in cheap talk, the message sent by him cannot be credible at all. In the CS model, the payoff function of the sender (S) as well as that of the receiver is single-peaked, so that, given θ, the favorite actions to S and R do not differ very much even if they do differ. This implies that, for some low θ, both S and R prefer one action to another, while the reverse is true for some other high θ. In other words, there is room for coordination between S and R and in effect cheap talk enables such coordination to occur by conveying the message whether θ is high or low. In this model, however, the assumption of single-peaked preferences is violated and all the types of S prefer a higher level of the receiver's action a. Thus, S would like to pretend to have observed as highest v as possible to induce R's highest action possible, regardless of his type.

---

by ignoring one message, if one requires the unbiasedness of the estimator at the very least. All of them are perfect unbiased estimators of θ. Our $b(\mathbf{m})$ is a summary statistic that can be obtained after two separate processes, the inference process and the estimation process based on the inference.

[9]Since even fully revealing strategies which are $v_i \neq s_i^*(v_i)$ reveal the truth in equilibrium, those strategies are literally truth-revealing, So, in fact, the words "fully-revealing" and "truth-revealing" could be exchangeable.

We now summarize with

Proposition 1 *If $n = 1$, there exists no communicative equilibrium.*

Proof. Suppose, in equilibrium, there exist two different observations v, v' such that $m \neq m'$ and $a \neq a'$ where $m = s^*(v)$, $m' = s^*(v')$, $a = \alpha^*(m)$ and $a' = \alpha^*(m')$. If we assume $a < a'$ without loss of generality, S who observes v will have an incentive to deviate to m' since $U^S(a, \theta) < U^S(a', \theta)$, $\forall\theta$.

However, if there is more than one sender, the above argument breaks down. Suppose there are two senders S_1, S_2 and the vector of messages (m_1, m_2) sent by them induces an action a, while (m_1', m_2') induces an action a' with $a < a'$. Then, we cannot conclude that S_i will prefer sending m_i' to m_i, because it does not necessarily induce a higher level of action a'. In the presence of more than a sender, one sender cannot be sure what message will be sent by the other sender.

In the next section, we will make a formal analysis of cheap talk with two senders in the case that v_1 and v_2 are noiseless, i.e., $v_1 = v_2 = \theta$. In section 5, the argument will be extended to the noisy case.

4 Noiseless Case

We first consider the case that $\sigma^2 = 0$ so that $v_1 = v_2 = \theta$. As it is well-known, there always exists a babbling equilibrium in which senders send a random message and the receiver ignores any vector of messages whatsoever. In this section, we will see whether there can exist a communicative equilibrium as well, in particular, a truth-revealing equilibrium where each type of sender reveals its true information.

Proposition 2 *In the noiseless case ($\sigma^2 = 0$), there exists the following communicative truth-revealing equilibrium in this game:*

(i) $s_i^*(v_i) = v_i$,

(ii) $b(\mathbf{m}) = \begin{cases} m = m_1 = m_2 & \text{if } m_1 = m_2 \\ \underline{m} \equiv \min\{m_1, m_2\} - |m_1 - m_2| & \text{if } m_1 \neq m_2, \end{cases}$

(iii) $\alpha^*(\mathbf{m}) = b$.

As the proposition says, this equilibrium is supported by the posterior belief $b(\mathbf{m}) = m$ if $m_1 = m_2 \equiv m$ and $b(\mathbf{m}) = \underline{m}$ if $m_1 \neq m_2$ where $\underline{m}$ is lower than the minimum of m_1 and m_2.[10] The proof is straightforward. If a sender with v_i sends $m_i > v_i$, then $m_i > v_i = v_j = m_j$, given that the other sender sends a truthful message $m_j = v_j$. Since $m_i \neq m_j$, $U^{S_i}(m_i) = u(\underline{m}) + \theta < U^{S_i}(v_i) = u(v_i) + \theta$, because $\underline{m} = v_i - |m_i - m_j| < v_i$. It is clear for a sender to have no incentive to send $m_i < v_i$. It is also clear that if the messages are the same, the receiver must believe them as the value of θ, i.e., $b(\mathbf{m}) = m$ if $m_1 = m_2 = m$, since senders do not lie in equilibrium and there is no noise in their information. If the messages differ ($m_1 \neq m_2$), any belief can be possible, because it is off the equilibrium path, so $b(\mathbf{m}) = \underline{m}$ is also a perfectly legitimate belief.[11]

Readers may wonder if off-the-equilibrium belief $b(\mathbf{m}) = \min\{m_1, m_2\}$ without subtracting $|m_1 - m_2|$ would suffice to support the above truth-revealing strategies as an equilibrium. In fact, the strategies could be an equilibrium with the off-the-equilibrium belief but only with the tie-breaking rule whereby S_i's indifference is resolved in favor of honesty.[12] Under our belief, S_i strictly prefers honesty to exaggerating the information.

[10] Although this crosschecking strategy is similar to those constructed by Krishna & Morgan (2001) or Battaglini (2002), there are slight differences. Our model is closer to the case that senders have like biases in Krishna & Morgan (2001), but in the model, they used the off-the-equilibrium belief $b(\mathbf{m}) = \min\{m_1, m_2\}$. Battaglini (2002) considers the case of opposing biases. His equilibrium strategy involves the minimum or maximum of the compact type set which is impossible in our model in which the type set is unbounded.

[11] We are fully aware that this equilibrium is not robust to even small noise. See discussions in the next section (Proposition 3). Battaglini (2002) made a similar argument by introducing noise in a way described in Footnote 5.

[12] This equilibrium can be eliminated by a stronger refinement such as a variation of trembling-hand perfect equilibrium in extensive form games which is adapted to games with continuum strategy space although the perfectness concept was originally defined for finite games. To illustrate, first discretize the message set of each sender to $\tilde{M} = \{\cdots, \theta - 2\delta, \theta - \delta, \theta, \theta + \delta, \theta + 2\delta, \cdots\}$ for some small $\delta > 0$ and take a perturbed strategy $\sigma_i = (\cdots, \epsilon_k^n, \cdots, \epsilon_k, 1 - \frac{2\epsilon_k}{1-\epsilon_k}, \epsilon_k, \epsilon_k^2, \epsilon_k^3, \cdots)$. Clearly, this is a totally mixed strategy converging to $m_i = \theta$, since $\sum_{n=1}^{\infty} \epsilon_k^n = \frac{\epsilon_k}{1-\epsilon_k}$ and that $\frac{\epsilon_k}{1-\epsilon_k} \to 0$ as $\epsilon_k \to 0$. Then, it is easy to see that $m_i = \theta$ (being honest) is weakly dominated by some inflation to $m_i' = \theta + \delta$, since the latter strategy is better when the other sender makes a mistake to $\theta + \delta$ with some positive probability. Therefore, $m_i = \theta$ cannot be a best response to the above perturbed strategy of the other. This implies that it cannot be a trembling-hand perfect equilibrium.

