**Reply to Referee Report 1**

Thank you very much for reading my paper so thoroughly. I appreciate your feedback and comments. I answer each of the comments below in a chronological order. The remarks of the Referee are italics and smaller fonts. To distinguish equations between the reply and the discussion paper I use Roman numerals in the responses.

***Comment:***

*In my opinion, in the introduction the Author should clarify the reasons why a search model with heterogeneous agents is needed for the evaluation of active labour market policy. For what I have understood, the heterogeneity among sectors and in the worker skills is used by the Author in order to split among employees that search on the job and employees that do not search. But probably the same result could be achieved by taking a random productivity per match and computing a threshold of the productivity that divides the employees in the two categories, as usual in search models. The main question is: what results adds an agent-based model to traditional search models, such as the one of [1], where analytical solutions related to the influence of the ALMP on endogenous variables are obtained?*

**Response:**

If I understand correctly, the Referee suggests that similar models may be built in traditional and agent-based methodology, then if we compare the results, the conclusions might be the same. I cannot agree with this approach. The main argument which I elaborate on below is that we cannot develop the model characterized by similar level of complexity in the traditional way. Even if it was possible, such model would be uncomputable and thus we cannot state that the result would be similar.

ABM approach allows a larger level of flexibility and diversity, because we do not need to find the solutions of equations but create routines for agents' behavior. We are able to write a routine for almost every behavior. Therefore, such complex systems are closer to real life, where actors are really heterogeneous and everyone must make decisions (Tesfatsion 2006). ABM can improve the realism of simulation and provide more accurate results of investigating social phenomena (Tesfatsion, Judd 2006).

The discussed agent-based model includes jobs from various sectors of economy and of different skills demands. I model three sectors and five skill levels, which implies 15 categories of job vacancies. Each category is characterized by different wage, production level, and cost. I also model unemployed who differ in the duration of unemployment, job preferences, skill level as well as productivity, wage requirements, and benefits. We can then observe the impact of the parameters not only within the sectors of the economy but also within the skill levels. The spatial aspect of the simulation is also important. Agents plan moves on the grid according to their resources as well as information gathered from the local labor market. Then they move and update the information for the next turn. Finally, agents have memory, for instance they make lists of firms they have visited, remember the wage they earned in the previous jobs, agents adapt to the labor market situation and can change their preferences. Their memory influences their future decisions and shapes the local labor market as a whole. The described system is, for obvious reasons, beyond reach of the classic models.

The next issue I want to refer to is that random productivity changes, implemented often via Markov chain in classic models, do not fit the dynamics of the agent-based models. Markov process is determined by its most recent value. On the contrary, the agents' decisions in ABM are dependent on the state of the whole system of more than one step ago (Tesfatsion, Judd 2006).

I believe that choosing the agent-based method for implementing labor market search theory can join the strengths of both worlds. The rigorous well-founded but stiff theory meets freedom and complexity of the agent-based model, which in turn appears to be even too flexible.

**Comment:**

*to my knowledge, all the value functions must be pre-multiplied by the interest rate (say r). For instance, the value of being unemployed (Eq. 2) is:* $rU_t^i = b_i + l_i + h_t^i[E(w)_t^i - U_{t+1}^i]$ *and not:* $U_t^i = b_i + l_i + h_t^i[E(w)_t^i - U_{t+1}^i]$. *The same for the all value functions. Has this error consequences on the results of the simulations?*

**Response:**

The Referee invokes the model in continuous time, where *r* could be perceived as the discount factor, thus *rU* is the discounted future value of *U*. The Bellman equations (value functions) in the paper are written in the discrete time, therefore values of the variables are computed for a given *t* and we do not discount them with *r* (e.g. Smith and Zenou 2003). Even if I would have to rewrite the value functions and multiply by *r*, it does not influence the simulation results – I discuss this point in details in the next response.

***Comment:***

*Workers employed "under their qualification" continue to search (Eq. 5). The value of an occupied job is therefore different if the worker that fill the job has an appropriate/inappropriate skill level (compare equations 6 and 7). This generates strong inconsistencies in the whole model and has strong consequences for the value of a vacancy. The Author assumes that the vacancy is always filled by an appropriate worker as in Eq. 3, where* $F(v)_t^i$ *and not* $\overline{F(v)}_t^i$ *is used in the value function. The same happens for unemployed (Eq. 1), that seem to consider only* $E(w)_t^i$*and not* $\overline{E(w)}_t^i$*in their value function. Therefore, I would like to known if this inconsistency (that must be corrected in the theoretical model) has consequences on the results of the simulations.*

**Response:**

I agree with the Referee's comment. In that case, two equations for the value of unemployment should be distinguished. The value of unemployment for job seekers with the lowest skill level looks like before because he or she cannot be employed below qualification:

$$U_t^i = b_i + l_i + h_t^i(E(w)_t^i - U_{t+1}^i) \quad (I).$$

In turn, the value of unemployment for the job seeker with skill level > 1, who can work below qualification should be written as:

$$U_t^i = b_i + l_i + \overline{h}_t^i(\overline{E(w)}_t^i - U_{t+1}^i) + h_t^i(E(w)_t^i - U_{t+1}^i) \qquad (II),$$

Where $\overline{h_t^i}$ is the probability of finding the job below qualifications; $\overline{E(w)_t^i}$ is the gain from working below qualification. Similarly $h_t^i$ and $E(w)_t^i$ are the same values for obtaining the more skill-fitted vacancy.

