### **Reply to referee report 4**

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### General

We do not understand how the referee does not see that our model consisting of several timedependent differential equations is a dynamic one. Algebraic equations are static ones and time dependent differential equations are dynamic ones that are used to model the time paths of variables we are interested in. Moreover, dynamics is a branch of mechanics in which accelerations are caused by forces. In this sense, our theory is the first one in economics that exactly follows the lines of dynamics developed in mechanics. We do believe that all sciences construct a family where similar modelling principles can be applied in homomorphic events in different specific sciences. Thus all modelling principles that have a sound basis are applicable in other sciences once the modelling events are observed to have a homomorphic structure.

In existing models of neoclassical economics the entrance of new firms into markets and their disappearance from the markets are mostly explained by words, not by using formal models. As the referee tells, these are stories, not exact models. In the neoclassical models, firms are always assumed to behave in their optimal way, and thus in these models every firm immediately jumps from an old equilibrium to a new one, which is not a realistic description of the behaviour of a firm in the real world. Every adjustment in the production flow of a firm takes time, which is included in our modelling. We stress that by definition, dynamics concerns motion (or equilibrium) of bodies under the action of forces (in kinematics there exist accelerations, but accelerations are not explained with forces).

We have defined an open optimization problem for every consumer so that we can define the marginal willingness to pay (MWP) for every consumer also outside his/her optimum situation, in contrary with the neoclassical theory that only models the optimum point. Thus everywhere outside a consumer's optimum the consumer compares his/her MWPs and the prices of goods, and adjusts his/her flows of consumption on this basis. Thus, outside the optimum point of a consumer, his/her consumption accelerates positively or negatively so that he/she will find his/her optimum. This has not been modelled earlier in the neoclassical framework that models only the equilibrium point of a consumer. This element really creates complexity in the model behaviour.

Of course, every parameter in our model can be assumed to be random which would make the model even more complex. Actually, in our simulations all initial values are chosen randomly; maybe we should state this more clearly in our article. More complicated randomizations can naturally be included in the model e.g. by assuming random shocks in costs for some firms due to technological changes. These are, however, possible future extensions of our model. There is no limit in adding complexity in our model, but the purpose of this first order approximation model is to demonstrate that our model works and the real world behaviour corroborates our theory.

In economics, there is a clear difference between price determination in perfectly competed markets and in monopolistic competition. In a perfectly competed market, which our model applies, none of the firms can affect the price alone but all firms take part in the price determination by their production flows similarly as consumers take part in the price determination by their consumption flows. Actually, our model has the same price determination process as in Samuelson (1941-42). However, in monopolistic competition, every firm defines its product price by using the knowledge of the sales function of their product. This kind of modelling we have applied in several other studies, see e.g. Estola (2017).

## **Detailed replies:**

Referee states that: Section 3 is peremptory because "the theories of a firm and a consumer are symmetrical ...". This proposition bodes ill for any research programme in complexity.

We stress that symmetry within the theoretical structure does not necessarily reduce complexity of the model. Detecting symmetries deepens understanding of the research topic, and they are powerful tools, e.g., in physics. In our model, every consumer adjusts his/her flows of consumption to increase his/her utility, similarly as every firm adjusts its flows of production to gain more profit. In this way the consumer and producer behaviour are symmetric.

# Referee states that: *DED ignore the fundamental aggregation problem that bedeviled general equilibrium macroeconomics for over a century by tagging subscripts to their physical concepts.*

We understand the aggregation problem being about counting different micro units together to formulate the macro-level quantities that characterize macro-level behaviour. We have done this in our article, e.g., in counting GDP (see our figure 2(a)), though we have not explained how GDP is calculated. We thank for pointing this out and will add explanation of GDP calculation in the article. In our article GDP is a sum of all produced goods multiplied with their respective prices. Similarly, total production and demand of a particular good are aggregates of productions of all firms and demands of all consumers that are heterogeneous agents.

