We thank the referee for his comments and suggestions, although we strongly disagree with virtually all of them.

Our paper sets out to do three things:

(1) To revisit the original FTPL universe (flexible prices, exogenous policy rate) and to rebut the arguments recently made to reassert its intellectual respectability. This includes our discussion of the FTPL as an equilibrium selection device, where we base our argument on Kocherlakota and Phelan (1999) and extend the FTPL world to the flexible price level, exogenous nominal money stock universe.

(2) To argue that Sims's (2011, 2016) attempts to revive the FTPL (or perhaps the policy message of the FTPL) in a New-Keynesian or Old-Keynesian setting fail. Imposing the IBC of the State, holding with equality, leads to an overdetermined model. Not imposing the IBC of the State and working with what then are quite conventional New- and Old-Keynesian models *and* imposing non-Ricardian fiscal-financial-monetary rules (often including some form of fiscal dominance plus financial repression) can easily lead to explosive public debt dynamics. Longer-duration debt of the State does not come systematically to the rescue of the non-Ricardian government. Any policy conclusions drawn from these models are therefore not robust.

(3) To argue that points (1) and (2) matter not only because it is our scholarly duty to expose fallacies, but also because the FTPL and Sims's fiscal dominance plus financial repression models are a very dangerous guide to policy. Should some unfortunate country have its policy makers stray onto the FTPL-Sims paths, at least a belated sharp fiscal correction and at worst a sovereign default or hyperinflation could result. Any policy conclusions drawn from these models are therefore both not robust and dangerous.

These three points make our paper are a seamless, if expansive web. It is long, but not too long. The paper is long also because it makes the arguments both analytically and intuitively. We want to reach the widest possible audience because the issues are so important. It does not often happen in economics that a logical howler makes it into the literature, especially with distinguished names attached. We can only think of two that meet the high standard set by the FTPL.

The first is Jacob Viner's description, in a footnote to his famous article "Cost Curves and Supply Curves", of his attempt to persuade a draftsman to make the long-run average cost curve a U-shaped envelope that consisted of the minimum points on all short-run average cost curves. The draftsman pointed that this was impossible[1] and could (fortunately) not be persuaded by Viner to attempt to do so.[2] A second well-known example involves incorrect proofs by David Levhari (1965) and Paul Samuelson (1962) that reswitching cannot occur if certain conditions are satisfied. Even Nobel laureates occasionally get it

---

[1] Unless the minimum points on all short-run average cost curves were at the same average cost level.

[2] *"My instructions to the draftsman were to draw the AG curve so as never to be above any portion of any ac curve. He is a mathematician, however, not an economist, and he saw some mathematical objection to this procedure which I could not succeed in understanding. I could not persuade him to disregard his scruples as a craftsman and to follow my instructions, absurd though they might be."* Viner (1931, p. 36, footnote 2).

wrong. Both graciously retracted their errors in Levhari and Samuelson (1965) (see also Samuelson (1966)).[3]

The referee's simple version of the model

We will use the referee's simple model to make a few important points. B_t is the nominal stock of one-period nominal bonds outstanding at the beginning of period t. The contractual price of this bond in terms of money is 1. The contractual price is the price that would prevail in the absence of default risk; S_t is the real primary (non-interest) surplus, including seigniorage, in period t; G_t is real public purchases in period t; T_t is the real value of taxes plus seigniorage in period t (the referee forgets to include real seigniorage in T); Y_t is real household income (before taxes); C_t is real household consumption in period t; i_t is the period t nominal interest rate; P_t is the general price level in period t.

The government (consolidated Treasury and central bank) period budget constraint is:

$$B_{t+1} = (1+i_t)B_t + P_t(G_t - \mathrm{T}_t)$$

Let $1+r_t = (1+i_t)P_t / P_{t+1}$ be the period t real interest factor.

