Date-Stamping US Housing Market Explosivity

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Abstract

In this paper we set out to date-stamp periods of US housing price explosivity for the period 1830 – 2013. We make use of several robust techniques that allow us to identify such periods by determining when prices start to exhibit explosivity with respect to its past behaviour and when it recedes to long term stable prices. The first technique used is the Generalized sup ADF (GSADF) test procedure developed by Phillips et al. (2013), which allows the recursive identification of multiple periods of price explosivity. The second approach makes use of Robinson (1994)'s test statistic, comparing the null of a unit root process against the alternative of specified orders of fractional integration. Our analysis date-stamps several periods of US house price explosivity, allowing us to contextualize its historic relevance.

Keywords:
GSADF, Bubble, Structural Breaks, Random Walk, Explosivity, Recursive process
JEL: C22, G15, G14

1. Introduction

In the wake of the recent global financial crisis, it became pertinently clear that bubbles in core asset markets can cause tremendous real economic consequences if abruptly corrected. It is becoming harder to argue that we can simply dismiss the need to intervene during such episodes and simply "mop-up" following corrections (as the previous Fed chairman once argued). This follows as the previously held belief that financial markets have become sufficiently self-stabilizing fails to hold at times, particularly in markets where price corrections do not happen as smoothly.

The steep rise and subsequent fall of US house prices in the late 2000s have been the subject of much debate over the last few years. This follows largely from its role as underlying asset class to many of the derivative instruments that contributed to the financial crisis of 2008.

Property markets and residential houses, in particular, constitute a key asset class to the portfolio of most households worldwide. Abrupt movements of house prices, therefore, have a very real impact on households' abilities to consume and save. This in turn significantly impacts the economy's production and job creation capacity. As such, policies that curb unstable and bubble-like expansions in prices of houses in the economy could be considered a core policy objective, as sharp and sudden corrections in such prices could dramatically impact general price stability in the economy.

Since Shiller (1980) introduced the idea that prices of assets could deviate significantly from their underlying fundamentals (however defined), a large literature has emerged that aims to explain, document and even suggest preventions for asset price bubble formations. Although some efficient market proponents dismiss such notions, most accept that high transaction costs and limits to short selling could indeed lead to prices diverging from fundamental levels. As noted in Glaeser et al. (2008), e.g., such market failures that hamper the ability of markets to correct price inefficiencies is particularly applicable to housing markets, where transaction costs are very high and short selling exceptionally difficult. This implies periods of price inefficiencies, and in particular periods of bubble-like behaviour, could feasibly exist with relatively little scope for arbitrage.

Our aim in this paper is to identify periods of bubble-like house price expansions over the last two centuries for the US market. This will serve to put the most recent bubble episode into historical perspective, and shed light on past price trends. We defer from making policy recommendations

on curbing such behaviour, instead focusing on defining historical periods of US house price explosivity.

The key research question is how to tell when rapidly rising house prices constitute a bubble. Case and Shiller (2003) defines a housing bubble as being driven by home buyers who are willing to pay inflated prices for houses due to their expectations that houses will keep experiencing unrealistic appreciation in the future. This notion might be based on high expected returns, with the "dividend" portion of holding the asset being the value of residing in the residence (or the rental income earned), and the capital gain the expected rising price of the home. In fact, both can be expected to experience periods of rapidly rising prices in the short run, which can fuel the demand for home-buyers and mortgage originators alike, as the value of the underlying asset rises. But, as seen in the US market in 2007, external factors might lead to costly corrections with very real economic impacts felt across income divides.

Indeed, house prices may also experience such costly corrections as a result of deteriorating macroeconomic factors, even though it might not have experienced a rapid increase before. It might also experience a gradual downward correction with little or no noticeable real costs. Our objective is not to estimate the costs or consequences of these periods of explosive price build-ups, but merely to document and contextualize their historic occurrences. We also do not attempt to distinguish between the type of bubble which occurred (albeit "irrational exuberance" or "credit-boom" driven), as we believe this falls outside the scope of our paper.

Our paper's contribution to the literature is to add estimates of past house-price bubbles that have not yet been applied to this asset class. We make use of two robust and efficient techniques that allow us to date-stamp periods of explosivity of these measures. The first technique that we will use is the Generalized sup ADF (GSADF) test procedure developed by Phillips et al. (2013), which is a recursive right-tailed unit root testing procedure that allows the identification of multiple periods of price explosivity. The second approach makes use of Robinson (1994)'s test of unit roots against the alternative of specified orders of fractional integration. We use the approach developed by Balcilar et al. (2015), which extends Robinson (1994)'s test statistic, to allow the identification of multiple periods of deviations from unit root behaviour in the presence of multiple endogenously determined structural breaks at unknown dates. This approach also provides the added benefit of testing a broader range of persistence than that which is measured using the unit root alternative in the first test.

Using these techniques, we identify several periods of explosivity for real US house prices that can be used in future studies.

Our paper is then structured as follows: Section 2 discusses literature relevant to our study. Thereafter, Section 4 outlines the methodologies used to identify periods of explosivity of US house prices, while Section 3 describes the data used in the study. Section 5 discusses our findings, while Section 6 concludes our study.

2. Review of relevant literature

2.1. House Price Bubble Literature

Our study is not the first to consider the historical underlying price trends of housing markets. Jordà et al. (2015) focuses on a similarly long-dated sample of house prices for 17 countries dating back to 1870. In their study, they focus on identifying periods of credit-driven house price bubbles and the economic consequences of such events.[1]

Our first challenge is identifying a fundamental level for house prices from which to deviate in order to define a bubble period. As we face a lack of historical data on measures that have previously been used to define fundamental house prices (including rental prices, construction costs and gross margins to home builders, as suggested by, among others, Himmelberg et al. (2005) and Glaeser et al. (2008), we use another broad measure to define a level to which prices converge.[2] Our premise is that house price movements tend, in the long run, to display stationary behaviour relative to broad price movements in the economy. We thus label periods of positive deviations from such stationarity for sustained periods as episodes of price explosivity. This can be motivated conceptually that during periods where house prices rise at a significantly higher rate than general prices in the economy, we can feasibly expect it to be experiencing inflationary pressures resembling explosive behaviour.

One concern of this approach in anchoring the fundamental level of house prices relative to the aggregate price of goods in the economy, is the heterogeneity in the relative value of houses over time. As discussed in Knoll

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[1]The authors provide persuasive evidence that un-leveraged equity market bubbles have a vastly different real economic impact than credit-driven house price booms.

[2]We also do not directly account for different interest rate regimes, as our focus remains on the historical time-series behaviour of house prices.

et al. (2014), the quality of houses and value of property increases over time. In our study, we make the plausible implicit assumptions that such changes occur over many decades and could be safely assumed to be less important in valuing homes in the short term viz-a-viz general aggregate prices.

Other studies have similarly used price trends to determine periods of high house prices. Studies by Lowe et al. (2002) and Goodhart and Hofmann (2008) define house price booms as periods where their real price indexes exceed some threshold relative to an Hodrick-Prescott (HP) filtered trend. Bordo and Jeanne (2002)'s definition, in contrast, attempts to calculate a long-run fair value, by measuring deviations of the 3-year moving average growth-rate from the series standard deviation. Other studies also focus on sustained peak-trough or trough-peak changes.[3] In a similarly long-dated study, Jordà et al. (2015) use a combination of the above, looking at the divergence of log real house prices from its trend rising above one standard deviation from the calculated HP filtered trend.

Our contribution to the above literature is to apply other novel techniques in defining possible periods of historical house price explosivity. A discussion of the literature on existing asset price bubble techniques follows.