5 Noisy Case

In Section 3, we assumed that $v_i = \theta + \epsilon_i$, where $\mathrm{Var}(\epsilon_i) = \sigma^2 > 0$. In this section, we analyze the noisy case.

For our purpose, let us concentrate on the following specific form of strategy profile;

(3-I) S_i with v_i announces $m_i = v_i$.

(3-II) R believes $b(\mathbf{m}) = \max\{m_1, m_2\}$ if $|m_1 - m_2| \leq \rho$ and believes $b(\mathbf{m}) = \min\{m_1, m_2\}$ if $|m_1 - m_2| > \rho$ for some $\rho > 0$.[13]

(3-III) R chooses $\alpha(\mathbf{m}) = b$.

R's action rule given by (3-III) will be called a "crosschecking strategy".[14] Note that there is no off-the-equilibrium message in this noisy case, because any message can occur even if both tell the truth, as long as ϵ_i follows a normal distribution over $(-\infty, \infty)$.

Now, consider the optimal strategy rules of senders. Sender 1 will maximize

$$\begin{aligned} U^{S_1}(m_1; v_1) &= \int_{-\infty}^{m_1-\rho} u(v_2)h(v_2 \mid v_1)dv_2 + \int_{m_1-\rho}^{m_1} u(m_1)h(v_2 \mid v_1)dv_2 \\ &\quad + \int_{m_1}^{m_1+\rho} u(v_2)h(v_2 \mid v_1)dv_2 + \int_{m_1+\rho}^{\infty} u(m_1)h(v_2 \mid v_1)dv_2. \qquad (1) \end{aligned}$$

The economic reasoning behind this formula goes as follows. Given that sender 2 announces truthfully, the first term and the last term represent the punishment that sender 1 would get when v_2 is very low ($v_2 < m_1 - \rho$) and when v_2 is very high ($v_2 > m_1 + \rho$) respectively. The second and the third terms indicate his utility when v_2 falls under a normal (reward)

[13] As we put in Footnote 8, these on-the-equilibrium beliefs are perfectly legitimate beliefs about θ, since the weak consistency condition of wPBE requires $b_1(m_1)$ and $b_2(m_2)$ to be consistent with the equilibrium messaging strategies of senders but does not impose any requirement on the estimator for θ based on $b_1(m_1)$ and $b_2(m_2)$.

[14] Navin Kartik commented on the monotonicity of our strategies. We could say that the receiver's strategy is monotonic if $\partial\alpha(m_1, m_2)/\partial m_i \geq 0$. It is not difficult to see that the cross-checking strategy is not monotonic. If we impose monotonicity of R's strategy, there would be no equilibrium other than the babbling equilibrium in this game.

region. Suppressing v_1, we have

$$\begin{aligned}\frac{\partial U^{S_1}}{\partial m_1} &= u(m_1-\rho)h(m_1-\rho)+u'(m_1)\int_{m_1-\rho}^{m_1}h(v_2)dv_2+u(m_1)(h(m_1)-h(m_1-\rho)) \\ &\quad +u(m_1+\rho)h(m_1+\rho)-u(m_1)h(m_1) \\ &\quad +\left[u'(m_1)\int_{m_1+\rho}^{\infty}h(v_2)dv_2-u(m_1)h(m_1+\rho)\right]. \qquad (2)\end{aligned}$$

The first term is the loss from being punished by increasing his announcement marginally (when v_2 is very low), and the last term is the gain from avoiding punishment (when v_2 is very high). The remaining terms are just the effect of utility increases in normal cases due to the inflated announcement.

If $u(a)$ is linear, the first term and the last term are cancelled out due to symmetry, so $\frac{\partial U^{S_1}}{\partial m_1} > 0$, for all m_1, but if u is strictly concave, the loss will be larger than the gain in absolute values, and thus, it may not be necessarily that $\frac{\partial U^{S_1}}{\partial m_1} > 0$ for all m_1.

Truthful revelation requires $\frac{\partial U^{S_1}}{\partial m_1}|_{m_1=v_1} = 0$. This implies that

$$\begin{aligned}\frac{\partial U^{S_1}}{\partial m_1}|_{m_1=v_1} &= h(v_1-\rho)(u(v_1-\rho)-u(v_1))+h(v_1)(u(v_1)-u(v_1)) \\ &\quad +h(v_1+\rho)(u(v_1+\rho)-u(v_1))+u'(v_1)\left(\int_{v_1-\rho}^{v_1}h(v_2)dv_2+\int_{v_1+\rho}^{\infty}h(v_2)dv_2\right) \\ &= 0. \qquad (3)\end{aligned}$$

By using $h(v_1-\rho)=h(v_1+\rho)$ and $\int_{v_1}^{v_1+\rho}h(v_2)dv_2 = \int_{v_1-\rho}^{v_1}h(v_2)dv_2$, equation (3) is reduced to

$$2h(v_1-\rho)\left[u(v_1)-\frac{u(v_1-\rho)+u(v_1+\rho)}{2}\right] = \frac{u'(v_1)}{2}. \qquad (4)$$

This equation implies that the equilibrium value for ρ must balance the expected net loss from inflating the message, which is the left hand side (LHS), with the direct gain from the inflated message, which is the right hand side (RHS).

If $u(\cdot)$ is linear, LHS is zero, which implies that a sender always has an incentive to inflate his message, since it incurs no net penalty in expected terms.

Proposition 3 *In the noisy case, there is no communicative equilibrium if the utility function $u(a)$ is linear.*

If sender 1 increases m_1, there are the gain due to a transition from the punishment interval to the reward interval $(u(m_1+d)-u(m_1))h(m_1+d)$ and the loss due to a transition from the reward interval to the punishment interval $(u(m_1-d)-u(m_1))h(m_1-d)$. The two conflicting effects due to region changes are cancelled out and thus only the positive effect of inflating information remains. It is surprising that introducing even a small noise would overturn the truth-revealing equilibrium. Even a small noise would make all messages possible in equilibrium by vanishing any off-the-equilibrium path thereby making it difficult to punish a sender who sends a high message by crosschecking strategy.

If $u''(\cdot) < 0$, however, equation (4) can have a solution for ρ (which is independent of v_1), since $\frac{u(v_1-\rho)+u(v_1+\rho)}{2} - u(v_1) < 0$ due to the concavity of u and $u'(v_1) > 0$. We will denote the solution by ρ^*.

The effect of an increase in σ^2 on ρ^* is ambiguous. First, note that RHS of equation (4) depends on neither σ^2 nor ρ. Now, suppose σ^2 gets larger, i.e., the probability that m_1 and $m_2(=v_2)$ fall outside the non-punishment region gets higher. Then, to maintain the expected loss (LHS) equal to the gain (RHS), one must choose a larger ρ^* to recover the penalty probability to the original lower level. That is, as information is less accurate, the receiver must use a more lenient strategy which allows a wider confidence interval.

