

Report on *Nash equilibria in all-pay auctions with discrete strategy space*, by Zheng Li

Economics

30th June 2017

1 Summary

The paper fully characterizes the set of Nash equilibria for a two-player complete information all-pay auction with bidding space restricted to positive integers lower than or equal to a given bidding cap which can be infinity. The author very briefly motivates the paper by saying that the literature has focused on the continuous bidding space for tractability, implicitly arguing that the discrete bidding space considered in this paper may be more realistic or relevant in certain applications. In addition, the author also restricts prize valuations to be positive integers.

2 Major comments

The paper is a brief technical note. The analysis is competent although the exposition can be substantially improved. My main concern is that the analysis lacks motivation.

1. I do not find the specific discrete bidding space considered in this paper to be of special interest per se, nor more realistic or relevant than the conventional continuous bidding space in any application that comes to mind. For instance, if bids are monetary it seems natural to assume a continuous bidding space. The author should discuss relevant applications that particularly fit the positive integer restriction, i.e., the “even spacing” between consecutive bids. I do not immediately see any. For instance, a discrete bidding space can be motivated in terms of cumulative indivisible investments (e.g., building capacity), but then why these investments should be of equal size? Moreover, why do we also restrict prize valuations to be positive integers? This implies a particular proportionality between bids and valuations which should be motivated. The author briefly mentions applications in experiments but should elaborate on this point.
2. To my understanding, the results in this paper are qualitatively in line with the analysis under the continuous bidding space assumption. For instance, the fact that

we do not obtain full dissipation (but we do obtain “approximate” full dissipation) when bids are restricted to positive integers is hardly surprising. Restricting the bidding space to positive integers does not seem to bring any novel insight to our understanding of all-pay auctions besides increasing the number of (approximately) payoff-equivalent equilibria. Similarly, introducing bidding caps into the model does not seem to change the picture. I believe the author should focus on results that are reversed (or at least substantially different) when we move from the continuous to the discrete bidding space case.

3. The structure of Nash equilibria crucially depends on whether the valuations of the contended prize are even or odd, but it is not immediately clear how this is of any practical significance. Why are we interested in the effect of odd vs even prize valuations? Note that these effects disappear under the continuous bidding space assumption, which suggests they are of secondary magnitude and can be discarded as “approximation” unless we have good reasons to particularly focus on integer prize valuations and integer bids.

3 Minor comments

1. The author introduces notation for bidders (1, 2) and for prize valuations (v_1, v_2) in the Introduction. This notation is never used again, as it is redefined in Section 2 into x, y and Q_x, Q_y respectively. The author should homogenize notation. I personally prefer the one in the Introduction which is more conventional.
2. I am not sure the notation about the bidding space $\{0, 1, \dots, C - 1, C\}$ is ideal given that C can be infinity. One alternative is to define the bidding space as $B_C := \{b \in \mathbb{Z}_+ : b \leq C\}$ for each possible bidding cap $C \in \mathbb{Z}_+ \cup \{+\infty\}$.
3. Notation is sometimes introduced without being defined. For instance, see $n, P_0^x, \dots, P_{Q-1}^y$ and V_x, V_y in Proposition 1. As this notation is crucial and carries on through the rest of the paper in the characterization of Nash equilibria under different restrictions, it is worth defining it properly once for all.
4. When commenting on the result on the tie-breaking rule, the author should be careful in saying that the equilibrium characterization becomes independent of whether the prize is even or odd, but it is not generally independent of the valuations of the prize.