2.2. Methodology literature

Accurately documenting the inflationary build up of asset prices has long interested economists and policy makers alike. A large literature has emerged that have tried to identify and explain the occurrence of asset price bubbles, leading to often divergent views on suitable policy responses following its detection (c.f. Gürkaynak, 2008) for an in-depth discussion of the performance of various bubble detection techniques). Often the difficulty in testing for the presence of bubble-like behavior in asset price series lie in correctly identifying and date-stamping multiple periods of explosivity. Traditional unit root and co-integration tests aimed at identifying such periods (as e.g. proposed by Diba and Grossman, 1988), fail to identify the existence of bubbles that periodically collapse. Evans (1991), e.g., points out that ordinary stationarity tests remain exposed to the possibility of identifying pseudo stationary behaviour when a series in fact displays periodically collapsing bubbles.

Various techniques have been proposed that allow the detection of multiple periods of collapsing speculative bubble in asset prices. Al-Anaswah and

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[3]c.f. Helbling et al. (2005) and Claessens et al. (2009) in this regard

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Wilfling (2011) and Lammerding et al. (2013), e.g., use Markov-switching models to differentiate between regimes of price stability and price explosivity (the latter authors also use a robust Bayesian estimation procedure). Another class of techniques use a sequential unit root testing procedure developed by Phillips and Yu (2011) and Phillips et al. (2011), which built on the indirect stationarity tests suggested by Diba and Grossman (1984) and Hamilton and Whiteman (1985). As noted by Bettendorf and Chen (2013), the key advantage of sequential identification procedures, particularly relevant to our analysis, is that it detects periods of explosivity despite potential misspecifications of the market fundamental process. In this study, we will make use of the generalized version of the sequential ADF tests, developed by Phillips et al. (2013) (PSY hereafter), which is robust to the identification of multiple collapsing bubble episodes. It has since gained ground in its broad empirical applications (c.f. inter alia Bettendorf and Chen, 2013; Etienne et al., 2014; Caspi et al., 2015) and allows consistent date-stamping for the origination and termination of multiple asset price bubbles.

A key challenge when using PSY's approach to identify asset price bubbles, is specifying the true definition of a fundamental level from which prices deviate. Typically, the return to holding the asset, in the form of dividend yields for equities (c.f. Phillips et al., 2013)) and the convenience yield for commodities (c.f. Pindyck, 1993; Lammerding et al., 2013; Gilbert, 2010; Shi and Arora, 2012), is first defined in a pricing equation. Then a bubble component is specified, which, at times, displays explosive behaviour. Although several papers critique this identification of bubble components (e.g. Cochrane, 2009; Pástor and Veronesi, 2006; Cooper, 2010, offer critical discussions on this), explosive or mildly explosive behaviour in asset price series indicate possible market exuberance during the inflationary phase of a bubble, a feature that can be uncovered from recursive testing procedures on time-series data (Phillips et al., 2013; Phillips and Magdalinos, 2007).

Caspi et al. (2015) also use the GSADF approach to identify periods where oil prices deviate from the general price level in the US, as well as levels of oil inventory supplies, respectively. Their use of these measures as proxies for the fundamental price of oil follow from a similar lack of data on historical oil price derivatives used to calculate the convenience yield. Instead, they study periods where the nominal price of oil displays periods of significant build-up relative to the general price level and stock of US oil supply, which both act as credible alternatives to the standard convenience yield.

The second approach that we will use in this study to identify periods

of explosivity tests the null of a unit root process against the alternative of fractionally integrated orders which exceed one. Several studies have in the past used a long memory process to test for explosivity in asset price series using a test statistic developed by Robinson (1994) (e.g. Cuñado et al., 2007; Gil-Alana, 2003, 2008; Balcilar et al., 2015). A key consideration in defining explosive periods are controlling for structural breaks, which, as highlighted by Perron (1989), may lead to the non-rejection of the unit-root hypothesis. Gil-Alana (2003) assumed known structural break dates in their analysis, while Gil-Alana (2008) employed a residuals sum squared approach where a single structural break date was allowed at an unknown time. Our approach follows that of Balcilar et al. (2015) in allowing multiple structural breaks at unknown dates. We then use Robinson (1994)'s LM test statistic to determine the fractional order of integration of the US house price series after controlling for endogenously determined level and trend shifts. We then recursively identify periods where the lower bound of the fractional order exceeds unity, and subsequently return to levels below unity, to allow us to identify explosive periods equivalent to those determined using PSY, (2013)'s GSADF approach. Both approaches are robust to multiple periods of periodically collapsing bubbles, less sensitive to the specific definition of the underlying fundamental process and able to provide recursive date-stamping of explosive periods in the underlying data.

3. Data Description

Our metric of interest in this study is the real house price (RHP) over the annual period of 1830-2013, with the start and end date being purely driven by data availability on house prices at the time of writing. The nominal house price index used is the Winans International U.S. Real Estate Index, which tracks the price of new homes back to 1830. This index is then deflated by the Consumer Price Index (CPI) to derive the real house price (RHP) index.[4] The RHP is then transformed into its natural logarithmic form.

Our motivation for the use of the Winans Real Estate Index (WIREI)follows from its robust design and tracking of a wide geographical sample of US house values. The emphasis of the index design is to allow researchers to study US

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[4]The house price index data was obtained from the Global Financial Database, while the CPI data was downloaded from the website of Robert Sahr (http://oregonstate.edu/cla/polisci/sahr/sahr).

real estate as an asset class. This is particularly useful to our analysis, as we seek to identify periods in the RHP series where US house prices as an asset class, broadly, experienced explosivity. The WIREI aggregates house prices across all the major geographic areas in the US, while aggregating price reports from the Census Bureau, Bureau of Labor and Statistics, as well as work done by Long (1869 - 1936), Gottlieb (1837 - 1868) and Riggleman (1830 - 1836).[5] The benefit of this is that future research can use our results for RHP explosivity to compare it to corresponding periods of explosivity in other asset classes (such as stocks and bonds). The dataset we use has a correlation of roughly 80% for overlapping periods with the shorter dated Shiller House price index, although the latter has a sharper rise and decline in the early 2000s than the WIREI.

From Figure 1 below we see that the CPI-deflated price of houses in the US remained roughly stable from the beginning of our sample until the mid-part of the 20th century.[6]

Knoll et al. (2014) comprehensively discuss the difficulties in defining an appropriate index for housing prices. Jordà et al. (2015) also alludes to the difficulty of distinguishing between the value of the structure and the underlying land in such indexes. We refer interested readers to these studies to gain insight into such challenges. The WIREI data set aggregates the prices of new homes in the US dating back to 1830.[7] We believe, in this regard, that new house prices provide a fair estimate of the value of existing (unsold) house prices, which would otherwise be exceedingly difficult to estimate without adding a long list of other potential calculation pitfalls.[8]

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[5]The WIREI data is annualized from 1830 - 1963, after which we annualized monthly house prices for the remaining 70 years.

[6]Historical datasets used by Jordà et al. (2015) and Knoll et al. (2014) confirm this general house price behaviour globally. The reader is referred to Knoll et al. (2014) for a discussion on plausible reasons for this phenomenon.

[7]For the sake of brevity, we omit a deeper discussion into possible alternative choices of house price indexes.