On the other hand, increasing ρ has another effect. Raising ρ not only lowers the penalty probability, but also increases the net loss from the penalty strategy itself, since the expected utility from increasing m_1, $\frac{u(m_1+\rho)+u(m_1-\rho)}{2}$, decreases in ρ. (See Figure 1.) If this effect dominates the former effect on the penalty probability, the expected loss due to an increase in ρ could be larger. Thus, in this case, ρ^* must be adjusted to a lower level if σ^2 is larger.

Now, consider the limiting case of σ^2. Given any fixed ρ, if σ^2 keeps falling, the penalty probability approaches zero, while the loss remains the same (because the loss is independent of σ^2). (See Figure 2.) Therefore, the expected net loss from inflating m converges to zero, implying that senders will have an incentive to inflate their messages; hence, no communicative equilibrium for low σ^2.

Example Let $u(a) = 1 - e^{-a}$. Note that $u'(a) = e^{-a} > 0$ and $u''(a) = -e^{-a} < 0$. Equation (4) which characterizes the first order condition of the incentive compatibility constraint can be written as

$$2h(v_1-\rho)\left[(1-e^{-v_1})-\frac{1-e^{-v_1+\rho}+1-e^{-(v_1+\rho)}}{2}\right]=\frac{1}{2}e^{-v_1}. \tag{5}$$

This is reduced to

$$4h(v_1-\rho)(\frac{e^{\rho}+e^{-\rho}}{2}-1)=1, \tag{6}$$

where $\frac{e^{\rho}+e^{-\rho}}{2}\geq 1$ with equality if $\rho=0$. This determines the equilibrium confidence interval ρ. Moreover, since $v_2(=v_1+\epsilon_2-\epsilon_1)$ has the distribution of $N(v_1,2\sigma^2)$, we have

$$\begin{aligned} H(v_1-\rho) &= \text{Prob}(v_2\leq v_1-\rho) \\ &= \Phi\left(\frac{v_1-\rho-v_1}{\sqrt{2}\sigma}\right) \\ &= \Phi\left(\frac{\rho}{\sqrt{2}\sigma}\right), \end{aligned}$$

where $\Phi(x)=\frac{1}{2}\left[1+\text{erf}\left(\frac{x}{\sqrt{2}}\right)\right]$. Note that this probability does not depend on v_1, so ρ^* does not, either. Figure 3 shows for this particular utility function that (i) there exists $\bar{\sigma}(>0)$ such that the first order condition is satisfied for some $\rho^*(\sigma)$ whenever $\sigma\geq\bar{\sigma}$, (ii) $\rho^*(\sigma)$ is increasing in σ for most of the values of σ, and (iii) there does not exist $\rho^*(\sigma)$ for very low values of σ. The appendix also shows that this solution satisfies the second order condition and global optimality.

The following proposition slightly generalizes this numerical example.

Proposition 4 *In the noisy case, there exists $\bar{\sigma}>0$ such that for any $\sigma\geq\bar{\sigma}$, there exists a truth-revealing equilibrium for some $\rho>0$ which is independent of v_1 and v_2 if the utility function u is any negative affine transformation of e^{-a}, i.e., $u(a)=\gamma-\beta e^{-a}$ where $\beta,\gamma>0$.*

This proposition says that the utility function $u(a)=\gamma-\beta e^{-a}$ satisfies the differential equation given by (4) for some ρ which is independent of v_i, implying that under this utility function, there is a possibility that there exists ρ^* characterizing the cross-checking strategy and moreover, it does not depend on v_1 and v_2. This utility function enables senders to reveal truth by making the punishment larger than the reward when a sender inflates his information.

A drawback of this proposition is that the existence of the truth-revealing equilibrium is not guaranteed if σ^2 is very low. The following proposition strengthens the result.

Proposition 5 *In the noisy case, there exists a truth-revealing equilibrium for some $\rho > 0$ which is independent of v_1 and v_2 if the utility function is $u(a) = \gamma - \beta e^{-a/\sigma}$ where $\beta, \gamma > 0$.*

This proposition says that if $u(a) = \gamma - \beta e^{-a/\sigma}$, truth-telling is an equilibrium for *any* σ, i.e., that no lower bound for σ exists for the truth-revealing equilibrium. The utility function reflects the reality that a sender's utility from consuming network goods is reduced by the noise of his information. So, we can interpret a/σ as the effective network size which is discounted by the noise of private information. In fact, this scaling has the effect of normalizing σ to one. This guarantees the existence of the optimal ρ^* which turns out to be $\rho^* = 1.697\sigma$.

6 Conclusion

We have shown that one sender can be disciplined by the presence of the other sender so that each sender will reveal its information truthfully for fear of being penalized by conveying false information. In reality, the information of the quality of a newly introduced experience good is diffused by word-of-mouth communication from existing users. This paper provides an explanation for why word-of-mouth communication should convey reliable information on the quality of network goods.

Even though the arguments in this paper have been made within a limited context of word-of-mouth communication about the quality of an experience good, the general insight behind them can be carried over to enormous economic situations in which multiple parties possess some information relevant to a certain decision-making. For instance, college professors may want more students of his own to be admitted to decent graduate schools - which can be thought of as network externalities. If a professor does not care about his reputation at all – this is usually the case for a professor from abroad –, he will always write the most favorable recommendation letters as he can. This is the reason why most graduate schools do not believe references from foreign countries. Of course, this is one equilibrium (babbling equilibrium). However, apart from the reputational consideration, a professor – even a foreign professor – sometimes writes a very sincere and fair letter for fear that his student may be rejected by the simple reason that his evaluation is too much different from another professor's evaluation.

Also, our result can be straightforwardly extended to n senders rather than two senders. With n senders, the cross-checking strategy will be of the form; $\alpha^*(\mathbf{m}) = b$ where

$$b(\mathbf{m}) = \begin{cases} \max\{m_1, m_2, \cdots, m_n\} & \text{if } \max\{|m_i - m_j| : i \neq j\} \leq \rho \\ \min\{m_1, m_2, \cdots, m_n\} & \text{if } \max\{|m_i - m_j| : i \neq j\} > \rho, \end{cases}$$

for some $\rho > 0$, although the computations for the equilibrium value of ρ will be very complicated. It will be left to readers.

An idea analogous with our insight, although using costly signals rather than cheap talk, was discussed in the industrial-organization literature by Bagwell and Ramey (1991). In their model, multiple incumbent firms face a potential entrant. They showed that one incumbent with unfavorable private information on the industry cost level could not pretend to be one with favorable information by deviating from its static Nash equilibrium price, since it could not coordinate its defection with the other incumbent sharing the information.

Some may suspect that our finding is not a good representation of the real world. The source of this suspicion is the assumption that the existence of the other speaker is common knowledge to both referees as well as the uninformed party. Thus, a more plausible scenario would be to assume that the number of referees is the private information of the uninformed party. This may be an interesting research agenda.