The government period budget constraint can be rewritten as:

$$b_{t+1} = (1+r_t)b_t + G_t - \mathrm{T}_t \tag{1}$$

By definition:

$$S_t = T_t - G_t \tag{2}$$

This can be solved forward recursively to yield:

$$b_t = \sum_{k=0}^{\infty}\prod_{j=0}^{k}\left(\frac{1}{1+r_{t+j}}\right)S_{t+k} + \lim_{k\to\infty}\prod_{j=0}^{k}\left(\frac{1}{1+r_{t+j}}\right)b_{t+k} \tag{3}$$

The solvency constraint of the government is:

$$\lim_{k\to\infty}\prod_{j=0}^{k}\left(\frac{1}{1+r_{t+j}}\right)b_{t+k} \leq 0 \tag{4}$$

Equations (3) and (4) imply the intertemporal budget constraint (IBC) of the government:

$$b_t \leq \sum_{k=0}^{\infty}\prod_{j=0}^{k}\left(\frac{1}{1+r_{t+j}}\right)S_{t+j} \tag{5}$$

---

[3] Reswitching is the phenomenon of switching back at a very low real interest rate to a set of production techniques that had seemed viable only at a very high real interest rate. See Samuelson (1966).

The period budget constraint of the household is:

$$b_{t+1} = (1+r_t)b_t + Y_t - C_t - T_t \tag{6}$$

Correcting the referees sign error, this implies

$$b_t = \sum_{k=0}^{\infty} \prod_{j=0}^{k} \left(\frac{1}{1+r_{t+j}} \right) (Y_{t+k} - C_{t+k} - T_{t+k}) + \lim_{k \to \infty} \prod_{j=0}^{k} \left(\frac{1}{1+r_{t+j}} \right) b_{t+k} \tag{7}$$

The household solvency constraint is:

$$\lim_{k \to \infty} \prod_{j=0}^{k} \left(\frac{1}{1+r_{t+j}} \right) b_{t+k} \geq 0 \tag{8}$$

Equations (3) and (4) imply the IBC of the household

$$b_t \geq \sum_{k=0}^{\infty} \prod_{j=0}^{k} \left(\frac{1}{1+r_{t+j}} \right) (C_{t+j} + T_{t+j} - Y_{t+j}) \tag{9}$$

If there are optimizing households and there is non-satiation in consumption (and possibly in real money balances too), the IBC of the household will hold with equality:

$$b_t = \sum_{k=0}^{\infty} \prod_{j=0}^{k} \left(\frac{1}{1+r_{t+j}} \right) (C_{t+j} + T_{t+j} - Y_{t+j}) \tag{10}$$

We impose goods market equilibrium:

$$Y_t = C_t + G_t \tag{11}$$

Equations(2) (10) and (11) then imply

$$b_t = \sum_{k=0}^{\infty} \prod_{j=0}^{k} \left(\frac{1}{1+r_{t+j}} \right) S_{t+j} \tag{12}$$

An implication of the IBC of the household holding with equality (because of optimizing behavior and non-satiation) and the assumption of goods market equilibrium is that the IBC of the government holds with equality.

The FTPL treats both the IBC of the household, holding with equality (equation (10)) and the IBC of the government, holding with equality (equation (12)) as separate equilibrium conditions, together with the goods market equilibrium condition (11). Under all except one set of conditions, this leads to an overdetermined system. Under the one set of conditions where the system is not overdetermined (a freely flexible general price level, a positive stock of nominal bonds outstanding and either an exogenous nominal interest rule with a non-negative nominal stock of money outstanding and/or an exogenous nominal money stock rule with a positive stock of money outstanding.

The referee's statement "The authors do not back up their insistence that the FTPL is logically inconsistent with a mathematical proof or demonstration, instead relying on repeated assertion" is demonstrably false. All inconsistencies and anomalies are stated and demonstrated in Section 3C (pp. 20-27). Inconsistency 1 can be found on pages 22 and 23; Anomaly 1 can be found on page 23 with the appropriate reference to the paper by Kocherlakota and Phelan (1999); Anomaly 2 can be found on pages 24 and 25; Anomaly 3 can be found on page 25; Anomaly 4 can be found on page 25; Anomaly 5 can be found on pages 25 and 26; Anomaly 6 can be found on pages 26 and 27. Inconsistency 2 can be found on pages 29 and 30. What more does the referee want?