[8]Case et al. (2005) argues that such an index may, in fact, underestimate actual house price trends - something that would only serve to strengthen our findings. Knoll et al. (2014) also lists some of the compromises needed in order to calculate a house price index based on more subjective property valuations.
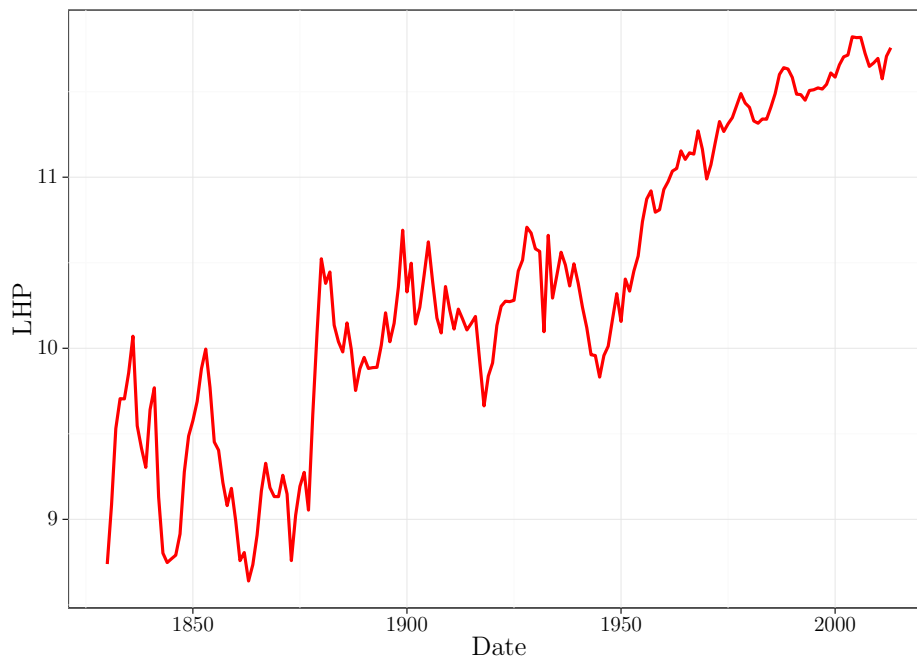
Figure 1: Log Real US House Prices

In light of the above concerns, the WIREI index adjusts the price data for average house size over time (price per square foot).

The first step in using the GSADF date-stamping procedure is to apply the summary right-tailed GSADF tests to the series. Table 1 shows that for both series, at the 5% level (with the smallest window size of 15), we find that our GSADF test statistics exceed the 10% and 5% right-tailed critical values respectively, rejecting the hypothesis in favour of a root exceeding unity at some point. This provides evidence that RHP experienced periods of explosivity for the full sample. Using this approach to locate the bubbles, we compare the SADF statistic sequence with the 95% SADF critical value sequence, obtained using Monte Carlo simulations.Details of this approach are contained in Phillips et al. (2013). The existence of possible structural breaks in the series would merely serve to strengthen the argument for roots exceeding unity, and so we do not control for such events here. As can be seen from Figure 2 in the appendix, RHP shows sustained growth in the post-war era, reaching its peak in 2005. Over the sample period, there were three episodes identified by the GSADF approach as explosive. Our fractional integration approach also provides evidence for the presence of several periods

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of explosivity for the RHP. These results and their economic relevance will be discussed in the next section.

4. Methodological Discussion

The first technique that we use to label episodes of price explosivity builds on the work pioneered by Phillips and Yu (2011) and Phillips et al. (2011), and in particular the generalized form of the sup ADF (GSADF) proposed by Phillips et al. (2013). This method uses a flexible moving sample test procedure to consistently and efficiently detect and date-stamp periods where a price series displays a root exceeding unity. Bubbles are so identified in a consistent manner with false identifications seldom given even in modest sample sizes.[9] The test procedure suggested by PSY recursively implements an ADF-type regression test using a rolling window procedure. Suppose the rolling interval begins with a fraction r_1 and ends with a fraction r_2, with the size of the window given as $r_w = r_2 - r_1$. Then, let:

$$y_t = \mu + \delta.y_{t-1} + \sum_{i=1}^{p} \phi_{r_w}^i \delta.y_{t-i} + \epsilon_t \tag{1}$$

where μ, δ and ϕ are parameters estimated using OLS. We then test null of $H_0 : \delta = 1$ against the right sided alternative $H_1 : \delta > 1$. The number of observations used in equation 1 is then $T_w = [r_w T]$, where [.] is the integer part. The ADF statistic corresponding to equation 1 is thus denoted by $ADF_{r_1}^{r_2}$.

Building on this approach, PSY formulated a backward sup ADF test where the end point of the subsample remains fixed at a fraction r_2 of the entire sample, with the window size expanding from an initial fraction r_0 to r_2. This backward sup ADF (SADF) procedure can thus be defined as:

$$SADF_{r_2}(r_0) = sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2} \tag{2}$$

PSY then suggested repeatedly implementing the SADF procedure of equation 2 for each $r_2 \in [r_0, 1]$, leading to a generalized form (GSADF) written as:

$$GSADF(r_0) = sup_{r_2 \in [r_0, 1]} SADF_{r_2}(r_0) \tag{3}$$

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[9]See PSY, (2013) for a deeper discussion and Monte-Carlo estimations testing the efficacy of this identification procedure.

The supremum form of the recursively estimated ADF is motivated by the observation that asset price bubbles generally collapse periodically.[10] In this scenario, the sup ADF test delivers efficient bubble detection capabilities where one or two bubbles emerge, with the generalized form performing well even in the presence of multiple bubble episodes.

The initial minimum fraction in the SADF approach of equation 2, $r_w = r_0$, is selected arbitrarily, keeping in mind the issue of estimation efficiency. Thereafter, we expand the sample window forward until $r_w = r_1 = 1$, the full sample, and we have a recursive estimate of ADF defined as $ADF_{r_k}, \forall k \in (r_0, r_1)$. From the sequence of ADF statistics (SADF) so produced, we can then identify the supremum value that can be used to test the null hypotheses of unit root against its right-tailed (mildly explosive) alternative by comparing it to its corresponding critical values. If the right tailed alternative to the unit root null is thus accepted, we can infer mild explosivity of the series, indicated by δ_{r_1,r_2}.

The generalized form of this approach defined in equation 3, uses a variable window width approach which allows both the starting and ending points to change within a predefined range, $[r_0, 1]$. This allows the identification of multiple periods of explosivity and allows us to consistently date-stamp the starting and ending points. The starting points are identified as the periods, T_{r_e}, at which the backward sup ADF sequence crosses the corresponding critical value from below. The corresponding ending point to an explosive period is similarly defined as the period, T_{r_f}, where the backward sup ADF sequence crosses the critical value point from above.

We can formally define identified periods of explosivity using the GSADF approach as:

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\begin{aligned}
\hat{r_e} &= \inf_{r_2 \in [r_0,1]} \left\{ r_2 : BSADF_{r_2} > cv_{r_2}^{\beta_T} \right\} \\
\hat{r_f} &= \inf_{r_2 \in [\hat{r_e},1]} \left\{ r_2 : BSADF_{r_2} > cv_{r_2}^{\beta_T} \right\}
\end{aligned}
\tag{4}
$$

Where $cv_{r_2}^{\beta_T}$ is the $100(1-\beta_t)\%$ critical value of the sup ADF statistic based on $[T_{r_2}]$ observations. We also set β_t to a constant value, 5%, as opposed to letting $\beta_T \to 0$ as $T \to 0$. The BSADF(r_0) for $r_2 \in [r_0, 1]$ is the backward

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[10]Evans (1991) pointed out that in samples with frequent bubble formations, conventional unit root tests have limited bubble detection power

sup ADF statistic that relates to the GSADF statistic by noting that:[11]

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1]} \{BSADF_{r_2}(r_0)\} \qquad (5)$$

The second approach that we use also tests the right tailed alternative to a unit root null hypothesis, but unlike standard right-tailed tests, focuses on the fractional order of integration. The approach that we follow is similar to Balcilar et al. (2015), who built on the procedure developed by Robinson (1994) in determining the fractional order of integration. They also allow for the identification of multiple endogenously determined structural breaks in the form of level and trend shifts at endogenously determined dates. The identification approach is based on the procedure suggested by Gil-Alana (2008) and built on the principles suggested in Bai and Perron (1998). Balcilar et al. (2015) also construct statistical tests for the different orders of fractional integration for each regime, using Robinson (1994)'s LM test to determine the most likely order of integration. To explain this procedure, consider the following multiple regression form:

$$y_t = \beta' z_t + x_t, \quad \forall t = 1, 2, ..., T \qquad (6)$$

where y_t is the house price index series, β a $k \times 1$ vector of unknown parameters and z_t a $k \times 1$ vector of observable variables, which includes a constant, polynomials in time trends (t) and structural break dummies, depending on the deterministic structure imposed. As noted in Balcilar et al. (2015), the presence of such deterministic regressors does not affect the limiting null and local distribution of the Robinson test statistic.