Appendix

Proof of the Solution for the Example:

(i) The Second Order Condition of the Incentive Compatibility Constraint: We have

$$\begin{aligned} \frac{\partial^2 U^{S_i}}{\partial m_i^2} = \; & h'(m_i - \rho)(u(m_i - \rho) - u(m_i)) + h(m_i - \rho)(u'(m_i - \rho) - u'(m_i)) \\ & + h'(m_i + \rho)(u(m_i + \rho) - u(m_i)) + h(m_i + \rho)(u'(m_i + \rho) - u'(m_i)) \\ & + u''(m_i)\left(\int_{m_i - \rho}^{m_i} h(v_j)dv_j + \int_{m_i + \rho}^{\infty} h(v_j)dv_j\right) \\ & + u'(m_i)(h(m_i) - (h(m_i - \rho) + h(m_i + \rho))), \end{aligned} \tag{7}$$

and thus,

$$\begin{aligned}\left.\frac{\partial^2 U^{S_i}}{\partial m_i^2}\right|_{m_i=v_i} &= h'(v_i-\rho)(u(v_i-\rho)-u(v_i+\rho)) \\ &+h(v_i-\rho)(u'(v_i+\rho)+u'(v_i-\rho)-2u'(v_i)) \\ &+u'(v_i)(-2h(v_i-\rho)+h(v_i))+\frac{1}{2}u''(v_i). \end{aligned} \tag{8}$$

By using $h(x)=\frac{1}{2\sqrt{\pi}\sigma}e^{-\frac{(x-v_i)^2}{4\sigma^2}}$ and $h'(x)=-\frac{x-v_i}{4\sqrt{\pi}\sigma^3}e^{-\frac{(x-v_i)^2}{4\sigma^2}}$, we get

$$\begin{aligned}\left.\frac{\partial^2 U^{S_i}}{\partial m_i^2}\right|_{m_i=v_i} &= \frac{\rho}{4\sqrt{\pi}\sigma^3}e^{-\frac{\rho^2}{4\sigma^2}}(e^{-v_i-\rho}-e^{-v_i+\rho}) \\ &+\frac{1}{2\sqrt{\pi}\sigma}e^{-\frac{\rho^2}{4\sigma^2}}(e^{-v_i-\rho}+e^{-v_i+\rho}-2e^{-v_i})) \\ &+e^{-v_i}(-\frac{1}{\sqrt{\pi}\sigma}e^{-\frac{\rho^2}{4\sigma^2}}+\frac{1}{2\sqrt{\pi}\sigma})-\frac{1}{2}e^{-v_i} \\ &= e^{-v_i}\frac{1}{2\sqrt{\pi}\sigma}(\frac{\rho}{2\sigma^2}(e^{-\rho}-e^{\rho})e^{-\frac{\rho^2}{4\sigma^2}}+(e^{\rho}+e^{-\rho}-4)e^{-\frac{\rho^2}{4\sigma^2}}+1-\sqrt{\pi}\sigma) \\ &= e^{-v_i}\frac{1}{2\sqrt{\pi}\sigma}(-\frac{\rho}{\sigma^2}e^{-\frac{\rho^2}{4\sigma^2}}\sinh\rho+(2\cosh\rho-4)e^{-\frac{\rho^2}{4\sigma^2}}+1-\sqrt{\pi}\sigma). \end{aligned} \tag{9}$$

Therefore, the second order condition requires

$$-\frac{\rho}{\sigma^2}e^{-\frac{\rho^2}{4\sigma^2}}\sinh\rho+(2\cosh\rho-4)e^{-\frac{\rho^2}{4\sigma^2}}+1-\sqrt{\pi}\sigma<0. \tag{10}$$

Figure 4 shows that (4) implies (10), i.e., if ρ satisfies the first order condition, then it also satisfies the second order condition. In Figure 4, the red curve is the region in which the first order condition is satisfied, and the interior of the green curve is the region in which the second order condition is met.

(ii) Global Optimality: Since it is clear that a sender will not deviate to $m_i < v_i$, we will check only the incentive to deviate to $m_i > v_i$.

Since $u(a)=1-e^{-a}$, equation (2) can be rearranged into

$$\begin{aligned}\frac{\partial U^{S_i}}{\partial m_i} &= e^{-m_i}(1-e^{-\rho})h(m_i+\rho)-e^{-m_i}(e^{\rho}-1)h(m_i-\rho) \\ &+e^{-m_i}\left[\int_{m_i-\rho}^{m_i}h(v_j)dv_j+\int_{m_i+\rho}^{\infty}h(v_j)dv_j\right]. \end{aligned} \tag{11}$$

We will show $\psi(m_i) \equiv (1-e^{-\rho})h(m_i+\rho)-(e^{\rho}-1)h(m_i-\rho)+\int_{m_i-\rho}^{m_i} h(v_j)dv_j+\int_{m_i+\rho}^{\infty} h(v_j)dv_j < 0,\ \forall m_i > v_i$.

Note that $(1-e^{-\rho})h(m_i+\rho)-(e^{\rho}-1)h(m_i-\rho)$ is decreasing in m_i when $m_i < v_i+\rho$ due to $h'(m_i-\rho) > 0 > h'(m_i+\rho)$. It is clear that $\int_{m_i-\rho}^{m_i} h(v_j)dv_j + \int_{m_i+\rho}^{\infty} h(v_j)dv_j$ is also decreasing in m_i. Since $\psi(v_i) = 0$, we can conclude that $\psi(m_i) < 0$, $\forall m_i \in (v_i, v_i+\rho)$, since $\frac{\partial \psi}{\partial m_i} < 0$.

To check the behavior of $\frac{\partial \psi}{\partial m_i}$ when $m_i \geq v_i + \rho$, let us differentiate ψ with respect to m_i. Without loss of generality, we assume that $v_i = 0$. By using $h(x) = \frac{1}{2\sqrt{\pi}\sigma} e^{-\frac{(x-v_j)^2}{4\sigma^2}}$ and $h'(x) = -\frac{x-v_i}{4\sqrt{\pi}\sigma^3} e^{-\frac{(x-v_j)^2}{4\sigma^2}}$, we obtain

