

Nash Equilibria in All-Pay Auctions with Discrete Strategy Space

Referee Report

Summary

The paper analyse equilibrium bidding behaviour in an all-pay auction with discrete bidding strategy space and possibly bidding caps. Ties are broken with equal probability. The author considered the following four cases.

1. Homogenous bidders without caps: In any equilibrium, both players put positive mass on all the bids strictly lower than the valuation. When the valuation of the players is an even number, there exists a continuum of equilibria, and each player assigns same mass on odd bids and same mass on even bids. When the valuation of players is an odd number, there exists a unique equilibrium in which both players assign the same mass on every bid strictly below the valuation.
2. Heterogeneous bidders without caps: In all equilibria, the player with the higher valuation only puts equal and positive mass on odd bids, and the player with the lower valuation only puts positive mass on even bids. Finally, the expected payoff of the lower valuation player is always zero.
3. Homogeneous bidders with caps: Again there exists a continuum of equilibria when the valuation of players is an even number, and when the valuation is an odd number there exists a unique equilibrium which is symmetric. In all equilibria, none of the players puts any mass on some bids right below the cap. This property is consistent with Che and Gale (1998) in which strategy space is continuous.
4. Heterogeneous bidders with caps: Similar to the no caps case, a player only puts mass on a bid if the other player does not put any mass on it. In all equilibria, none of the players puts any mass on some bids below the cap.

Comments:

1. This paper thoroughly examines Nash equilibrium bidding behaviour in all-pay auctions with discrete bidding strategy space. All-pay auctions have been used to study competitive behaviour in the political arena, research and development, labour market, education, etc. Most of all-pay auction studies focus on continuous bidding strategy space for its technical easiness and tractability. However, actual decisions in terms of, e.g., how much to invest in R&D or bribing a politician, are usually choices from discrete sets, e.g., the amount of money to spend or hours to be spent on preparing for exams. The aforementioned applications of all-pay auction theory call for analysis regarding discrete bidding strategy space, since it is always important to check robustness of the results derived from continuous strategy space settings. The current paper indicates that such results are not always robust when the strategy space changes from continuous to discrete. The current paper then shows some interesting

(yet unintuitive) bidding behaviour in equilibria that has not been discovered.

2. However, I am a bit concerned about the applications and importance of the results in the current paper. Does equilibrium behaviour with discrete strategy space provide any insights that are missing in the literature on all-pay auctions with continuous strategy space? Are there any empirical/experimental evidences suggest the equilibrium behaviour in discrete bidding strategy space setting fits better to the data? If the continuous strategy space is good enough in describing actual behaviour in reality, then the results in the current paper is not very important. Indeed, some of the equilibrium behaviour, e.g., the high valuation player only bid on odd numbers whereas the low valuation player only bid on even numbers, are not intuitive and I find it difficult to imagine people behaving in such a way.
3. Nevertheless, the current paper makes important contributions to the all pay auction and contests literature. It shows that contest researchers need to be cautious when applying theories based on all-pay auctions with continuous strategy space to explain empirical/experimental evidence.
4. When bidding strategy is discrete, equilibrium behaviour is sensitive to tie-breaking rules. For example, Dechenaux et al. 2006 characterized equilibria of all-pay auction with discrete strategy space and homogeneous players when none of the players receives any payments in ties. The authors also show that the equilibrium behaviour depends on whether the cap is an odd or even number. To connect to the small literature on discrete strategy space and be clear about the contribution of the current paper, I suggest the author discuss the effect of tie-breaking rules and caps on equilibrium bidding behaviour.
5. Finally, in the introduction (page 1 paragraph 2 line 7), the current paper may not be the first to examine asymmetric equilibria for symmetric players when bidding space is discrete. Dechenaux et al. 2006 shows asymmetric equilibria may exist for symmetric players when bidding space is discrete. If this is the case, please cite the paper and make the relevant correction.

Overall, I suggest the author revise the paper according to the above comments and resubmit to the journal. In particular, what are the new insights that discrete strategy space setting can provide and the discussion about tie-breaking rule as well as caps. For the former, the author can refer to Dechenaux et al. 2006 which provides some evidence suggests subjects' behaviour in an experiment might be better explained by the model with discrete strategy space. For the latter, the author can make some conjectures about how tie-breaking rule and caps should affect the equilibrium bidding behaviour.

Reference

E Dechenaux, K Dan, V Lugovskyy, "Caps on bidding in all-pay auctions: Comments on the experiments of A. Rapoport and W. Amaldoss", *Journal of Economic Behavior & Organization*, 2006, 61 (2) :276-283