# Referee states that: *I did not get the definition of money. There is a term that is the sum of income and wealth. What is wealth? (I thought that net worth depended on the sign of capital gains rather than the other way around). Consumers have access to credit but there is no credit market.*

There exists term:  $M = M_0 + rW$ . Wealth (W) is a stock concept whose unit is  $\in$ . Income (M and  $M_0$ )

is a flow concept, its unit is  $\notin$ /time, and *r* is the interest rate with unit 1/time (note that interest rate multiplied with wealth (stock concept) is a flow concept and additive to income). Excess money over consumption is added into wealth, and consuming with credit diminishes wealth. In our paper money is a neutral quantity that is used in economic actions, like energy is used in physical actions in physics.

In physics, energy can be defined as the capacity for doing work. It may exist in potential, kinetic, thermal, electrical, chemical, nuclear, or other various forms. This applies for money in our paper, see Eq.(3,4) for definition of work. The work for consumer is calculated similarly as the work for firm, but not expressed explicitly in the paper. We ask whether we should add the work calculation for a consumer in the article.

# Referee states that: Consumers have access to credit but there is no credit market.

To incorporate a credit market in our model would make it more complicated, but actually it would behave in the same way. Every consumer either borrows or lends money, and we can add the aggregate borrowing and lending to create the excess demand of borrowed money. However, because we assume that central bank defines the interest rate, the excess demand or supply of borrowed money does not affect the interest rate in the case central bank uses its reserves to cover this excess demand or supply. This is what we assume, and so central bank can define the interest rate and we do not have to model the money market behaviour at all.

Referee states that: Based on a statement on page 9 that most of the interesting variables "come from outside" and "go outside" the system but that the system could still be "closed", the need for specifying exogenous and endogenous that had been growing impacted powerfully on me at this

#### point.

An open system, by definition, has an environment and there exist flows from the environment to the system and vice versa (system and environment interact with each other). A closed system has no environment, all flows are inside the system, and all interactions are within the system. Thus, our model is an open system due to the money flows (lending, borrowing, interest incomes or payments and profits). If these flows only occurred inside the system (e.g., consumer incomes are paid by firms via costs and capital income from profits, and there exists a money market inside the system), then the system would be closed. Thus, it is possible to make our model a closed one with these constraints, but as a first approximation model we have not done it. We would like to point out the fact that at least in physics open and closed systems behave in a fundamentally different way, and thus studying truly closed system could reveal fundamentally new observations about economic behaviour. We stress that nowadays there exists at least one closed economic system: the whole global economy.

# Referee states that: *The central bank is also an outsider, setting 'the' interest rate "basically based on the bankruptcy rate of firms"*. *What does this mean? There is no inflation and, therefore, no inflation target.*

Please see our figure 3: In our model there exists price inflation (and deflation), and thus overheating of economy that is a seed of economic crisis. We have not used such central bank behaviour that in overheating situation interest rate is raised because our purpose was to study and mimic real central bank behaviour under the "new normal" central bank policy in western countries. Of course, fighting inflation can be added in the model if we like to simulate the effects of central bank behaviour on the economy.

Referee states that: For the purpose, the labour market will need to be considered seriously and from there we can move to unemployment, structural stagnation, and matters of pressing interest.

This model and simulations is the first order approximation. The labour market can be added in our theory and model as it is done elsewhere (see Estola, 2017 about how Newtonian labour market works).

In conclusion, adding these suggestions would make this model very complicated and increase the page number by about one magnitude. Of course, it can be done, but in the first order approximation it would be quite inconvenient.

### References

P. A. Samuelson, Econometrica 9(2), 97–120 (1941).
P. A. Samuelson, Econometrica 10(1), 1–25 (1942).
M. Estola, Newtonian Microeconomics: A Dynamic Extension to Neoclassical Micro Theory Palgrave Macmillan, 2017.