$$\begin{aligned}
\frac{\partial \psi}{\partial m_i} &= -(1-e^{-\rho})\frac{m_i+\rho}{4\sqrt{\pi}\sigma^3} e^{-\frac{(m_i+\rho)^2}{4\sigma^2}} + (e^{\rho}-1)\frac{m_i-\rho}{4\sqrt{\pi}\sigma^3} e^{-\frac{(m_i-\rho)^2}{4\sigma^2}} \\
&\quad + \frac{1}{2\sqrt{\pi}\sigma} e^{-\frac{{m_i}^2}{4\sigma^2}} - \frac{1}{2\sqrt{\pi}\sigma} e^{-\frac{(m_i-\rho)^2}{4\sigma^2}} - \frac{1}{2\sqrt{\pi}\sigma} e^{-\frac{(m_i+\rho)^2}{4\sigma^2}} \\
&= \frac{1}{2\sqrt{\pi}\sigma} e^{-\frac{{m_i}^2}{4\sigma^2}} [-(1-e^{-\rho})\frac{m_i+\rho}{2\sigma^2} e^{-\frac{2\rho m_i+\rho^2}{4\sigma^2}} + (e^{\rho}-1)\frac{m_i-\rho}{2\sigma^2} e^{-\frac{-2\rho m_i+\rho^2}{4\sigma^2}} \\
&\quad + 1 - e^{-\frac{2\rho m_i+\rho^2}{4\sigma^2}} - e^{-\frac{-2\rho m_i+\rho^2}{4\sigma^2}}] \\
&= \frac{1}{2\sqrt{\pi}\sigma} e^{-\frac{{m_i}^2+\rho^2}{4\sigma^2}} [-(1-e^{-\rho})\frac{m_i+\rho}{2\sigma^2} e^{-\frac{2\rho m_i}{4\sigma^2}} + (e^{\rho}-1)\frac{m_i-\rho}{2\sigma^2} e^{\frac{2\rho m_i}{4\sigma^2}} \\
&\quad + e^{\frac{\rho^2}{4\sigma^2}} - e^{-\frac{2\rho m_i}{4\sigma^2}} - e^{\frac{2\rho m_i}{4\sigma^2}}] \\
&= \frac{1}{2\sqrt{\pi}\sigma} e^{-\frac{{m_i}^2+\rho^2}{4\sigma^2}} \phi(m_i), \qquad (12)
\end{aligned}$$

where $\phi(m_i) \equiv -(1-e^{-\rho})\frac{m_i+\rho}{2\sigma^2} e^{-\frac{2\rho m_i}{4\sigma^2}} + (e^{\rho}-1)\frac{m_i-\rho}{2\sigma^2} e^{\frac{2\rho m_i}{4\sigma^2}} + e^{\frac{\rho^2}{4\sigma^2}} - e^{-\frac{2\rho m_i}{4\sigma^2}} - e^{\frac{2\rho m_i}{4\sigma^2}}$.

Observe that $\frac{\partial \psi}{\partial m_i} > 0$ for all sufficiently large m_i and $\psi \to 0$ as $m_i \to \infty$. This leads us to conclude that $\psi(m_i) < 0$, $\frac{\partial \psi}{\partial m_i} > 0$ for all sufficiently large m_i. Now, if $\psi(m_i) > 0$ for some $m_i \geq \rho$, there must be at least three solutions for $\frac{\partial \psi}{\partial m_i} = 0$ in $(0, \infty)$. Now, ϕ can be further reduced to

$$\phi = (px+q)e^{x} - (rx+s)e^{-x} + t, \qquad (13)$$

where $x = \frac{\rho m_i}{2\sigma^2}$, $p = e^{\rho}-1$, $q = -1$, $r = 1$ and $t = e^{\frac{\rho^2}{4\sigma^2}}$ for $\sigma \geq \bar{\sigma}$. ($\bar{\sigma}$ is determined in Proposition 4.)

Claim 1 *There exists ρ such that (ρ, σ) for $\sigma \geq \bar{\sigma}$ satisfies the first order condition and $\phi(x) = 0$ has at most two solutions such that $x > 0$.*

It is reduced to show that $(px+p+q)e^{2x} = -rx+r-s$ has at most one zero such that $x > 0$. In the case that $p+q \geq 0$, $(px+p+q)e^{2x}$ is increasing for $x > 0$ and $-rx+r-s$ is decreasing for $x > 0$. Hence, it has at most one zero such that $x > 0$. Now, we can assume that $p+q < 0$. Note that $p+q = e^{\rho}(r-s)$ and $p = e^{\rho}r$. Using this, we have

$$\begin{aligned} & (px+p+q)e^{2x} = -rx+r-s \\ \Leftrightarrow\ & (px+p+q)e^{2x} - rx - r + s = -2rx \\ \Leftrightarrow\ & (px+p+q)(e^{2x} - e^{-\rho}) = -2rx. \end{aligned}$$

The y-intercept of $(px+p+q)(e^{2x}-e^{-\rho})$ is $(p+q)(1-e^{-\rho})$ which is negative. $-2rx$ is decreasing for $x > 0$ and passes $(0,0)$. So it is enough to show that $(px+p+q)(e^{2x}-e^{-\rho})$ is convex for $x > 0$, i.e., its second derivative is positive. (Then $(px+p+q)(e^{2x}-e^{-\rho}) = -2rx$ would have only one solution.) Define $\psi(x) = (px+p+q)(e^{2x}-e^{-\rho})$. Then, we have

$$\psi' = p(e^{2x}-e^{-\rho}) + 2e^{2x}(px+p+q),$$

$$\psi'' = 2pe^{2x} + 2pe^{2x} + 4e^{2x}(px+p+q) = 8pe^{2x} + p + q \geq 9p + q,$$

whenever $x > 0$.

So it is enough to show that $9p+q > 0$ i.e. $9\frac{e^{\rho}-1}{\rho} - \frac{\rho(e^{\rho}-1)}{2\sigma^2} - 1 > 0$. Note that the upper region of the green curve in Figure 5 satisfies $9p+q > 0$. This completes the proof.

Figure 6 shows the global optimality of $m_i = v_i$, i.e., $\frac{\partial U^{S_i}}{\partial m_i} < 0$ for all $m_i > v_i$.

Proof of Proposition 4:

We can rewrite $u(a)$ as following:

$$u(a) = \gamma - \beta e^{-a} = \gamma(1 - \frac{\beta}{\gamma}e^{-a}) = \gamma(1 - e^{-(a - \ln\beta/\gamma)}). \tag{14}$$

The graph of $u(a)$ is obtained simply by scaling of the vertical axis and transition of the a-axis. This does not change the first order condition and second order condition given by (6) and (10).

Proof of Proposition 5:

Let $X = x/\sigma$. Since $\mathrm{Var}(X) = \mathrm{Var}(x/\sigma) = 1$, the proof is immediate.

References

[1] Ambrus, A. and S. Takahashi, 2008, Multi-Sender Cheap Talk with Restricted State Spaces, Theoretical Economics 3, 1-27

[2] Austen-Smith, D., 1993, Interested Experts and Policy Advice: Multiple Referrals under Open Rule, Games and Economic Behavior 5, 3-43

[3] Bagwell, K. and G. Ramey, 1991, Oligopoly Limit Pricing, RAND Journal of Economics 22, 155-172

[4] Battaglini, M., 2002, Multiple Referrals and Multidimensional Cheap Talk, Econometrica 70, 1379-1401

[5] Charkraborty, A. and R. Harbaugh, 2010, Persuasion by Cheap Talk, American Economic Review 100, 2361-82

[6] Crawford, V. and J. Sobel, 1982, Strategic Information Transmission, Econometrica 50, 1431-1451

[7] Ellison, G. and D. Fudenberg, 1995, Word-of-Mouth Communication and Social Learning, Quarterly Journal of Economics 110, 93-125

[8] Epstein, D., 1998, Partisan and Bipartisan Signalling in Congress, forthcoming in Journal of Law, Economics and Organization

