Title: A very good question

Reply: It is really worthy discussing how to distinguish the mistake of Theorem 5 of Levy (1992). For discussion purposes, the relevant contents of Levy (1992) and Meyer (1989) are listed at the end of document. In order to answer this question that whether the mistake of Theorem 5 of Levy (1992) is a theoretical faultiness or it is only an error in description for missing out the restriction to the transformation in Meyer (1989), we must first ascertain that transformations mentioned in Theorem 5 of Levy (1992) are "the most general transformations" or "the increasing, continuous, and piecewise differentiable transformations". If the answer is the former, then the discussion paper by Gao and Zhao has proved that the mistake of Theorem 5 of Levy (1992) is a theoretical faultiness.

I believe that transformations mentioned in Theorem 5 of Levy (1992) are "the most general transformations", and the main reasons are as follows.

1. Although it seems that Theorem 5 of Levy (1992) is a summary of Meyer's result from the sentence "Meyer's result is summarized in the following theorem", we must notice that the sentence "Meyer (1989) and Brooks and Levy (1989) deal with the most general transformation m(X) and n(X)" indicates that transformations mentioned in Theorem 5 of Levy (1992) are "the most general transformations".

As we have already known, Meyer (1989) indeed deals with the increasing, continuous, and piecewise differentiable transformations. The main reason for this contradiction may be that the restrictions to transformations in Meyer (1989) are displayed in neither Theorem 1 nor Theorem 2.

2. From the content point of view, transformations mentioned in Theorem 5 of Levy (1992) are "the most general transformations".

It is easy to prove that the first part of Theorem 5 in Levy (1992) does hold for "the most general transformations", so it is nature to think that the second part will still hold.

3. Pay attention to the fact that if transformations mentioned in Theorem 5 of Levy (1992) are increasing, continuous, and piecewise differentiable, then condition (12) is sufficient and necessary for m(X) dominating n(X) by SSD. However, Theorem 5 of Levy (1992) only shows the sufficiency. Obviously, there is a big distinguish between Levy's Theorem 5 and Meyer's result. This truth indicates that transformations mentioned in Theorem 5 of Levy (1992) are "the most general transformations".

4. Let's consider this question from the logical relationship. Levy has already pointed that Sandom (1971) and Hadar and Russell (1974) discussed the stochastic dominance relations for particular transformations, then the following part should deal with "the most general transformations". That is, transformations mentioned in Theorem 5 of Levy (1992) should be "the most general transformations".

1. The description of Theorem 5 in Levy (1992)

Levy and Sarnat (1971b) prove that if X dominates Y by the various SD rules, then αX also dominates αY by the corresponding SD rule where $\alpha > 0$. Hadar and Russell (1974) analyze the condition under which the transformation $aX + b$ dominates the original distribution X. Sandmo (1971) analyzes the particular transformation $X + \Theta$ (where Θ is a constant) to determine the comparative statics effect of an increase of the random output price on the firm's optimal output decisions. Meyer (1989) and Brooks and Levy (1989) deal with the most general transformation $m(X)$ and $n(X)$. Meyer's result is summarized in the following theorem.

THEOREM 5. *Given a random variable X with the density $f(X)$ and support in the interval $[a, b]$, the random variable $Y = m(X)$ dominates the random variable $z = n(X)$ in the first degree if $\{m(X) - n(X)\}f(X) \geq 0$ for all X in $[a, b]$. Similarly, the dominance condition for SSD is given by the condition,*

$$\int_a^x \{m(t) - n(t)\}f(t)dt \geq 0 \qquad \text{for all } x \text{ in } [a, b] \tag{12}$$

(*for proof, see Meyer* 1989).

2. The description of Theorem 1 and Theorem 2 in Meyer (1989)

Throughout the analysis it is assumed that x is a continuously distributed random variable with support in the interval [a,b]. Its density function is denoted f(x) and its CDF by F(x). The transformations are assumed to be defined at all points in [a,b], and to be nondecreasing, continuous, and piecewise differentiable. Thus, the

Theorem 1: Given random variable x with density f(x) and support in the interval [a,b], random variable y = m(x) dominates z = n(x) in the first degree if and only if $(m(x) - n(x))\, f(x) \geq 0$ for all x in [a,b].

Theorem 2: Given random variable x with density f(x) and support in the interval [a,b], random variable y = m(x) dominates z = n(x) in the second degree if and only if

$$\int_a^x [(m(s) - n(s)]\cdot f(s)ds \geq 0 \quad \text{for all } x \text{ in } [a,b].$$