Response to Referee #2

Thank you again for your diligence, your comments, and your suggestions. They were all helpful to me as I prepared the revisions. In what follows, I put excerpts from your report in bold. Here is how the paper was revised in response to your suggestions:

1. **I do not find the specific discrete bidding space considered in this paper to be of special interest per se, nor more realistic or relevant than the conventional continuous bidding space in any application that comes to mind. For instance, if bids are monetary it seems natural to assume a continuous bidding space. The author should discuss relevant applications that particularly fit the positive integer restriction, i.e., the "even spacing" between consecutive bids. I do not immediately see any. For instance, a discrete bidding space can be motivated in terms of cumulative indivisible investments (e.g., building capacity), but then why these investments should be of equal size? Moreover, why do we also restrict prize valuations to be positive integers? This implies a particular proportionality between bids and valuations which should be motivated. The author briefly mentions applications in experiments but should elaborate on this point.**

   I address this point you raised below:

   (a) The discrete strategy space does not mean "restrict prize valuations to be positive integers". To clarify the notation, I added the following sentences into the first paragraph of section 2 (Theoretical Model):

   "When player could only choose bids from set $\{0, g, 2g, \ldots, ng, \ldots, C | n \in \mathbb{Z}_+\}$, we call the strategy space is discrete, where g is grid of the strategy space and C is a bidding cap. We use $|C/g|$ to represent the maximum integer N which satisfies $Ng \leq C$. Since $(|C/g| + 1)g > C$, we only need to consider strategy space $\{0, g, 2g, \ldots, ng, \ldots, |C/g|g\}$. Obviously, the strategy space $\{0, g, 2g, \ldots, ng, \ldots, |C/g|g\}$ is equivalent to $\{0\ 1\ 2, \ldots, |C/g|\}$. Therefore, without loss of generality, we can always assume $g = 1$, $Q_i \in \mathbb{Z}^+$ and $C \in \mathbb{Z}^+ \cup \{+\infty\}$. Moreover, we say the valuation of the prize Q_i is an odd (even) number, means $Q_i \in 2\mathbb{Z}^+ (Q_i \in 2\mathbb{Z}^+ + 1)$. In other words, the number of player i's strategies could be rationalized is even (odd)."

   (b) Strategy space is always discrete in reality. Specially Dechenaux et al. (2006)point out that some experimental results are related to possible multiple equilibria, especially asymmetric equilibria, in all pay auction with discrete strategy space. Therefore, studies for all-pay auction with discrete strategy space is valuable to understand experimental data and subjects' strategic behaviors in real life.

References:

- Emmanuel Dechenaux, Dan Kovenock, and Volodymyr Lugovskyy. Caps on bidding in all-pay auctions: Comments on the experiments of a. rapoport and w. amaldoss. *Journal of Economic Behavior & Organization*, 61(2):276–283, 2006

2. **To my understanding, the results in this paper are qualitatively in line with the analysis under the continuous bidding space assumption. For instance, the fact that we do not obtain full dissipation (but we do obtain "approximate" full dissipation) when bids are restricted to positive integers is hardly surprising. Restricting the bidding space to positive integers does not seem to bring any novel insight to our understanding of all-pay auctions besides increasing the number of (approximately) payoff-equivalent equilibria. Similarly, introducing bidding caps into the model does not seem to change the picture. I believe the author should focus on results that are reversed (or at least substantially different) when we move from the continuous to the discrete bidding space case.**

To motivate my paper, I added the following sentences into the second paragraph of section 1 (Introduction)

"However, strategy space is always discrete in reality. Specially, in experiments of all-pay auction, different with predictions under continuous strategy space, overbidding is common (Fehr and Schmid, 2010; Gneezy and Smorodinsky, 2006), and subjects' bids are not uniform (Ernst and Thöni, 2013; Liu, 2014). Besides, there are huge heterogeneity among subjects in experiments(Davis and Reilly, 1998; Deck and Sheremeta, 2012; Klose and Kovenock, 2013; Mago and Sheremeta, 2012). Dechenaux et al. (2006) point out that these findings are related to possible multiple equilibria, especially asymmetric equilibria, in all pay auction with discrete strategy space. Therefore, studies for all-pay auction with discrete strategy space is valuable to understand experimental data and subjects' strategic behaviors in real life."

References:

- Dietmar Fehr and Julia Schmid. Exclusion in the all-pay auction: An experimental investigation. *Available at SSRN 1815001*, 2010

- Uri Gneezy and Rann Smorodinsky. All-pay auctions — an experimental study. *Journal of Economic Behavior & Organization*, 61(2):255–275, 2006

- Christiane Ernst and Christian Thöni. Bimodal bidding in experimental all-pay auctions. *Games*, 4(4):608–623, 2013

- Tracy Xiao Liu. All-pay auctions with endogenous entry timing: An experimental study. *Working Paper*, 2014

- Douglas D Davis and Robert J Reilly. Do too many cooks always spoil the stew? an experimental analysis of rent-seeking and the role of a strategic buyer. *Public Choice*, 95(1-2):89–115, 1998