Anomaly and inconsistency

Let's provide a simple example based on the little model in this rejoinder. Assume real household output and real public purchases are exogenous and constant: $Y_t = \bar{Y} > G_t = \bar{G}$. Assume that real taxes including seigniorage are also constant, so the real augmented primary surplus is constant: $S = \bar{S}$ Assume that households have a time-separable utility functional with a constant pure rate of time preference ρ (and for completeness a constant exogenous nominal interest rate). Assume that the price level is freely flexible. It follows that $r_t = \rho$ and the equilibrium of the model can be written as follows:

$$\bar{Y} = C + \bar{G}$$
$$P_{t+1}/P_t = \frac{1+\bar{i}}{1+\rho} \qquad (13)$$
$$b_t = \left(\frac{1+\rho}{\rho}\right)\bar{S}$$

To be able to handle the exogenous money supply case I also add in a standard monetary equilibrium condition; M_t is the nominal money stock at the beginning of period t. The interest rate on money is zero.

$$\frac{M_t}{P_t} = \frac{\alpha}{\bar{i}}C, \ \alpha \geq 0 \qquad (14)$$

This system is overdetermined if the bonds are index-linked. In an open economy version of the model, it would be overdetermined if bonds are either foreign-currency-denominated or index-linked.

Only if the bonds are nominally denominated and the price level is freely flexible can there be an FTPL equilibrium. But, first, this equilibrium is only not overdetermined because in the conventional (correct or non-FTPL) approach, the model either has nominal indeterminacy (when the nominal interest is exogenous) although real variables other than the real value of the stock of nominal bonds are determinate, or both nominal and (limited) real indeterminacy (when the nominal money stock is exogenous)). And, second, this equilibrium is so anomalous as to be unacceptable economic grounds. We write the last equation of (13) as:

$$\frac{B_t}{P_t} = \left(\frac{1+\rho}{\rho}\right)\bar{S} \tag{15}$$

And yes, using the IBC of the household (holding with equality as an implication of optimization and non-satiation) to derive optimal consumption behavior (and the equilibrium condition determining the real rate of interest) and then using it again as in equation (15) *is* using the same equilibrium condition twice, *pace* the referee's assertion at the bottom of page 2 of his comments. In general, using the same equilibrium condition twice (or once as an equilibrium condition and once as an equilibrium selection device (which is formally equivalent)) leads to an overdetermined model. This is the case for the FTPL except in the case of a freely flexible price level, a positive nominal bond stock and either an exogenous nominal interest rate or an exogenous nominal money stock. When the nominal interest rate is pegged, there is nominal indeterminacy although all real variables are uniquely determined. When the nominal money stock is exogenous, we have nominal indeterminacy and also a restricted form of real indeterminacy: the rate of inflation is indeterminate. In the money-exogenous variant, we normally impose the equilibrium selection criterion that if there exists a stationary solution for stationary fundamentals, that is the equilibrium we select. Most of the time this gives a unique solution.

When the nominal interest rate is pegged, can the FTPL serve as equilibrium selection mechanism? We argue: no, because of the anomalies the FTPL generates even when the price level is flexible and the nominal interest rate is exogenous.

An obvious anomaly is that if $\mathrm{sgn}(B_t) \neq \mathrm{sgn}(S_t)$ we have a negative price level. Note also that, although the model can incorporate money and seigniorage revenue, every equation remains unchanged if there is no money in the model, except as a pure numeraire; just set $\alpha = 0$ in equation (14). So the model permits the determination of the price of something of which there is and always will be, a zero quantity outstanding: phlogiston. The "logic" of the model can also be applied to the IBC of an individual household, if we assume that an optimizing government always follows Ricardian fiscal-financial-monetary rules which ensure that its IBC is satisfied with equality. This means the FTPL can be turned into a Mrs. Jones Theory of the Price level, which is economic nonsense. Even an "internally consistent" model (as many equations as unknowns is a necessary condition for this) is only as good as the sum total of its logical implications. When even one of these implications is nonsensical, the model should be scrapped.