We consider the general case where z_t includes a constant, a linear time trend and $m = 2k$ level, as well as trend shift dummies, $DLT_{t,i}^{tl} = (DL_{t,i}^{tl}, DT_{t,i}^{tl})'$ at the dates $i = T_{b,1}^{tl}, ..., T_{b,k}^{tl}$. We then set $DL_{t,i}^{tl} = 1$ if $t > T_{b,i}^{tl}$ and zero otherwise, and also $DT_{t,i}^{tl} = t - T_{b,i}^{tl}$ and zero otherwise. Here we will also follow the notation of Balcilar et al. (2015) by defining T_k as the set of disjoint break dates, $T_k = \{T_{b,1}^{tl}, ..., T_{b,k}^{tl}\}$. We also define $\beta' z_t$ as follows:

$$\beta' z_t = \mu + \delta.t + \sum_{i=1}^{k} \left(\Phi_i DL_{t,i}^{tl} + \Theta_i DT_{t,i}^{tl} \right) \qquad (7)$$

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[11]The recursive GSADF estimations were done using Caspi (2013)'s routine in Eviews.

with the regressor errors, x_t, assumed driven by the following process as:

$$(1 - L)^d x_t = u_t \tag{8}$$

with L the lag operator, u_t covariance stationary, integrated of order zero, $I(0)$, and having a spectral density function that is positive and finite at zero frequency. Allowing for a fractional order of integration in equation 8, implies that d can assume any value on the real line.

The model structure above is based on the least squares principle first proposed by Bai and Perron (1998). The estimation is carried out as follows: first, a grid of values for the fractional integration parameter, d, is chosen as, e.g., $d_0 = [0.00, 0.01, ..., 1.20]$. The least squares estimates of μ, δ, ϕ_i and θ_i in equation 8 are then obtained for each k-partition of $\{T_1, ..., T_k\}$, denoted as $\{T_k\}$, by minimizing the sum of squared residuals in the d_0 difference models. This implies, minimizing the following residuals sum of squares (RSS):

$$\sum_{t=1}^{T} (1 - L)^{d_0} \left(y_t - \mu - \delta.t - \sum_{i=1}^{k} \left[\phi_i.DL_{t,i}^{tl} + \Theta_i DT_{t,i}^{tl} \right]^2 \right) \tag{9}$$

over all value of $T_1, ..., T_k$, yielding the parameter estimates $\hat{\mu}, \hat{\delta}, \hat{\phi}_i$ and $\hat{\theta}_i, \forall i \in [1, ...k]$, and also the break dates, $\{\hat{T}_k\}$. We also employ Schwarz' (1978) Bayesian information criterion (BIC) to select the number of breaks, k, prior to running the procedure.[12] We then calculate the test statistic of Robinson (1994) for each value of d_0 in the grid, a procedure that can be summarized as follows (following again the notation of Balcilar et al. (2015)).

In order to test the null hypothesis:

$$H_0 : d = d_0 \tag{10}$$

Robinson (1994) developed the following score statistic:

$$\hat{r} = \left[\frac{\sqrt{T}}{\hat{\sigma}^2} \right] \sqrt{\hat{A}} \hat{a} \tag{11}$$

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[12]The number of breaks is selected by minimizing the criterion: $\text{BIC}(k) = \ln\left[\frac{\text{RSS}(\hat{T}_k)}{T-n}\right] + \frac{2n \ln(T)}{T}$

where

$$\hat{a} = -\frac{2\pi}{T}\sum_{j=l}^{T-1}\Psi(\lambda_j)g(\lambda_j;\eta); \quad \hat{\sigma^2} = \frac{2\pi}{T}\sum_{j=1}^{T-1}g(\lambda_j;\hat{\eta}I(\lambda_j))$$

$$\lambda_j = \frac{2\pi j}{T}; \quad I(\lambda_j) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T}\hat{u}_t e^{i\lambda_j t}\right|$$

$$\hat{A} = \frac{2}{T}\left[\sum_{j=1}^{T-1}\Psi(\lambda_j)\Psi(\lambda_j)' - \sum_{j=1}^{T-1}\Psi(\lambda_j)\hat{\xi}(\lambda_j)' \times \left(\sum_{j=1}^{T-1}\hat{\xi}(\lambda_j)\hat{\xi}(\lambda_j)'\right)^{-1} \times \sum_{j=1}^{T-1}\hat{xi}(\lambda_j)\Psi(\lambda_j)'\right]$$

$$\hat{\xi}(\lambda_j) = \frac{\delta}{\delta\eta}\log(g(\lambda_j;\hat{\eta})); \quad \Psi(\lambda_j) = \mathrm{Re}\left\{\frac{\delta}{\delta\gamma}\log\phi(e^{-i\lambda_j};\gamma_0)\right\} \tag{12}$$

with $I(\lambda_j)$ the periodogram of $\hat{u}_t$. Parameter estimates for $\hat{\eta}$ are derived from the Whittle Maximium Likelihood (WML) method:

$$\hat{\eta} = \mathrm{argmin}_{\eta \in \Lambda}\frac{2\pi}{T}\sum_{j=1}^{T-1}g(\lambda_j;\eta)I(\lambda_j) \tag{13}$$

with $g(\lambda_j;\eta)$ the known function of the parametric spectral density of u_t. The model in equation 6 is completed by specifying a parametric form for u_t. In our analysis, we choose a general specification for u_t nested within an Autoregressive Moving Average (ARMA) model. This implies that by definition that x_t is characterized by a fractionally integrated ARMA (ARFIMA) model, which is a commonly used parametric specification for measuring long memory. The ARMA(p,q) model is denoted as:

$$\phi(L)u_t = \Psi(L)\varepsilon_t$$

while the ARFIMA(p,d,q) model for x_t can be written as:

$$\phi(L)(1-L)^d x_t = \Psi(L)\varepsilon_t \tag{14}$$

where ε_t is a white noise process with variance, σ^2, and $\phi(L) = 1 - \sum_{j=1}^{p}\phi_j L^j$ and $\Psi(L) = 1 - \sum_{j=1}^{q}\Psi_j L^j$ are polynomials in the lag operator L, with degrees of freedom p and q respectively. Furthermore, we assume that $\phi(Z)$ and $\Psi(Z)$ share no common roots and $\phi(Z) \neq 0$ and $\Psi(Z) \neq 0, \forall Z \leq 1$. The spectral density functions of these models, respectively, are given by:

$$f(\lambda;\sigma^2,\eta) = \frac{\sigma^2}{2\pi}\left|\frac{\Psi(e^{-i\lambda})}{\phi(e^{-i\lambda})}\right|, \quad \pi < \lambda \leq \pi \tag{15}$$