The corrected value function of the vacant job with skills demands < 5, which can be also settled by over-educated workers can be now written as:

$$V_t^i = -c_i + \overline{r_t^i}(\bar{F}(v)_t^i - V_{t+1}^i) + r_t^i(F(v)_t^i - V_{t+1}^i) \qquad (III),$$

Where $\overline{r_t^i}$ is the probability of matching with job seeker with skill level > skill demand, $\bar{F}(v)_t^i$ is the potential firm gain from employing mismatched worker. Consequently $r_t^i$ is the probability of matching with skill-fitted worker and $F(v)_i^t$ is the firm profit from employing skill-fitted worker.

The most skilled vacancies (level 5) can be settled only with job seekers with the highest skill levels. In that case, the value function remains unchanged:

$$V_t^i = -c_i + r_t^i(F(v)_t^i - V_{t+1}^i) \qquad (IV).$$

However, **the modification in the theoretical model should not influence the implemented behavioral routine**. At this point, it is necessary to explain in details how the agent-based implementation of the value functions works.

Eq. II and Eq. III should fit in an agent-based world. First of all, we do not solve the equation but we must think of behavior, which stands for the formula. We write the job-seeker behavior routine and firm behavior routine in pseudocode format for the value of unemployment and value of the vacant job in a given time period (Pseudocode 1).

**Pseudocode 1. The agent-based implementation of the value of the unemployment and value of vacant job equation**

```
Ask job-seekers {
If meet-vacancy with [sector = my-preferences and skills-demand < my-skills]
        then [scan neighborhood in search of other-free-vacancies] with
                [sector = my-preferences and skills-demand <= my-skills] {

                        If ANY other-free-vacancies   {
                                [ set value-of-unemployment =
                                benefits + leisure + match-probability * max(future-payoff)]

                                [set value-of-employment =
                                payoff - payoff-lost-probability + on-the-job-search-payoff ] }

                        If NOT ANY other-free-vacancies or skills-demand = my-skills {
                                [set value-of-unemployment = benefits + leisure ]
                                [set value-of-employment = payoff - payoff-lost-probability ] }

                If value-of-employment > value-of-unemployment  [ set can-work? TRUE]

                                else [set can-work? FALSE] }
                                }
Ask vacancies {
If meet-job-seeker with [preferences = my-sector and skills-level > my-skill-demands]
        Then [scan neighborhood in search of other-job-seekers] with
                [preferences = my-sector and skills-level <= my-skill-demands]
```

```
If ANY other-job-seeker  {
        [ set vacant-value = – cost + match-probability * max(future-payoff) ]
        [ set filled-value =
        payoff – wage-offer – payoff-lost-probability + future-payoff-probability ] }

If NOT ANY other-job-seekers or skills-level = my-skills {
        [ set vacant-value = -cost ]
        [ set filled-value = payoff – wage-offer – payoff-lost-probability ] }

If filled-value > vacant-value [ set can-hire? TRUE ]

        else [set can-hire FALSE] }
Ask agents {
if can-work=TRUE and can-hire=TRUE [ start-work ]

    else [continue-to-search]
        }
```

According to the Pseudocode 1, if a job-seeker finds a vacancy with lower skills demands he or she scans the neighborhood searching for better-fitted jobs which can be reached next turn. Then he or she calculates the present gain and potential future gain from a match with a better job, which enhances the value of unemployment for the next turn (second part of the Eq II). If the job seeker does not find better-fitted job his or her value of unemployment for the next turn will include only benefits and leisure. The current payoff is used to compute the value of employment. As a consequence, the value of unemployment would be bigger among higher-skilled job-seekers who apply for lower-skilled jobs, which is coherent with the Eq II.

The same with value function for vacancies: if a company meets an over-educated job-seeker, it scans the neighborhood in order to find a better-fitted job-seekers which can be hired in the next turn. If a company finds a future candidate(s) it calculates gain from the present candidate and gain from the future candidate, which enhance the value of vacant job for the next turn. Finally, calculations of the both agents must fulfill the Nash rule (Eq. 8) in order to start cooperation (sign a job contract).

