Replies to Reviewer 2

Manuscript title: Dynamic Pricing with Reference Price Dependence
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Let me first thank Reviewer 2 for his careful reading of the article and for his insightful remarks. Below is the point-by-point list of replies to the remarks.

Replies to Reviewer's Comments

Reviewer 1

1) *The adjusting reference price causes demand to be more elastic. It would be nice to see some discussion of whether this is a natural outcome or not. Intuitively, a high reference price should increase demand whereas a low reference price should decrease demand. Why does the latter effect appear to dominate here? Potentially because of the second assumption about the general demand function that I question below.*

Reply: A result from the paper is that if demand is sensitive to a reference price, then the demand is more elastic. In other words, the monopoly looses some market power in the sense that it can extract less rent from the demand, and it has to charge less to maximize profits. The reason is not that a high reference price has more effect in increasing demand that a low reference price in decreasing demand. Indeed, in this model and as you noticed, there is no loss aversion (consumers are not more sensitive to a loss than to a gain of same magnitude). The reason of loss of market power by the monopoly (or of elasticity being more elastic) is that past prices play a role in determining current demand. That is the monopoly, while setting prices today, is constraint by past prices that he proposed to the consumer himself. This additional constraint, demand being sensitive to past prices, explains the loss of market power and greater elasticity of demand. I fully agree with you that such explanations should be incorporated in the revised version of the manuscript. This incorporation would improve the economical relevance of the paper.

2) *The author assumes that, when the reference price is higher (everything else equal), demand decreases more for any increase in the reference price. This is not immediately obvious to me and I would like to see some discussion of why this is the case. Diminishing sensitivity in the context of Kahneman and Tversky (1979) prospect theory appears to suggest the opposite. Moreover, I understood that this effect is responsible for the (somewhat surprising) result that a higher reference price does not always lead to an optimal higher selling price. If so, then the author should elaborate on this assumption. More generally, it should be more clear which assumption this (somewhat surprising) result depends on.*

Reply: Thanks for asking for this clarification. You are correct claiming that in this model the sign of the cross derivative of demand with respect to selling price and reference price ($Dpr=<0$, with $=<$ for equal or inferior) drives the "(somewhat surprising) result that a higher reference price does not always lead to an optimal higher selling price." (Technically, $Dpr<0$ is a submodularity assumption of the demand function with respect to p and r.) Economically, $Dpr=<0$ captures the fact that demand is more difficult to increase by reducing the selling price when the reference price is high that when it is low. Recalling $Dp<0$ and $Dr>0$, note that: $Dpr=0$ is verified for any demand function additively separable in p and r (as the linear demand function (3), which is widely used in the literature); $Dpr<0$ is checked for any demand function multiplicatively separable in p and r (like the demand function that I give as example, but also like isoelastic (Cobb-Douglas) and exponential demand functions). Because the literature mainly used the linear demand function (with additive separability properties), the cross effect Dpr vanishes and plays no role. In this situation, the results follow the intuition: the

dynamics of the selling price has to follow the dynamics of the reference price. A role of the model presented in the current article is to highlight the impact of the cross effect. First this effect exists and it is negative; second this effect (if large enough) may cause a negative selling-reference price relationship. The possibility of a negative relationship is counter-intuitive. And it is the interest of any modeling to explain counter-intuitive phenomena. In the revised version of the article, I will explain clearly as you ask, where the counterintuitive phenomenon comes from, that is from *Dpr<0*. I will elaborate also more on that assumption and the structural properties of the demand function.

Also, you are perfectly correct writing "Diminishing sensitivity in the context of Kahneman and Tversky (1979) appears to suggest the opposite" (of *Dpr=<0*). See on that point the modeling of prospect theory by Popescu and Wu (2007), which assume supermodularity of the demand function (that is *Dpr>0*). Just note that Kahneman and Tversky (1979) (as the cumulative version of prospect theory by Tversky and Kahneman 1992) deals with decision under risk. In my modeling, there is no risk since demand is deterministic. My modeling is thus closer to the work on decision under certainty of Tversky and Kahneman (1991), which proposes the deterministic version of prospect theory.

Anyway, and as you noticed, I only model one behavioral effect, namely the reference price effect. This is why I donnot link the modeling to prospect theory for which there are several behavioral effects (reference price, loss aversion, diminishing sensitivity). But, I recognized that calling to prospect theory at the end of the conclusion was misleading. This call to prospect theory has to be removed from the conclusion to improve clarity.