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and

$$f(\lambda; \sigma^2, \eta) = \frac{\sigma^2}{2\pi} \left| \frac{\Psi(e^{-i\lambda})}{\phi(e^{-i\lambda})} \right|^2 \left| 1 - e^{-i\lambda} \right|^{-2d}, \quad \pi < \lambda \leq \pi \qquad (16)$$

with η a $l \times 1$ vector of unknown parameters estimated by maximum likelihood, assuming that the orders p, q are known a priori.[13] Note also that the fractional parameter, d, is fixed under the null, thus equation 15 above is relevant to our empirical estimations. Our approach can thus be summarized as follows. We select a value d_0 in the grid $d_0^1 + i\Delta_d$, with Δ_d the grid increment and $i = 1, ..., s$. Then an initial disjoint break date, T_1, is selected and the residuals, $\hat{u}_t = (1 - L)^{d_0}$, $\hat{x}_t = (1 - L)^{d_0} y_t - \hat{\beta}'[(1 - L)^{d_0} z_t]$, are thus obtained. This is then used to calculate the $\hat{r}$ statistic of equation 11, with break dates then updated using the Bai and Perron (1998) algorithm. These steps are then repeated until $\sum_{t=1}^{T} \hat{u}_t^2$ is minimized, and done for all the grid increments. At each step in the process, we minimize the $\text{RSS}(\hat{T}_k)$ for a given d_0, with the parameters $\hat{\beta}$ and nuisance parameters $\hat{\eta}$ estimated sequentially.

An approximate one-sided test of $H_0 : d = d_0$ is then rejected in favor of $H_a : d > d_0 (d < d_0)$ at the $100\alpha\%$ level when $\hat{r} > z_\alpha (\hat{r} < -z_\alpha)$, with α the probability that the standard normal distribution exceeds z_α. In the empirical implementation, we allow structural breaks in the full sample estimation. We use this procedure in the same fashion as the rolling window ADF regression of Phillips et al. (2013). In the rolling implementation, the sample interval begins with a fraction r_1 and ends with a fraction r_2, with the size of the window given as $r_w = r_2 - r_1$. We do not allow structural breaks in the rolling estimation since a small window size of r_w is unlikely to include structural break impacts.

5. Empirical Results

Below follows a discussion of the explosive periods identified using the approaches outlined above. The purpose of this section is not to fully discuss the causes and reactions to these periods of explosivity (this would be an interesting follow up to this paper), but instead to concisely summarize the environment surrounding these episodes and highlight the extent of real house

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[13]For the ARMA model, $\eta = (\phi_1, ..., \phi_p, \Psi_1, ..., \Psi_q)'$ and for the ARFIMA model, $\eta = (d, \phi_1, ..., \phi_p, \Psi_1, ..., \Psi_q)'$, with $l = p + q + 1$, implying that $g(\lambda_j; \eta) = |\frac{\Psi(e^{-i\lambda_j})}{\phi(e^{-i\lambda_j})}|$

price declines following such periods. Our second technique identifies more periods of explosivity in the post-war period (4), whereas the first technique identifies only three such periods in total.

Figure 2 displays the results of the GSADF procedure over the sample period, with starting periods of explosivity labeled when the green line (BSADF sequence) exceeds the blue line (95% critical values), and ends where it dips below the blue line. These periods of explosivity are summarized in table 2. We see that for the RHP series, there are three periods of explosivity with relatively short durations.[14] The first episode of explosivity was preceded by the five year depression following the panic of 1873, and saw the US Congress require a form of quantitative easing in the late 1870s.[15] This was followed by a spike in asset prices broadly, with real housing prices rising by 149% between 1878 and 1880. After this period of explosivity, real prices declined by roughly a third within three years.

The second period was between 1956 and 1957, where real house prices rose by over 43% between 1955 - 1957. This was driven by a decade of prosperity where the US economy grew significantly and employment were at all-time lows. This period was followed by a house price correction of 12% within two years, before again experiencing a sustained price increase (with weak explosivity identified by the end of the 1960s again).[16].

The last episode of explosivity is identified between 2004 and 2006. This follows a period where real house prices rose by roughly 26% from 2000–2006. The explosive episode identified was preceded by the Fed funds rate being lowered significantly[17], and characterized by sharply increasing house prices, large scale deregulation of institutions able to provide mortgage products, and a proliferation of investment vehicles designed by leveraged institutions to magnify the property market returns. This culminated in a period of credit-driven mortgage price increases. Prices corrected within two years by more than 15%, and by 21.5% within five years.

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[14]As noted by Phillips et al. (2011), periods of explosivity of short lengths should be excluded, which in our study we cut-off at a minimum of 2 periods for explosivity.

[15]The Bland-Allison Act of 1878 saw the US Congress require Treasury to buy up silver and in so doing inject liquidity into the economy.

[16]Weak explosivity here implies a single month of explosivity identified in 1964 and 1968)

[17]The Fed funds rate was lowered from 6.5% to 1.75% in 2001, following fears of a deflationary trap following the DotCom crash.

The next technique used in this study to label periods of explosiveness is the procedure proposed by Balcilar et al. (2015). The estimation was carried out as follows: for each chosen value of d we use the statistic for $\hat{r}$, given in equation 11, to test whether the fractional parameter, d, exceeds 1. This would be indicative of an explosive period, making it comparable to the sequential unit root tests above. We first test for various fractional orders d in the full ample. In our estimation for, the full sample we use two deterministic structures for z_t, with $z_{1,t}$ corresponding to a constant and trend, and $z_{2,t}$ corresponding to the general case in equation 7. The estimation procedure detailed in section 4, identified two endogenously determined linear trend and level breaks (denoted $DT_{t,i}^{t,l}$ and $DL_{t,i}^{t,l}$, respectively), which occurred at 1877 and 1954. These breaks correspond to periods of explosivity defined using the GSADF approach. The procedure is then used in a rolling estimation fashion with fixed window size of $r_w = 15$. Rolling estimation does not allow structural break dummies, since a small window size does not suffer from structural break impacts.

The fit of the structural break model for the full sample can be viewed in Figure 3 in the appendix. As can be seen, the model tracks the broad trend of the data rather well. Table 3 provides the estimated full sample fit of the structural break model, using deterministic structure $z_{2,t}$.

From table 3, we see that nearly all of the parameters for the second model structure, $z_{2,t}$, are significant. The significant structural break dummy estimates confirm the existence of significant breaks in both trend and levels of the RHP series at 1877 and 1954.

In order to validate the use of the GSADF procedures earlier (as structural breaks could lead to the shifting up of orders of integration), we also include $z_{1,t}$'s estimates in table 4. From it we see firstly that when not controlling for the structural breaks, the lower bound of significance for the fractional order of integration estimate exceeds unity at the 1% level. When controlling for the structural breaks using $z_{2,t}$, we see non-rejection covers the range 0.94 to 1.00 at the 5% level, and 0.92 to 1.01 at the 1% level.[18] This indicates that there is strong evidence that RHP experienced periods of explosivity, when comparing the null of a unit root to the more flexible test of a fractional order of integration, even when not controlling for structural breaks. This

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[18]Despite not rejecting range of values above 1 at the 1% level, it is clear that the lower bound is at the very least highly persistent and close to unity.

validates the use of PSY, (2013)'s approach, as it indicates that such breaks do not significantly account for explosivity in the full sample for RHP.

In order to date-stamp periods of explosivity using this approach, we employ a rolling window procedure to calculate the $\hat{r}$ statistic. We use a fixed length window size of 15 sequentially from the beginning to the end of the sample, adding a single observation and dropping the last at each step. We then calculate at each step a range of $\hat{r}$ statistics, enabling us to estimate a lower and upper bound limit for d (using a one sided test with 5% significance level). The benefit of using this approach to identify periods of explosivity is that, firstly, it allows for a changing structure of the underlying data, and secondly it is robust to possible structural breaks. This implies we use the rolling window identification technique on the $z_{1,t}$ deterministic form, as opposed to the form accounting for the breaks explicitly. Although there are differing views on the appropriate size of such fixed window techniques[19], our chosen window size reflects our desire to optimize the representativeness of the model, particularly as we identified two breaks in the series. Figure 4 shows our rolling window estimations.

