July 30, 2017

Response to Referee #1

Thank you again for your diligence, your comments, and your suggestions. They were all helpful to me as I prepared the revisions. In what follows, I put excerpts from your report in bold. Here is how the paper was revised in response to your suggestions:

1. **This is an interesting paper and the author derives relevant results. However, I believe the paper may benefit from some important changes in its exposition. In general, the results are only described in mathematical terms and no intuition or description in words is provided (see e.g. the paragraph after Prop 2). This does not facilitate the reader's understanding, and the author should exert more effort guiding the reader towards a good understanding of the results.**

   Thank you for the suggestions and revising the paper based on these suggestions have made it clearer and better. I have made the following changes in the revised version:

   (a) Compare the differences of expected bid and payoff with continuous strategy space to that with discrete strategy space, after Propositions 1 and 2.

   (b) Add statement that, with discrete strategy space, though player x's strategy choices do not affect her expected payoff $V_x$, she could choose equilibrium strategy to affect player y's expected payoff $V_y$; therefore, we could study player x's kindness and higher order belief though $V_y$ under different experiment design.

   (c) After Proposition 5, emphasize that under favor-one-sided tie-breaking rule, equilibrium is independent to the parity of the value of object, and since only two probabilities of bidding strategy is undetermined, we could test whether players' behavior in the experiment is consistent with predications of Nash Equilibrium.

2. **The author states that most theoretical models assume a continuous strategy space for tractability concerns. Despite a continuous strategy space often provides a more tractable setting, tractability does not have to be the main reason for this choice by the literature, and it is not clear what strategy space (continuous or discrete) is more suitable for each application.** I address this point you raised below:

   (a) Tractability is one important concern for continuous strategy space setting but not the only reason: even for discrete strategy space, we could refine the strategy space repeatedly, and its limitation would be a Nash Equilibrium with the continuous strategy space(Dasgupta and Maskin, 1986).

(b) However, strategy space is always discrete in reality. Specially, in experiments of all-pay auction, different with predictions under continuous strategy space, overbidding is common (Fehr and Schmid, 2010; Gneezy and Smorodinsky, 2006), and subjects' bids are not uniform (Ernst and Thöni, 2013; Liu, 2014). Besides, there are huge heterogeneity among subjects in experiments(Davis and Reilly, 1998; Deck and Sheremeta, 2012; Klose and Kovenock, 2013; Mago and Sheremeta, 2012). Dechenaux et al. (2006) point out that these findings are related to possible multiple equilibria, especially asymmetric equilibria, in all pay auction with discrete strategy space. Therefore, studies for all-pay auction with discrete strategy space is valuable to understand experimental data and subjects' strategic behaviors in real life.

References:


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- Dietmar Fehr and Julia Schmid. Exclusion in the all-pay auction: An experimental investigation. *Available at SSRN 1815001*, 2010
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- Emmanuel Dechenaux, Dan Kovenock, and Volodymyr Lugovskyy. Caps on bidding in all-pay auctions: Comments on the experiments of a. rapoport and w. amaldoss. *Journal of Economic Behavior & Organization*, 61(2):276–283, 2006

3. **When the author introduces the condition Q=2n, the variable n has not been defined. If n is just a natural number, the author should mention that the condition requires Q to be an even number.**

   Thank you very much for your suggestion. To clarify the notation, I have made the following changes in the revised version:

   (a) I added the following sentences into the first paragraph of section 2 (Theoretical Model)