[9] Farrell, J., 1993, Meaning and Credibility in Cheap-Talk Games, Games and Economic Behavior 5, 514-531

[10] Farrell, J. and G. Saloner, 1985, Standardization, Compatibility, and Innovation, Rand Journal of Economics 16, 70-83

[11] Galeotti, A., C. Ghiglino and F. Squintani, 2013, Strategic Information Transmission Networks, Journal of Economic Theory 148, 1751-1769

[12] Gibbons, R., 1989, Learning in Equilibrium Models of Arbitration, American Economic Review 78, 896-912

[13] Gilligan, T. and K. Krehbiel, 1989, Asymmetric Information and Legislative Rules with a Heterogeneous Committee, American Journal of Political Science 33, 459-490

[14] Kartik, N., 2009, Strategic Communication with Lying Costs, Review of Economic Studies 76, 1359-1395

[15] Krishna, V. and J. Morgan, 2001, A Model of Expertise, Quarterly Journal of Economics, 116, 747-775

[16] Li, Z., H. Rantakari and H. Yang, 2016, Competitive Cheap Talk, Games and Economic Behavior 96, 65-89

[17] Li, M., 2010, Advice from Multiple Experts: A Comparison of Simultaneous, Sequential, and Hierarchical Communication, The BE Journal of Theoretical Economics 10, 1

[18] Lipnowski, E. and D. Ravid, 2018, Cheap Talk with Transparent Motives, Mimeo

[19] McGee, A., and H. Yang, 2013, Cheap Talk with Two Senders and Complementary Information, Games and Economic Behavior 79, 181-191

[20] Satterthwaite, M., 1979, Consumer Information, Equilibrium Industry Price, and the Number of Sellers, Bell Journal of Economics 10, 483-502

[21] Seidmann, D., 1990, Effective Cheap Talk with Conflicting Interests, Journal of Economic Theory 50, 445-458

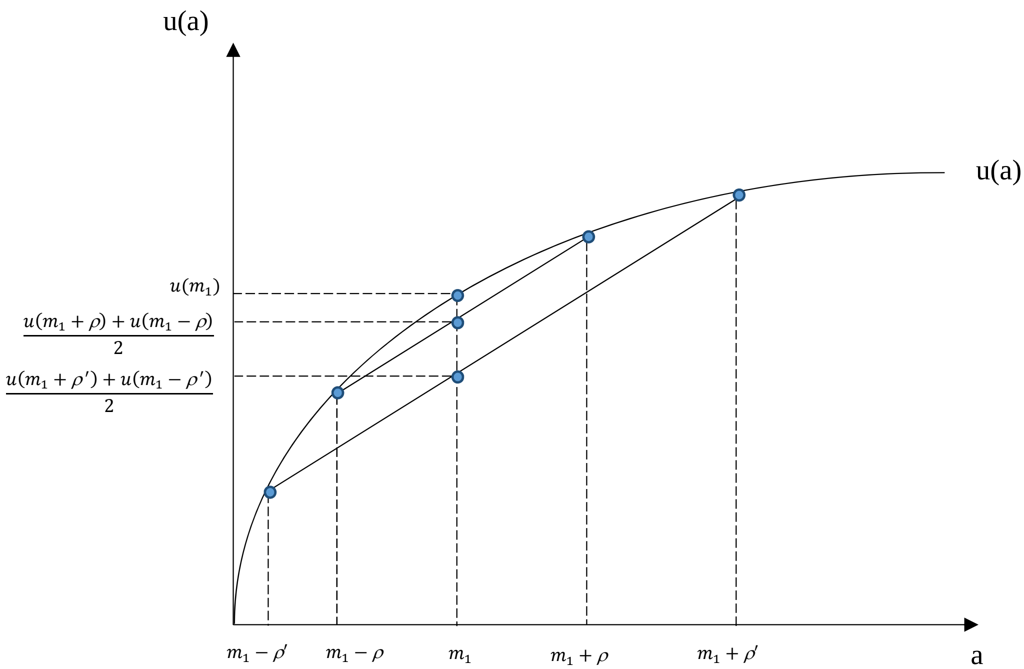

Figure 1: The effect of an increase in ρ ($\rho' > \rho$)