From Figure 4, we identify periods of explosivity as starting when the lower bound (blue line) cross 1, and ends when it dips below 1. Table 5 summarizes the periods of explosivity so identified. As before, we ignore episodes shorter than 1 period in duration, while also excluding periods of potential negative explosivity, as our focus is on price build-ups.

From table 5, we see that the fractional integration rolling window approach offers greater insight into periods of explosiveness during the 1800s, particularly as there is a much shorter burn-in period. We see, e.g., several periods of explosiveness in real house prices during the $1849 - 1855$ California gold rush[20], which saw an increase in real house prices of over 70% during this period.[21] The US gold rush continued until 1864, during which time another period of explosivity can be identified towards the latter part. The next period of RHP explosivity is labeled between $1866 - 1873$, right after the US civil war which ended in 1865. Prices during this phase peaked in 1867, 81% higher than during the war in 1864. The next episode of explosivity identified is from $1926 - 1929$, during which time real house prices rose

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[19]c.f. Pesaran and Timmermann (2005) for a deeper discussion.

[20]c.f. Santos, R. L. (1998). The Gold Rush of California: A Bibliography of Periodical Articles. *Cass Publications.*

[21]RHP peaked in 1853 at 194% above the level in 1848.

by over 48%. This coincided with unprecedented asset price inflation across nearly all US asset classes. Real house prices peaked in 1928, and were down 85% by 1932, following the start of the Great Depression in 1929.

This approach then identifies two short-lived periods of explosivity during the mid 1980s and late 1990s. The first episode transpired in the build up to what is today known as the Savings and Loan crisis, which started in 1986, and saw credit-driven real house prices rise by roughly 30% from 1984–1987. This was caused in part by large scale deregulation of lending standards and a reduction in capital reserve requirements in the US, which both served to drive large scale credit creation, particularly in financing mortgages. The 1990s saw RHP first decline substantially (after peaking in 1989, it fell by roughly 21% by 1993), while picking up in the late 1990s and reaching its 1989 peak again in 2001. Real prices then surged in the early 2000s, peaking in 2004 at 27% higher than in 2000. The RHP correction came after the 2008 global financial crisis, with a sharp turnaround in RHP between 2009 – 2011, identified as a period of significant decline in RHP using our statistics. These results echo the findings by Shiller (2015), who show real estate asset price increased during the early and mid 1980s and mid-1990s, with a subsequent contraction in the late 2000s.

The periods identified as bubbles by the fractional integration and GSADF methods differ in terms of number of bubbles and the bubble periods to some extent. We further examine this and explain these differences. Detection and prediction of bubbles have fundamental definitional issues. In our application, the definition of bubbles builds on statistical notions. The methods we employed identify the start of a bubble with the initiation of an explosive behavior of process and the end of bubble is identified with the ceasing of the explosive price behavior, leading the definition of a bubble as "periods of explosive behavior. Although definition of an explosive price behavior does usually coincide with periods identified as bubbles based on event based studies (see e.g. Zhang et al., 2016), economists conceptualize bubbles as periods where the price of an asset grows faster than the asset's fundamental value. For instance, Shiller (2015) define the bubble as "a situation in which news of price increases ... despite doubts about the real value of an investment. This conceptualization of bubble based on a "fundamental value is problematic since it is not easy to measure or define what constitutes fundamental value of an asset. Therefore, in order to identify a bubble one needs to define a metric and there is little agreement about what these metrics might be (Contessi and Kerdnunvong, 2015).

Given the difficulties of defining proper metrics for identifying bubbles based on economic theory and, as Shiller (2015) profoundly emphasizes the bubbles can be seen as "price increases spurs investor enthusiasm, which spreads by psychological contagion from person to person, in the process amplifying stories that might justify the price increases and bringing in a larger and larger class of investors and such behaviors gives rise to financial bubbles makes the identification of development of a bubble in real time based on econometric methods difficult. As Gürkaynak (2008) points out "for each paper that finds evidence of bubbles, there is another one that fits the data equally well without allowing for a bubble. The problem pointed out by Gürkaynak (2008) arises not only because of conceptual differences across econometric methods used, but also by features of such methods. Each method has certain advantages and disadvantages and display sensitivity to certain conditions. Among the methods used for identifying bubbles, log-periodic power law of Johansen et al. (2000), GSADF, exponential curve fitting (EXCF) method of Watanabe et al. (2007a,b) are commonly used. In this study, we compare the rolling fractional integration test with GSADF since they both define price explosivity based on the unit root behavior. In empirical analyses, each approach has certain robustness and sensitivity to deviation from assumptions and these largely explain the differences in findings. Our paper aims to compare the rolling fractional integration due to several advantages it offers. As Michaelides et al. (2016) in their paper, the modelling process of bubbles and underlying econometric methods, which uses advanced mathematical and statistical theory, is still a young filed and ongoing research area. Thus, the analysis in this paper uses new tools that may help better understand the detection and modelling of bubbles. The unit root testing approach of the GSADF test may suffer from both power and size distortion, and also might be sensitive to the treatment of the deterministic component, particularly in the presence of structural breaks. The GSADF procedure is based on the augmented Dickey-Fuller unit root test (ADF). As shown long before by Cochrane (1991), the ADF might have arbitrarily low power in finite samples. Moreover, there are unit root processes with likelihood function that are arbitrarily close to likelihood functions of stationary processes with root local to unity and *vice versa*. Schwert (1989), using Monte Carlo simulation, shows that unit toot tests are sensitive to model misspecification and display size distortions. Since the seminal paper by Perron (1989), it is well known that unit root tests are low powered against misspecification in the trend component due to structural breaks.

Given these issues with unit root testing, the efficient fractional integration test offers some advantages. First, it has a standard limiting distribution and efficient test with good power properties and no significant size distortions. Second, it is robust to the deterministic trend specification and not effected by how structural breaks are treated. Third, it identifies the explosive behavior based on not an estimate of a parameter but rather as a result of a sequential testing procedure that is efficient. Fourth, the method takes into account of both short and long memory properties of the data and, therefore, robust to effect of long memory effects. As noted Gürkaynak (2008), there is already a quite large disagreement among the various bubble detection approaches as noted by Gürkaynak (2008)[22] On the econometrics front, these features of the fractional integration method and issues relating to the ADF test do create the differences in bubble periods detected by these methods. The fractional integration method detects five more bubble periods than the GSADF procedure. Zhang et al. (2016) show that the GSADF procedure detects fewer bubbles than the LPPL and EXCF methods. This is also valid in the empirical application in this study. To the best of our knowledge, there is no study comparing the bubble detection power of available methods. Zhang et al. (2016) uses reality checks on the bubble periods detected by examining whether these periods correspond to the actually known historical explosive price behavior or whether they correspond to the known bubbles from the literature. The reality check is also a feasible approach in our case as external information is available is available about all bubbles detected in sample periods. As we shown in Table 5, all periods classified as explosive price growth periods by the fractional integration method correspond to historically known events. Among these events, 1850s boom-and-boost cycle, panic of 1873, great crash of 1929, 1982-1992 housing cycle, and 2000-2010 housing cycle are already well documented periods of housing market booms and crashes. Thus, the GSADF may indeed be missing some of the explosive price growth periods. Another issue with the bubble periods detected by two methods is the non-overlapping bubble years for two bubbles cases detected by both methods. The GSADF indicates bubbles in three periods 1879-1880, 1956-1957, and 2004-2006. One reason for the non-overlapping periods in Table 2 and Table 5 is due to a rule we use,

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[22]See also Rosser (2008) about the large consensus on this issue in the economic literature.

i.e., not classifying a period of one year explosive price growth as bubble. We have done so in these cases, because the bubble develops and dies within the same year. If consider one year period bubbles, the fractional integration method identifies a bubble during 1880. There is already a bubble identified by the fractional integration method in 2002-2004. Since the bubble dates identified by both methods are only estimates and not exact, considering there might be sampling variability, we conclude that these methods do not agree about an explosive price behavior only during 1956-1957. Indeed, house price explosivity is not documented in the united states during 1950s.