- Cary Deck and Roman M Sheremeta. Fight or flight? defending against sequential attacks in the game of siege. *Journal of Conflict Resolution*, 56(6):1069–1088, 2012

- Bettina Klose and Dan Kovenock. The all-pay auction with complete information and identity-dependent externalities. *Economic Theory*, pages 1–19, 2013

- Shakun D Mago and Roman M Sheremeta. Multi-battle contests: An experimental study. *Available at SSRN 2027172*, 2012

- Emmanuel Dechenaux, Dan Kovenock, and Volodymyr Lugovskyy. Caps on bidding in all-pay auctions: Comments on the experiments of a. rapoport and w. amaldoss. *Journal of Economic Behavior & Organization*, 61(2):276–283, 2006

3. **The structure of Nash equilibria crucially depends on whether the valuations of the contended prize are even or odd, but it is not immediately clear how this is of any practical significance. Why are we interested in the effect of odd vs even prize valuations? Note that these effects disappear under the continuous bidding space assumption, which suggests they are of secondary magnitude and can be discarded as "approximation" unless we have good reasons to particularly focus on integer prize valuations and integer bids.**

   Potters et al. (1998) point out that when the value of object is even, there are multiple equilibria in all-pay auction. Therefore, experimental data can hardly test whether subjects' behavior deviates from Nash Equilibrium. To solve this problem, Potters et al. (1998) set the value of object as 13 to ensure the uniqueness of the symmetric equilibrium, and find that subjects' behavior is not significant different with Nash Equilibrium. Our paper finds that when the value of object is odd, asymmetric equilibria do not exist, and in other words, the symmetric equilibrium is the only Nash Equilibrium, which theoretically supports Potters et al. (1998)'s experiment.

   References:

   - Jan Potters, Casper G De Vries, and Frans Van Winden. An experimental examination of rational rent-seeking. *European Journal of Political Economy*, 14(4): 783–800, 1998

4. **The author introduces notation for bidders $(1, 2)$ and for prize valuations (v_1, v_2) in the Introduction. This notation is never used again, as it is redefined in Section 2 into x,y and Q_x, Q_y respectively. The author should**

homogenize notation. I personally prefer the one in the Introduction which is more conventional.

Thank you very much for your suggestion. I replaced bidders$(1, 2)$ into bidders(x, y) in the paper to be consistent with Bouckaert et al. (1992).

References:

- Jan Bouckaert, Hans Degryse, and Casper G De Vries. Veilingen waarbij iedereen betaalt en toch iets wint. *Tijdschrift voor economie en management*, 37(4):375–393, 1992

5. **I am not sure the notation about the bidding space $\{0, 1, \ldots, C-1, C\}$ is ideal given that C can be infinity. One alternative is to define the bidding space as $B_C := \{b \in \mathbb{Z}_+ : b \leqslant C\}$ for each possible bidding cap C $\mathbb{Z}_+ \cup \{+\infty\}$**

Thank you very much for your suggestion. I have changed the wording accordingly.

6. **Notation is sometimes introduced without being defined. For instance, see $n, P_x, \ldots, P_y$ and V_x, V_y in Proposition 1. As this notation is crucial and carries on through the rest of the paper in the characterization of Nash equilibria under different restrictions, it is worth defining it properly once for all.**

Thank you very much for your suggestion. I added the following sentences into the first paragraph of section 2 "Following Bouckaert et al. (1992), V_i is the expected payoff of player i; P_i^n is the probability for player i to bid n, $n \in \{0, 1, \ldots, C\}$. "

7. **When commenting on the result on the tie-breaking rule, the author should be careful in saying that the equilibrium characterization becomes independent of whether the prize is even or odd, but it is not generally independent of the valuations of the prize.**

Thank you very much for your suggestion. I have changed the wording accordingly.

Thank you again for your helpful and careful comments. I hope I have satisfactorily addressed all your concerns.

References

Jan Bouckaert, Hans Degryse, and Casper G De Vries. Veilingen waarbij iedereen betaalt en toch iets wint. *Tijdschrift voor economie en management*, 37(4):375–393, 1992.

Douglas D Davis and Robert J Reilly. Do too many cooks always spoil the stew? an experimental analysis of rent-seeking and the role of a strategic buyer. *Public Choice*, 95 (1-2):89–115, 1998.

Emmanuel Dechenaux, Dan Kovenock, and Volodymyr Lugovskyy. Caps on bidding in all-pay auctions: Comments on the experiments of a. rapoport and w. amaldoss. *Journal of Economic Behavior & Organization*, 61(2):276–283, 2006.

Cary Deck and Roman M Sheremeta. Fight or flight? defending against sequential attacks in the game of siege. *Journal of Conflict Resolution*, 56(6):1069–1088, 2012.

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Shakun D Mago and Roman M Sheremeta. Multi-battle contests: An experimental study. *Available at SSRN 2027172*, 2012.

Jan Potters, Casper G De Vries, and Frans Van Winden. An experimental examination of rational rent-seeking. *European Journal of Political Economy*, 14(4):783–800, 1998.