Referee report on "When unionisation is profitable for firms in network industries" MS number 2103

The authors consider a network industry in which the surplus of an agent increases with the number of agents who buy the same good from the same firm. Furthermore, they consider a duopoly that can choose to be unionised or not. They claim that under some configurations of parameters, firms prefer to be unionized.

Main comments

1. My main concern is about the second order conditions that are not checked. For instance, result 2 does not take into account the restrictions $q^{EB/EB} > 0$, which requires, from (8), that $n < \frac{b(\theta-1)+3}{2}$ which is smaller than n^* (if $\theta < 1$). In other words the condition $q^{EB/EB} > 0$ (which requires probably the same restriction as the second order condition) prevents n to be larger than n^*. Thus, the peculiar case that is highlighted in the paper and in the title of the paper (unionisation (EB) that dominates perfect competition on the labor market (PM) if $n > n^*$) is not feasible. My fear is that a similar critique is applicable in section 3.

2. My second concern is about the motivation of the paper, which I do not understand precisely. What is the aim? Why is it important to deal with unionization in network industries? Do you have specific examples to illustrate your case? Try to be more convincing.

3. My third concern is related to the modeling strategy. By assuming that $y_i = q_i$, (see page 4), you can rewrite the inverse demand function (1) as $p_i = a - (1-n)(q_i + q_j)$. Now I do not see the difference between this demand and a classic demand function like $p_i = a - b(q_i + q_j)$. A change in n (interpreted here as a change in the network effect) is the same as a change in b, the slope of the demand. If this is correct, then I do not see any difference between what is studied in the paper and the classic analysis on unions. In other words, you should be able to perform the same analysis without having to limit yourself to network industries.

Other comments.
4. The results on pages 5-6 should be presented more clearly. Examples:
- $\Delta\pi^1$ is defined as $\pi^{PM/PM} - \pi^{EB/EB}$. But $\pi^{PM/PM}$ and $\pi^{EB/EB}$ are not defined (only $\Pi^{EB/EB}$ and $\Pi^{EB/EB}$ are defined, and I presume that they are equivalent)
- $\Delta\pi^2$ is defined as $\pi^{PM/EB} - \pi^{EB/EB}$. But $\pi^{PM/EB}$ and $\pi^{EB/EB}$ are not defined. Is $\pi^{PM/EB}$ equal to $\Pi_1^{PM/EB}$ or equal to $\Pi_2^{PM/EB}$? As far as I understand, $\pi^{PM/EB} = \Pi_1^{PM/EB} = \Pi_2^{EB/PM}$ but this should be clearly explained (same comment for $\Delta\pi^3$).
- In result 1 what do you mean with "the more likely bargaining with unions may emerge"? Does it mean that both firms are unionized (EB/EB) or that

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at least one firm is unionized (EB/PM, PM/EB, EB/EB)? This should be clarified.

   - If it means that both firms use EB (the EB/EB case) then clearly state the conditions for a SPNE, i.e., $\Pi_1^{EB/EB} - \Pi_1^{PM/EB} > 0$ and $\Pi_2^{EB/EB} - \Pi_2^{EB/PM} > 0$, which are equivalent because $\Pi_1^{PM/EB} = \Pi_2^{EB/PM}$. Thus the condition requires $\Delta\pi^2 < 0$.

   - If it means that at least one firm uses EB (the EB/EB, EB/PM and PM/EB cases) then clearly state the conditions for a SPNE, i.e., $\Pi_1^{EB/PM} - \Pi_1^{PM/PM} > 0$ or $\Pi_2^{PM/EB} - \Pi_2^{PM/PM} > 0$, which are equivalent.

   - Clearer explanations would make the description of Figure 1 easier to understand.

   - Why using "$\Longleftarrow$" in the proof instead of "$\Longleftrightarrow$"?

   - In the proof of result 2, I think that we should read $n \lesseqgtr n^*$ instead of $n \gtreqless n^*$.

   5. In Section 3, you should explain how $\Pi_1^{EB/EB}$ is computed. Is it equal to $\Pi_2^{EB/EB}$? I suppose it is not, because firm 1 is the first firm to bargain over the employment level, and firm 2 follows. In such a Stackelberg setting, firms do not sell the same quantities. But this is not clear in the paper... and I am not sure that this is how the profits were computed.

   6. You should at least discuss the hypothesis that firms either bargain over wages and employment with a bargaining power exogenously given by $1 - b$, or are able to hire labor on a perfect labor market, which amounts to a bargaining power equal to 1. Why do you consider two discrete cases instead of a continuous one in which you would perform a comparative statics on the firms' bargaining power b?

   7. Page 3: y_i is not a "market share" but the output because you assume (page 4) that $y_i = q_i$.