Statistical methods may identify explosive behavior in prices, but they do not show underlying reasons why explosive price changes develop or end with crashes in prices. If one defines a bubble based on the deviation of house prices from the fundamental value, then other information for reality check and assessments the validity of bubbles detected based on statistical methods should be used. There might be various reasons that induce the exponential growth in prices and one can use other information to gain insight on why the bubble developed. There might be reasons for explosive behavior that are not necessarily due to unjustified behavior. Reinhart and Rogoff (2014) identifies distinct common features that appear to be precursors of most financial crises, some of which are identified as the end of asset price bubbles. They list a number of common changes before the onset of financial crises, such as the slow run-up of asset prices, significant reductions in output growth rates, notable increase in government debt to GDP ratio, and large capital inflows. Goodman and Thibodeau (2008) evaluates how much of the 2002-2006 house price appreciation in the US can be attributed to fundamental economic factors. They consider both the demand factors and supply factors and argue that inelastic supply was partly responsible for increased house prices. However, their simulations based on the estimated supply elasticities showed that the speculation was a major cause of the house price appreciation and house prices has grown much above the level that can be attributed to fundamentals, implying a housing bubble during the 2000-2005 period. Glaeser et al. (2008) argues that the observed higher volatility in housing prices relative to fundamentals is due to inelastic housing supply. They further present a housing bubble model where inelastic supply leads more explosive price growth and longer bubble developments.

Lastly, we comment on the predictive power of models and their capacity to signal end of a bubble. Econometric modeling of process of bubbles, their detection and prediction is a is a young field of research and mathematical and

statistical theory are still in development stage. To the best of our knowledge, these models cannot yet successful predict the explosive price growth periods before they start (see, e.g. Jiang et al., 2010). However, they will be able to indicate whether the process of explosive behavior has already started (Zhang et al., 2016). Among existing bubble detection models the LPPL approach of Johansen et al. (2000) and Sornette et al. (2009) was shown to have predictive power in a few cases (see e.g. Jiang et al., 2010; Yan, 2011; Sornette, 2017).[23] However, not all bubble predictions were accurate and some of these were not reproducible (Li, 2010). The partial success of bubble models should, however, not be overemphasize. Persistent boom-and-boost cycles in asset markets have significant economic welfare and social effects. Prevention and mitigation of these bubbles is a challenge to policy makers and market participants as they are hard to predict. Models, such as the ones used in this study, serve to identify whether explosive price growths have developed and whether these are followed by market crashes. One should keep in mind that these models cannot be used as crystal balls as they cannot predict whether the price bubble will develop in a certain period. They do, however, help to detect whether certain current asset prices show explosive behavior, evaluate whether bubbles have occurred, and consequently help policy makers to prioritize which asset markets require attention. Thus, bubble detection may help designing policies for bubble mitigation and preventions. Therefore, the use information from the bubble tests may also help to prevent future price bubbles, particularly when the statistical information is combined with other information to identify the potential factors behind the development of bubbles. The bubble models also have partial success in predicting the end of explosive price behavior and hence may signal forthcoming market crashes. In Table 2 and Table 5, we report first period where a slowdown in the exponential price growth is detected or whether prices actually started to decline. These price change reversals might be used as a signal of forthcoming crashes. For instance, the fractional integration model signals a crash two year before the price collapse for the end 1850s boom-and-boost cycle and panic of 1857 and five years before the Panic of 1873.

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[23]For instance, LPPL based models are reported to successfully predict the August-2009 crash in the Chinese equity market (see Jiang et al., 2010)

6. Conclusion

This paper set out to identify periods of US house price explosivity from 1830 – 2013. In order to identify house price fundamentals, we make use of the general price level (measured as the US CPI index). The implicit assumption thus made is that house prices tend to reflect general movements in prices across the economy. Large deviations from past levels could therefore be considered as explosive in the short term as it could feasibly lead to higher allocation towards houses as assets experiencing high capital growth. This, in turn, feeds into more demand and even higher prices, potentially driving an episode of unsustainable asset price increases, particularly as a result of factors inherent to property purchases (such as typically high transaction costs and low ability to short-sell) that make it uniquely prone to bubble-type episodes. Although other measures have been suggested for use as fundamentals, we are constrained by data availability for our long dated sample.[24]

The first technique used to identify periods of explosivity, is the recursive GSADF test suggested by Phillips et al. (2013). This test allows the effective date-stamping of periodically collapsing bubble-like periods, allowing us to label several historical periods of significant real house price build-ups. For the RHP measure, we define three short periods of explosivity, during the late 1800s, mid 1950s and the mid 2000s.

The second measure used to test right tailed alternatives to unit root testing, focuses on the fractional order of integration, d. The procedure uses Robinson (1994)'s $\hat{r}$ statistic to define confidence bands for likely values of d. We also allow for the identification of multiple periods of endogenously determined structural breaks in the form of level and trend shifts at unknown dates. We then use a rolling window approach to date-stamp periods of likely explosivity in the series, identified as periods where the lower bound of the 95% confidence interval of d exceeds unity. The periods so identified suggest several periods of explosivity during the 1800s, particularly surrounding the US gold rush, as well as immediately following the Civil War. Significant and unsustainable build-ups in real house prices are then also observed in the 1920s shortly before the Great Depression, the 1980s during the period preceding the S&L crisis, as well as during the late 1990s and early

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[24]Despite this, we maintain the appropriateness of these measures as proxying an essentially immeasurable fundamental level.

2000s, with a correspondingly significant negative price adjustment following the global financial crisis. Our results suggest that the more flexible, long memory approach of using fractional integration to test the alternative hypothesis, provides a richer set of dates of where prices likely deviated from mean reversion toward aggregate prices in the US.

In summary, our analysis provides a thorough investigation of the time-series characteristic of US house prices over the last two centuries, novel in its coverage as well as use of fractional integration in determining house price explosivity.

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7. Appendix

Table 1: Right Tailed ADF Test

Sample : 1830 2013			
Included observations: 184			
Lag Length: Fixed, lag=0			
Window size: 15			
H_0: RHP has a unit root			
		t-Statistic	Prob.
GSADF		0.050	0.033
Test critical values:	99% level	0.622	
	95% level	-0.167	
	90% level	-0.519	

Table 2: GSADF explosive periods: RHP

Sample : 1830 - 2013			
Included observations: 184			
Starting Date	Ending Date	Duration (Years)	First Period of Crash Signal
1879	1880	2	1881
1956	1957	2	1957
2004	2006	3	2005

Table 3: Estimates of deterministic and structural parameters

Constant	8.9579*** (0.0525)	9.4866*** (0.073)
Trend	0.0149*** (0.0005)	-0.0096*** (0.003)
$DL_{t,1}^{t,l}$		9.944*** (0.114)
$DT_{t,1}^{t,l}$		0.003** (0.001)
$DL_{t,2}^{t,l}$		8.963*** (0.295)
$DT_{t,2}^{t,l}$		0.015*** (0.002)
MA(1)	-0.032 (0.074)	0.043 (0.074)
MA(2)	0.016 (0.074)	0.176** (0.073)
BIC	-1.014	-1.337
$\hat{\sigma}$	0.355	0.248

Notes:

The table reports the parameter estimates of the model defined in equation 6 and explained thereafter, at minimum absolute values of the $\hat{r}$ statistic given in equation 11.
Standard errors of the estimates are given in parentheses.
***, ** denote significance at 1% and 5% levels, respectively.
$\hat{\sigma}$ is the standard error of the estimate and BIC the Bayesian Information Criterion.