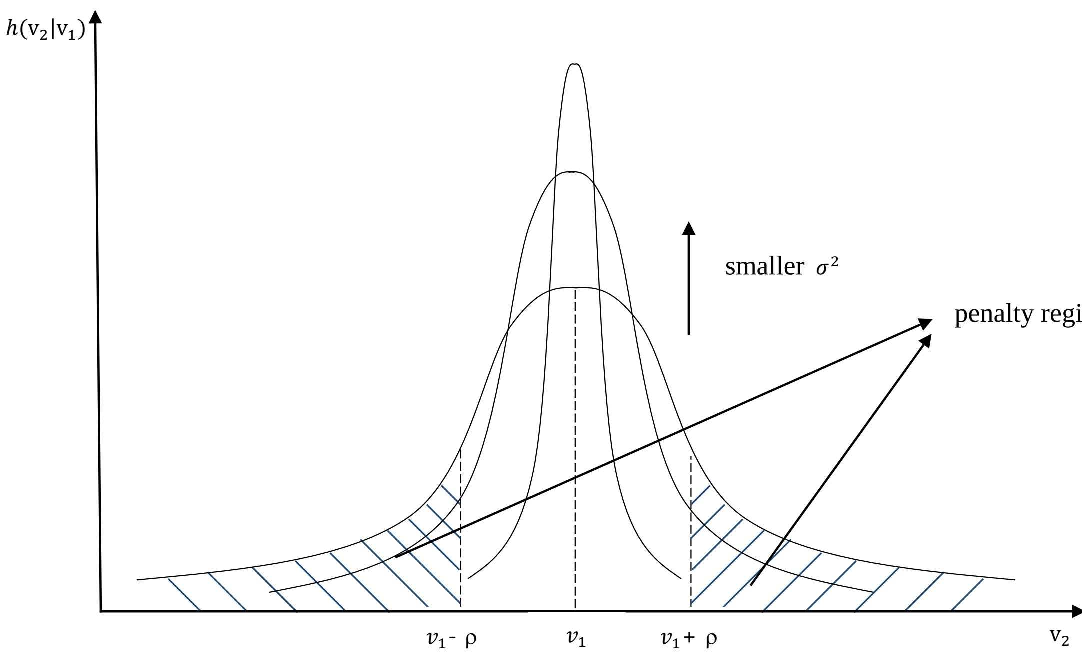

Figure 2: The effect of a fall in σ

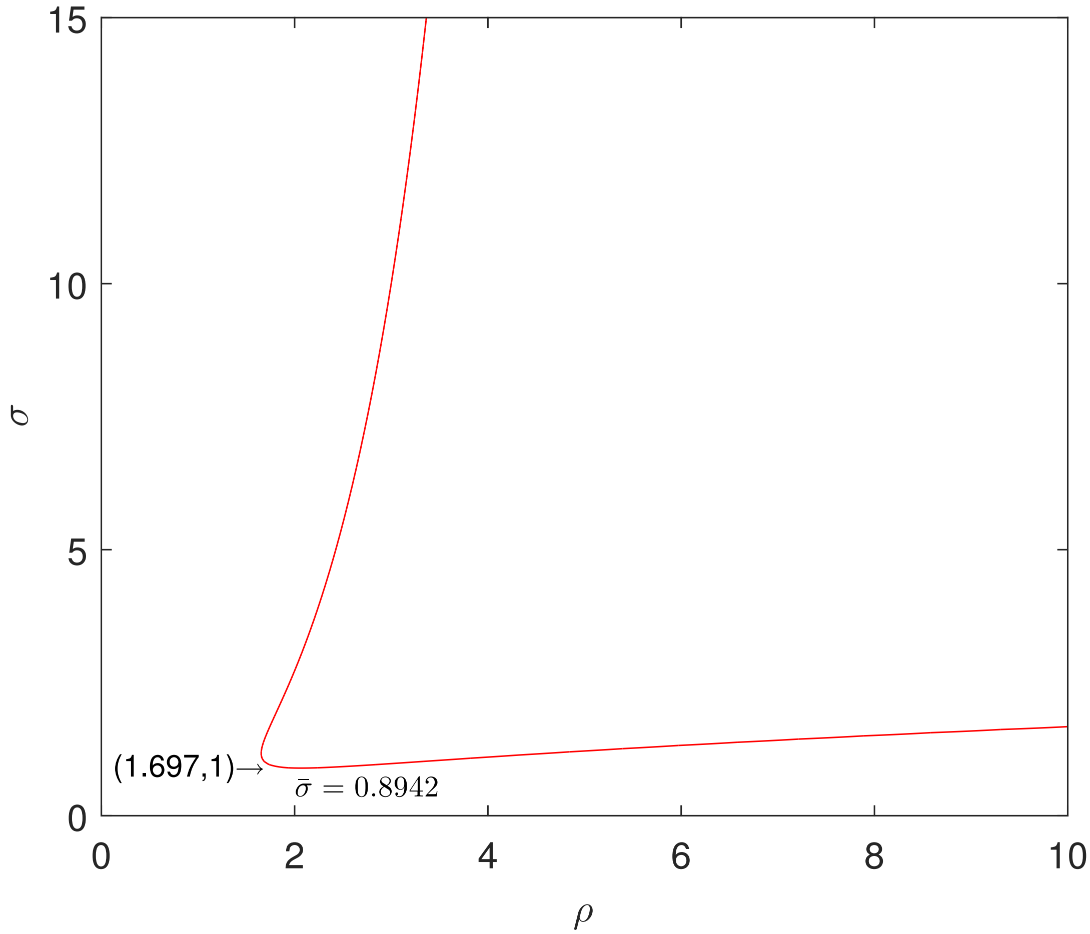

Figure 3: Optimal ρ^* for various values of σ

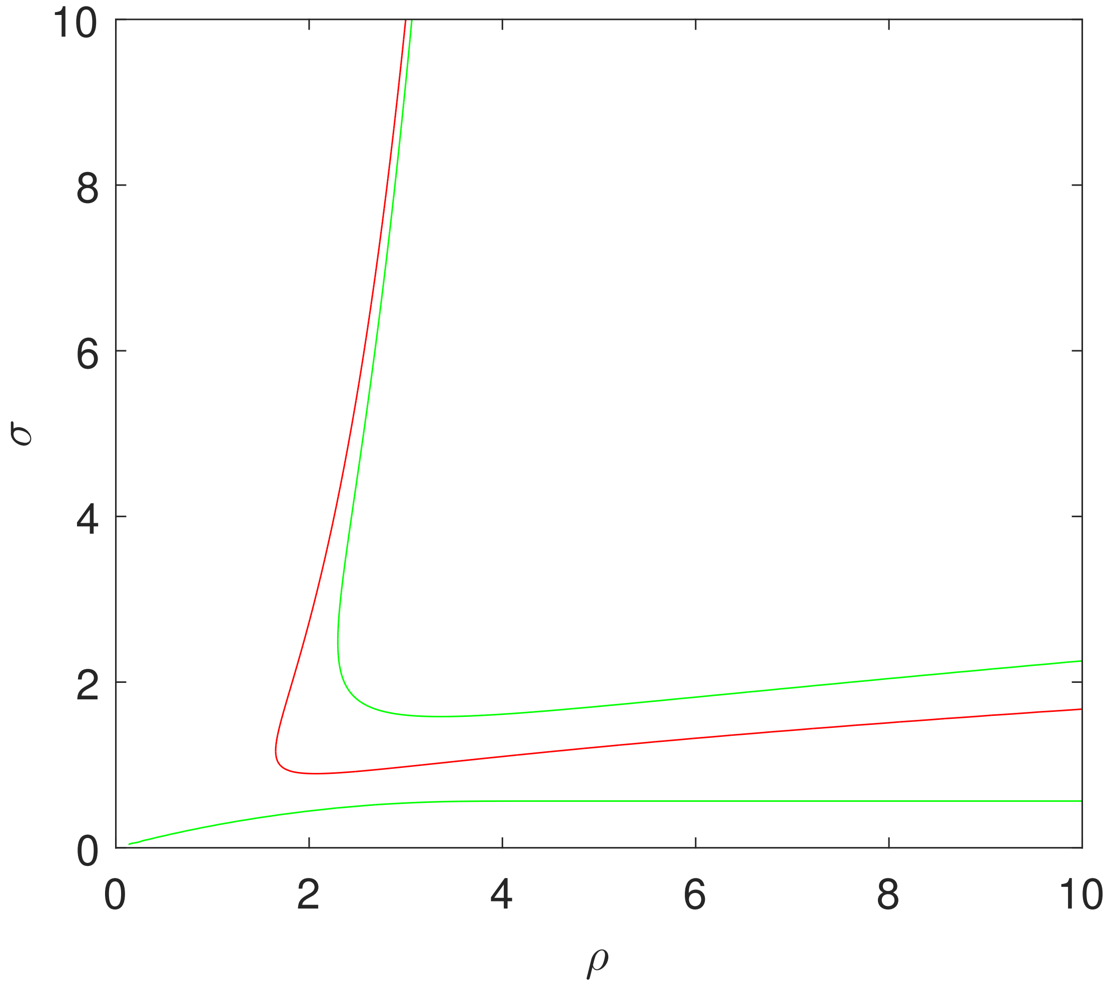

Figure 4: Region where the second order condition is satisfied