With very little additional work it can also be shown that in any model with a predetermined general price level, the FTPL (that is, imposing the IBC of the State, holding with equality, while also using the IBC of the household, holding with equality, to characterize optimal consumption) results in an overdetermined system. Sims's attempt at resurrecting the FTPL in Sims (2011, 2016) fails. Sims now argues, and we are happy to go along with that, that the IBC of the State is not to be applied in either his Old-Keynesian or his New-Keynesian model, which turns them into conventional economic models with specific non-Ricardian fiscal-financial-monetary programs – often fiscal dominance with financial repression.

The referee suggests we cite the existing literature on index-linking and duration but he does not cite any publications that might suggest that we have failed to cite literature that weakens our case. We are convinced there is no such literature. We believe we have cited all relevant literature in the 55 references we include. The referee suggests we explore the implications of an extended model with the "debt

revaluation factor" D_t included. This is not the purpose of our paper. The primary purpose of our paper is *destructive*: to make the case that the FTPL is a logical nonsense. A secondary purpose is to warn against the kind of fiscal-financial-monetary policy behavior that might be encouraged by a belief that the FTPL is correct. We make the minimum use of the debt revaluation factor necessary for our purposes. A fully articulated " D_t framework" can already be found in Buiter (2002, 2005), but this is not relevant to the issues addressed in our paper. In our paper we show, using the D_t notation, when it is appropriate to treat the IBC of the State (holding with equality) as an equilibrium government bond pricing equation. The distinction between the market value of the bonds and their contractual value is central here. The FTPL, however, confuses the contractual value of government bonds with their market value and makes much of the one set of conditions (flexible prices, positive stock of nominal bonds) where the general price level appears to be able to play the role of the debt revaluation factor. But one way or the other, this leads us back to the unacceptable anomalies and, when we leave the flexible price level/positive stock of nominal bonds world, the inconsistencies of the FTPL.

References

Buiter, Willem H. (2002), "The Fiscal Theory of the Price Level: A Critique," *Economic Journal,* CXII, 459–480.

Buiter, Willem H. (2005), "New developments in monetary economics: Two ghosts, two eccentricities, a fallacy, a mirage and a mythos, "*Economic Journal* 115(502), Mar. 2005, C1-C31, presented as the Royal Economic Society 2004 Hahn Lecture.

Levhari, David (1965), "A Nonsubstitution Theorem and Switching of Techniques", *The Quarterly Journal of Economics*, Vol. 79, No. 1 (Feb., 1965), pp. 98-105

Levhari, David and Paul A. Samuelson (1966), "The Nonswitching Theorem is False", *The Quarterly Journal of Economics*, Volume 80, Issue 4, 1 November 1966, Pages 518–519, https://doi.org/10.2307/1882912

Samuelson, Paul "Parable and Realism in Capital Theory: The Surrogate Production Function," Review of Economic
Studies, XXIX (June 1962), 193-207.

Samuelson, Paul A. (1966), "A Summing Up", *The Quarterly Journal of Economics,* Vol. 80, No. 4 (Nov., 1966), pp. 568-583

Sims, Christopher A. (2011): "Stepping on a Rake: The Role of Fiscal Policy in the Inflation of the 1970's," *European Economic Review* 55 (2011) 48ã˘A ¸S56, 55, 48–56.

Sims, Christopher A. (2016): "Active fiscal, passive money equilibrium in a purely backwardlooking model," Discussion paper, Princeton University, http://yftp/OldKeynesianFTPL/BackwardAFPM.pdf.

Viner, Jacob. [1931] 1953. "Cost Curves and Supply Curves", Zeitschrift für Nationalökonomie 3:23-46. In *Readings in Price Theory*, edited by George J. Stigler and Kenneth E. Boulding, 198-232. London: Allen and Unwin