Table 4: Fractional integration estimations using Robinson (1994)'s statistic

d_0	$z_{1,t}$	$z_{2,t}$	d_0	$z_{1,t}$	$z_{2,t}$
0.81	$21.32^{*,\dagger}$	$8.88^{*,\dagger}$	1.01	0.64	$-1.95^{\dagger}$
0.82	$19.60^{*,\dagger}$	$8.12^{*,\dagger}$	1.02	0.09	$-2.30^{*,\dagger}$
0.83	$17.99^{*,\dagger}$	$7.38^{*,\dagger}$	1.03	-0.44	$-2.63^{*,\dagger}$
0.84	$16.48^{*,\dagger}$	$6.67^{*,\dagger}$	1.04	-0.94	$-2.95^{*,\dagger}$
0.85	$15.06^{*,\dagger}$	$5.99^{*,\dagger}$	1.05	-1.42	$-3.26^{*,\dagger}$
0.86	$13.73^{*,\dagger}$	$5.34^{*,\dagger}$	1.06	$-1.87^{\dagger}$	$-3.55^{*,\dagger}$
0.87	$12.47^{*,\dagger}$	$4.71^{*,\dagger}$	1.07	$-2.30^{*,\dagger}$	$-3.84^{*,\dagger}$
0.88	$11.29^{*,\dagger}$	$4.10^{*,\dagger}$	1.08	$-2.71^{*,\dagger}$	$-4.11^{*,\dagger}$
0.89	$10.17^{*,\dagger}$	$3.52^{*,\dagger}$	1.09	$-3.09^{*,\dagger}$	$-4.37^{*,\dagger}$
0.9	$9.12^{*,\dagger}$	$2.96^{*,\dagger}$	1.1	$-3.46^{*,\dagger}$	$-4.61^{*,\dagger}$
0.91	$8.12^{*,\dagger}$	$2.42^{*,\dagger}$	1.11	$-3.81^{*,\dagger}$	$-4.85^{*,\dagger}$
0.92	$7.18^{*,\dagger}$	$1.90^{\dagger}$	1.12	$-4.15^{*,\dagger}$	$-5.08^{*,\dagger}$
0.93	$6.29^{*,\dagger}$	$1.40^{\dagger}$	1.13	$-4.46^{*,\dagger}$	$-5.30^{*,\dagger}$
0.94	$5.45^{*,\dagger}$	0.92	1.14	$-4.76^{*,\dagger}$	$-5.51^{*,\dagger}$
0.95	$4.65^{*,\dagger}$	0.46	1.15	$-5.05^{*,\dagger}$	$-5.71^{*,\dagger}$
0.96	$3.89^{*,\dagger}$	0.02	1.16	$-5.32^{*,\dagger}$	$-5.90^{*,\dagger}$
0.97	$3.17^{*,\dagger}$	-0.41	1.17	$-5.58^{*,\dagger}$	$-6.09^{*,\dagger}$
0.98	$2.49^{*,\dagger}$	-0.82	1.18	$-5.83^{*,\dagger}$	$-6.26^{*,\dagger}$
0.99	$1.84^{\dagger}$	-1.21	1.19	$-6.06^{*,\dagger}$	$-6.43^{*,\dagger}$
1	1.22	-1.59	1.2	$-6.28^{*,\dagger}$	$-6.59^{*,\dagger}$

Notes:
* and † indicate the non-rejection at the 1% and 5% levels, respectively, when comparing the $\hat{r}$ statistic to the standard normal critical values for a one sided test.

$z_{1,t}$ indicates a deterministic structure with no structural breaks, while $z_{2,t}$ has two endogenously identified linear trend and level breaks.

Table 5: Rolling $\hat{r}$ explosive periods: RHP

| Sample : 1830 - 2013				
Included observations: 184				
Starting Date	Ending Date	Duration (Years)	Event	First Period of Crash Signal
1850	1852	3	1850s boom-and-boost cycle (Goldschein, 2012)	1852
1858	1863	6	1850s boom-and-boost cycle and panic of 1857 (Goldschein, 2012)	1861
1866	1873	8	Panic of 1873 (Goldschein, Goldschein)	1868
1926	1929	4	Great crash	1929
1984	1985	2	1982-1992 housing cycle (Glaeser et al., 2008)	1985
1998	1999	2	Post 1996 boom (Glaeser et al., 2008)	1999
2002	2003	2	Post 1996 boom (Glaeser et al., 2008)	2003
2009	2011	2	2000-2010 housing cycle (Goldschein, Goldschein)	2011
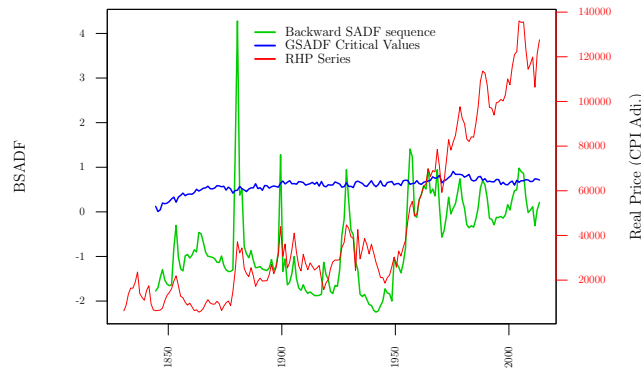
Figure 2: Backward SADF procedure: Real House Price
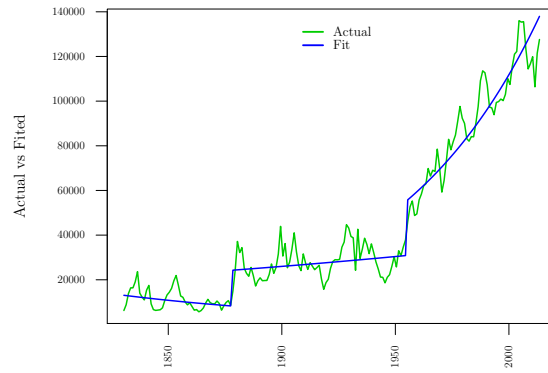
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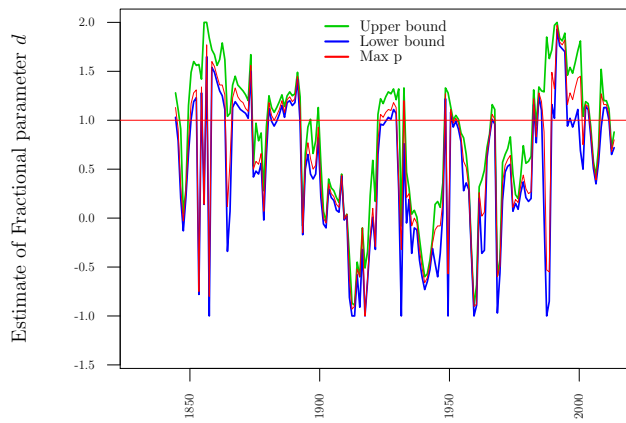Figure 3: Actual versus Fitted values of fractional integration model

Figure 4: Rolling estimations of the $\hat{r}$ statistics