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A New Approach of Stochastic Dominance for Ranking Transformations on the Discrete Random Variable (by Jianwei Gao, Feng Zhao)

In the reviewed paper the authors propose new techniques to rank random variables (*transformations* of random variables) based on Stochastic Dominance (SD hereafter) criteria. Although there is a long and well known literature on SD, the authors really offer novel and original research in their work, specially with regards to:

1. They focus on discrete random variables, which have received little attention in the existing literature
2. They base their measures on the properties of the functions that define the transformations of the random variables, and not on their probabilistic features (i.e. cumulative or probability functions)

The paper is, in general, well written and well structured. The presentation of the results flows in a natural way, starting with the most natural questions regarding the issue at hand (necessary conditions) and then building up, step by step, a discussion on what characteristics must be considered to derive some sufficient conditions.

After and introductory section, in which the authors review the more relevant literature on the topic and present their goals, and a "preliminaries" section where the context and main notation is introduced, the research is presented as follows:

- A section discuses the necessary and sufficient conditions for First Order Stochastic Dominance (FSD hereafter, as in the paper) between transformations of random variables.
- Another section closely replicates the preceding section, but for Second Order Stochastic Dominance (SSD) in this case.
- Then, a comparison between the new and the existing SD rules is supplied.
- Finally, a numerical example and a some concluding comments and summary close the paper.

The paper is easy to read and the author's proposed contribution becomes clear and understandable. The mathematical results are well proved and illustrated by mean of graphical interpretations. We find no flaw on this. The numerical example in the next-to-last section nicely puts the issue in perspective and facilitates a global understanding of the contributions of the paper.

To the best of our knowledge, the authors' contribution is original, meaningful and will become useful. We thus believe that it deserves to be known. We agree with the author's two main claims:

- Most of the existing results on SD are for continuous random variables and thus cannot be directly applied to discrete random variables.
- The existing conditions for SD are based on the probabilistic characteristics of the random variables (the CDFs or PDFs), which often results in difficult or tedious computations.

Prior to a possible publication, though, we believe that some minor points should be addressed:

- In page 3, when the main notation is introduced, the authors recursively define

  $$F^{(n)}(x) = \int_a^x F^{(n-1)}(x)dx \; (n = 2, 3, \cdots) \text{ and } F^{(1)}(x) = F(x)$$

  and then immediately define U_n as the class containing all the functions u with $(-1)^{k+1}u^{(k)} \geq 0 \; (k = 1, 2, \cdots, n)$
  Although a knowledgeable reader will have no problem in understanding this, we believe that this notation is somehow misleading. One might think than, because of the similarity in the notation, $u^{(k)}$ is to be understood as $F^{(n)}$ (defined just two lines above) instead of the $k-$th derivative of the function u.
  Also, for the sake of completeness, it would be good to indicate that "u is a real-valued function"

- In page 4, at the beginning of Section 3, the authors introduce the short hand notation m_i (and similarly n_i) for $m(x_i)$ ($n(x_i)$ respectively) where $m(X)$ and $n(X)$ are the transformed random variables and $\{x_1, \cdots, x_n\}$ the support of X.
  We believe that, for consistency, either one or the other notation should be used throughout the paper.
  For instance, the proof of Theorem 3 (page 8) starts by saying "If $m(x_i) \geq n(x_i)$ ..." whereas two lines below it continues "...let $\Phi = \{i | 1 \leq i \leq n, \text{ and } m_i < n_i\}$..."

- In page 5, the proof of Theorem 1 ends with the expression:

  $$\sum_{i=1}^{n} u'(\xi_i)(m_i - n_i) \geq 0$$

  Being this the first time than ξ appears in the paper, it should be mentioned that it is a value between n_i and m_i.

- In formal logic, it is not customary to use the term “derive” in the statement of a Theorem as it is usually reserved for its proof. We suggest to replace the final sentence in the statement of Theorem 2, “..., we derive that the random variable...”, by “.... Then the random variable...”
  Also, this phrasing is more in line with how other theorems in the paper have been stated.
- Table 3 in page 16 is difficult to read. We suggest adding some horizontal lines to clearly separate the 4 different rows.

To summarize, this paper offers simple yet rigorous techniques to rank (transformations of) random variables which, in our view, show promise of being useful in the analysis of risk and uncertainty in many fields of business and economics. We believe that it deserves to be published.