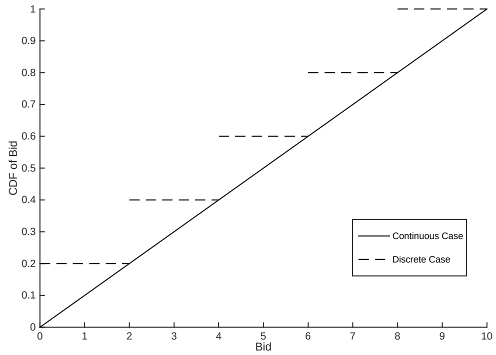
   *"When player could only choose bids from set $\{0, g, 2g, \ldots, ng, ..., C|n \in \mathbb{Z}_+\}$, we call the strategy space is discrete, where $g$ is grid of the strategy space and $C$ is a bidding cap. We use $|C/g|$ to represent the maximum integer $N$ which satisfies $Ng \leq C$. Since$(|C/g| + 1)g > C$, we only need to consider strategy space $\{0, g, 2g, \ldots, ng, ..., |C/g|g\}$. Obviously, the strategy space $\{0, g, 2g, \ldots, ng, ..., |C/g|g\}$ is equivalent to $\{0\ 1\ 2, ..., |C/g|\}$. Therefore, without loss of generality, we can always assume$g = 1$, $Q_i \in \mathbb{Z}^+$ and $C \in \mathbb{Z}^+ \cup \{+\infty\}$. Moreover, we say the valuation of the prize $Q_i$ is an odd (even) number, means $Q_i \in 2\mathbb{Z}^+ (Q_i \in 2\mathbb{Z}^+ + 1)$. In other words, the number of player i 's strategies could be rationalized is even (odd)."*

   (b) Replace $Q_i = 2n(Q_i = 2n + 1)$ into $Q_i \in 2\mathbb{Z}^+ (Q_i \in 2\mathbb{Z}^+ + 1)$ in the paper.

4. **P's and V's of Proposition 1 are not defined. This sharply breaks the flow of the paper, because it forces the reader to look for their definition. The original papers from which part of the result is taken (Bouckaert et al 1992 and Schep 1985) are not written in English, and thus they do not help understand the author's notation. The explanation of the Proposition at page 3 clarifies (too late) the meaning of the notation.**

   Thank you very much for your suggestion. I added the following sentences into the first paragraph of section 2 "Following Bouckaert et al. (1992), $V_i$ is the expected payoff of player $i$; $P_i^n$ is the probability for player $i$ to bid $n$, $n \in \{0, 1, ..., C\}$. "

5. **A graphical representation of the distribution of mass of Proposition 1 might greatly help the reader understand the results. After Prop 1, a comparison with the case of continuous strategy space might be helpful.** This is an excellent idea. I added the following sentences and graph after Proposition 1: Proposition 1 means that Nash equilibrium strategy with discrete strategy space could be widely different with the continuous strategy space case. For example, when $Q = 10, V_y = 1$, following figure shows the CDF for probability that player x bids. Specifically, with discrete strategy space, the probability that player x bids 0 (8) is 20% larger (lower) than the continuous strategy space case.




6. **At page 4, "When $Q_y = 2n + 1$, it is always [...]" lacks the conclusion that one should draw from that inequality.**

   Thank you for the suggestion. I added the following sentences "This means that, when the valuation of the prize for player y is an odd number, her expected bids are always lower than that with continuous strategy space." Similarly, I added the following sentence after "it is $\frac{Q_y^2+1}{2Q_y} > \frac{Q_y}{2}$ with $Q_y \in 2\mathbb{Z} + 1$", "which means that player x's bids are also always lower than that with continuous strategy space".

7. **The author repeatedly says that the Nash equilibrium depend on the parity of the reward size. The meaning of this remains cryptic until one enters into the details of the paper. I suggest clarifying from the beginning its meaning, rephrasing for instance to: the valuation of the prize is an odd or an even number.**

   Thank you for the suggestion. I have changed the wording accordingly, and added the following sentences into the first paragraph of section 2 "We say the valuation of the prize $Q_i$ is an odd （even） number, means $Q_i \in 2\mathbb{Z}^+ (Q_i \in 2\mathbb{Z}^+ + 1)$, In other words, the number of player i 's strategies could be rationalized is even (odd)."

8. **In the proofs, the author uses "suggest" for formal results, such as"Lemma 4 suggests $V = 0$". I believe that"implies" is a more appropriate word for analytical results. Additionally,"we get" should probably be substituted by "we obtain" .**

   Thank you for the suggestion. I have changed the wording accordingly.

Thank you again for your helpful and careful comments. I hope I have satisfactorily addressed all your concerns.

# References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