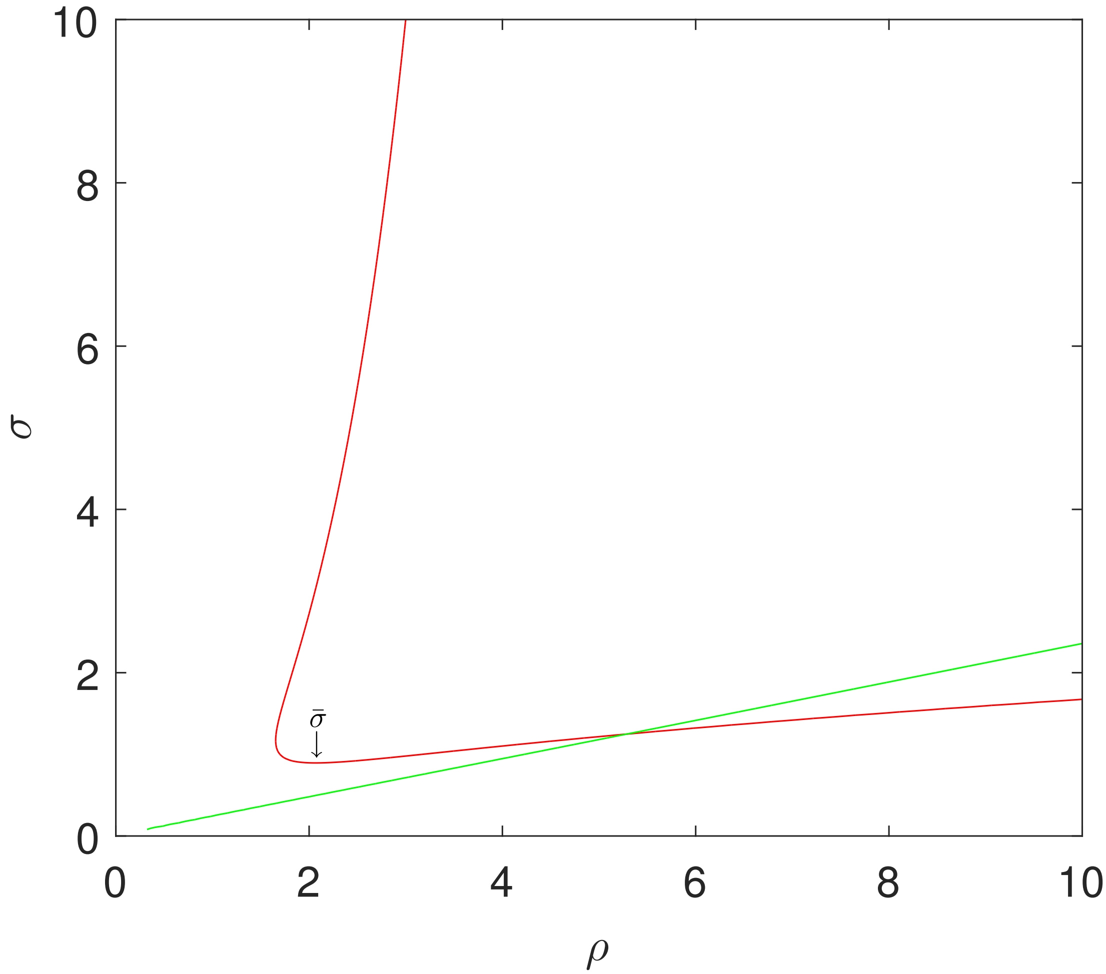

Figure 5: Region where $9\frac{e^{\rho}-1}{\rho} - \frac{\rho(e^{\rho}-1)}{2\sigma^2} - 1 > 0$ is satified

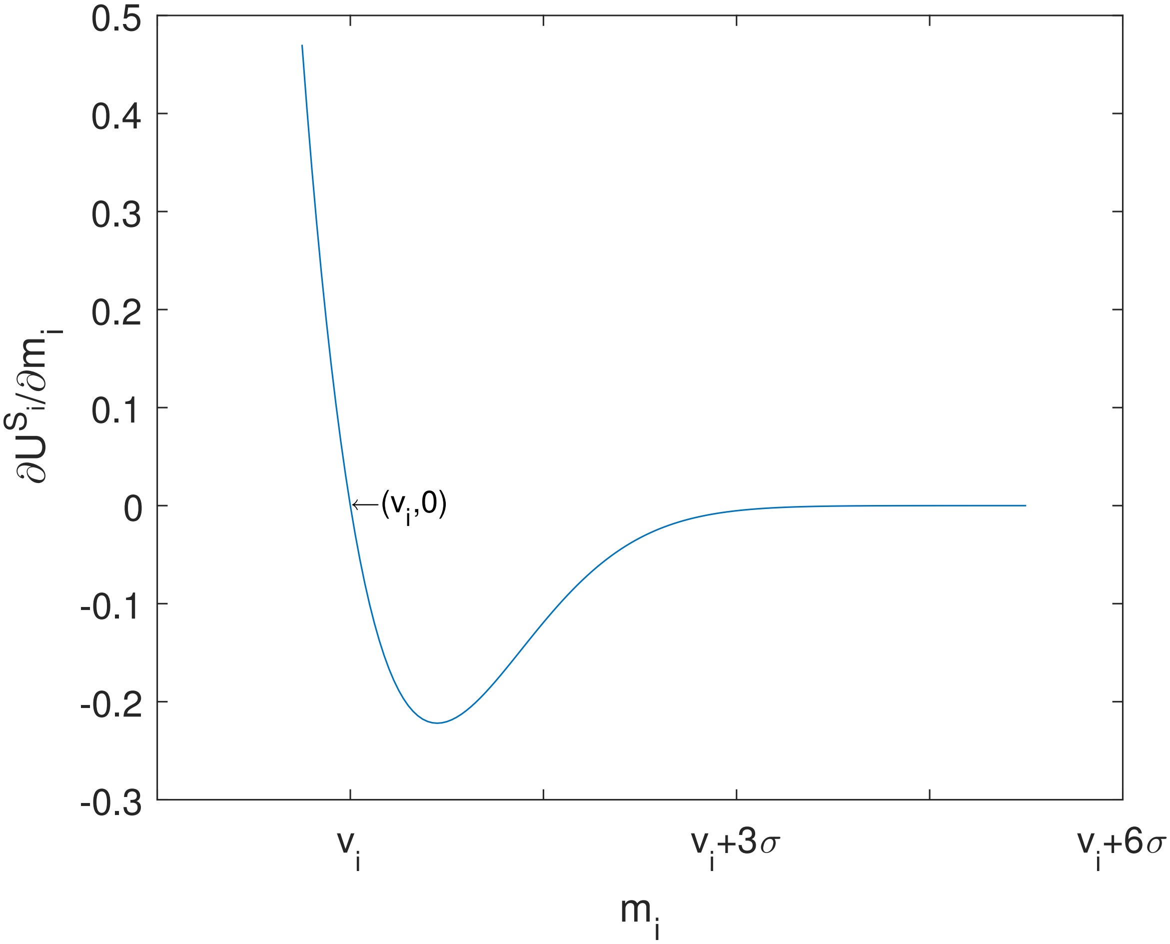

Figure 6: Global optimality of $m_i = v_i$