This is my idea for a behavioral routine which captures the concept of Bellman (1957) dynamic optimization equations as written in Eq.I, Eq.II, Eq.III, and Eq. IV.

***Comment:***

*The value function of being unemployed $U_t^i$ defines the wage rate (Eq. 11) and therefore the value functions for all the states. But $U_t^i$ (Eq. 2) is endogenous. How the Author deals with this point? How does he set the wage if $U_t^i$ is not calculated as an explicit function?*

**Response:**

I think it is an important issue which I did not put enough attention to. The wage determination process is described below.

At the stage of seeking for a trading partner and bargaining procedure, job-seekers and companies make use of an additional exogenous variable called *wage-offer* which is assigned to each vacancy. The *wage-offer* consists of the *minimum-wage* in the economy (parameter) and *random-float* variable, which value depends on skill demands of the given vacancy. The higher the skill demands, the more increased the upper boundary of the *random-float* variable. For instance, the upper boundary of the *random-float* variable of vacancies with skill demands = 1 is 0.5. In the case of vacancies with skill demands = 5 is 1.5. Such solution implies that *wages-offer* of more skilled vacancies are on average higher. The variable *wage-offer* enables computing temporary values for U for the bargaining. The real wage of the worker is computed only if he or she matches the proper vacancy and starts producing. Then the real wage is updated at each period of the simulation according to the wage equation obtained from the Nash solution (Eq. 11). As wages depend on productivity, the positive relation between skill demands and wages is still fulfilled.

***Comment:***

*How do active labour market policies influence the theoretical model? I imagine that they change $h_t^i$ and $r_t^i$. Both the probability of finding a job and the one of finding a worker depend on the tightness in the labour market, $\theta_t^i$. Therefore, one should think that the world with ALMP should influence positively the search effectiveness, indicated by s. Nevertheless, it is well known from theoretical job search model that in an increase in search effectiveness reduces unemployment. I wonder if the whole discussion on ALMP can be synthesized by an increase in s... . But the footnote 11 defines A (not s) as the efficiency parameter and treats it as exogenous.*

*Finally, a clear relationship between the number of search units and the probability of finding a job must be specified (the footnote 7 does not highlights the point)*

**Response:**

As the Referee noticed, in theoretical job search model increase in the search intensity should reduce unemployment. However, the search model proposed by Cahuc and Le Barbanchon (2010) proves that in some situations policy that enhances search intensity may indeed induce unemployment. In chapter V Pissaridess (2000) suggests that if search intensity increases, the vacancies/unemployment ratio increases but the probability of vacancy – worker match decreases. As a result, there are two potential compensating effects of the policies that improve search intensity. The impact of such policy depends on model specification and calibration. It should also be kept in mind that search intensity is only one of 16 parameters that influence the economy and positive/negative interactions or compensating effects must be taken into account. In fact, as shown in figure 11, cross effects are the majority of the total impact of ALMP parameters.

The results obtained with the agent-based model shows that the influence of the job-search assistance program is not so straightforward because as shown it can lower the unemployment rate and at the same time it can increase the LTU rate (even if LTU participate in the ALMP program). Therefore, the total unemployment would not change and even may become more persistent. The model allows detailed measurement of this impact.

In pointed footnote I present a general shape of aggregate matching function, which was used as a basis for developing a behavioral matching routine. As the Referee noticed the equation lacks search effort. With the presence of this variable the aggregate matching function should be:

$$M = A(su^{\alpha}v^{1-\alpha}) \qquad (V).$$

The relation between the number of search units and the probability of matching a job is described on page 9 (the second and third paragraph). Below we provide pseudocode 2, which presents the matching routine for a representative agent, which could be useful for describing some details of the procedure.

**Pseudocode 2. The search and match routine for the agents**

```
Ask job-seekers {
If [searching? = TRUE] [update my-list-of-firms in the neighborhood]
[set search-units = random or if ALMP=TRUE [set search-units = random + ALMP-bonus] ]
        {
        while [search-units > 0] [face nearest firm and move forward 1 step]
                set search-units = search-units – 1

                if any? firm-here with free-vacancies [check skill-demands and sector]
                        if skill-demands=my-skills and sector=my-preferences {

                        [stop searching]
                        [ go to bargaining-routine] }
                        }
                }
```

Every period, job seekers draw search units at random from a distribution with upper boundary dependent on unemployment duration and ALMP participation. Then they look around and update the list of firms they can visit this turn. Then they move towards the nearest firm. Each step is one search unit. They move as long as they have search units or find the firm characterized by the desired sector of economy and skill demands.

In the end, I would like to highlight that I appreciate your contribution to my paper and hope that my responses resolve your doubts. If you accept the above elucidations I will provide you the revised version of the paper soon. I think that the pseudocode of the model should also be placed (instead of figure 1) in the revised version of the paper, as it may shed more light on some details of the model design.

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