3) *I was also left with an open question about the price the firm started off with-shouldn't that matter for the price dynamics? I did not understand what is the assumption with respect to the starting price.*

Reply: Thanks for asking to clarify this point. The firm set the selling price *p* over the planning horizon *[0,T]* applied the pricing rule (11b). Recall that the selling price is a control variable (as opposed to the reference price *r*, which is a state variable). So there is no assumption about *p* at *t=0*; there is an assumption about *r* at *t=0* (this assumption is explicit after (2) as it is standard in articles using optimal control). Because I analyzed in this paper the optimal path, the value of *r* at *t=0* is not of importance. But the value of *r* at *t= 0* would be important if I analyzed a steady state. So there is no assumption with respect to *p* at *t=0*, and the assumption with respect to *r* at *t=0*, which is explicit in the article, does not impact the results linked to the optimal path. (Or the only impact is that if *r* at *t=0* is larger than *p* at *t=0*, then the reference effect is positive at *t=0*. The opposite will hold if *r* is lower than *p* at *t=0*.)

4) *When I hear the word reference dependence, I think of loss aversion a la Kahneman and Tversky (1979); however, the author only models a type of anchor as there is no kink in demand where the actual price exceeds the reference price. That said, I think adding loss aversion, one of the most robust behavioral observations about risk preferences, would make the analysis much more interesting. While there exists experimental evidence for anchoring, the psychology is much less well supported and I think that loss aversion is first-order important in the context of consumer pricing. Thus, I expect the results to be very different, if loss aversion were introduced. It strikes me as weird that the author says "when customers are subject to reference effects in the spirit of prospect theory" given that three of the four major ingredients of prospect theory are left out. Purely semantically, it would be most precise to just use the word anchor in my opinion.*

Reply: You are perfectly correct. In the article, I only analyze the effects of reference dependence (comparison of a result with respect to a benchmark, which determine the situation of a gain or a loss). I disregard other important behavioral elements such as loss aversion (gains weighing more heavily than losses) and diminishing sensitivity (gain and loss satiation). That is, in my view, there are three main ingredients of prospect theory in riskless situations in the articles of Tversky and

Kahneman (1991) (See the modeling of these three aspects of prospect theory by Popescu and Wu 2007). The fourth element is the probability weighting function that is used when there is uncertainty (Kahneman and Tversky 1979, Tversky and Kahneman 1992). This element does not apply when demand is deterministic, like in the present model.

Note that loss aversion is possible only if there is a reference effect. That is the reference effect comes prior to loss aversion. Also, loss aversion implies the discontinuity of the demand function at the point of status quo. This point has already been studied by Fibich et al (2003) with a linear demand function. Further, prospect theory (in the sense of reference effect, loss aversion, and decreasing sensitivity) has been modeled by Popescu and Wu (2007) for a general demand function.

So a goal of the article was to write a short paper explaining that the dynamics of the selling price does not always have to follow the dynamics of the reference price. A goal was not to model prospect theory (with loss aversion and decreasing sensitivity). But I agree with you that loss aversion and decreasing sensitivity may challenge the results presented here.

Eventually, I agree with you that I should not mention "prospect theory" in the article, because this call for other behavioral elements that I ignore here. As you say, I should use the concept "anchor" (or benchmark or reference price).

5) *It would make sense to have the reference price adjustment depend on past demand. If past demand is high then presumably more individuals updated their reference price and vice versa. It would be nice to see a discussion there.*

Reply: This is a bright idea, which I have never seen in the literature. Conceptually, this idea relies on the notion of internal reference price, that is a past price experienced by the consumers when they bought the product. This is different from the external reference price, a price simply seen by consumers, as is assumed implicitly in the literature. So many thanks for the suggestion. As you say, this point deserves discussion.

6) *It would be interesting to know how much profit is lost and whether the profit loss is a first-order effect. Given that loss aversion is absent in the model, I presume it is not.*

Reply: I agree that this is an interesting point that has to be discussed in the article.

7) *It would be nice to have the standard model as a special case of the current model. beta= 0 is the no-adjustment benchmark.*

Reply: Thanks for this suggestion. In the revised version, I will add this no-adjustment baseline-model as a benchmark for the adjustment case. The differences in both models will be clearer.

8) *I don't know the surveys of Kalyanaram and Winer (1995) and Mazumdar et al. (2005) and thus the citations appeared strange to me as the first citations about reference dependence.*

Reply: The surveys mentioned above come from the marketing literature (Marketing Science and Journal of Marketing). Their interest is to provide an overview of the reference effect (what I model), and not of prospect theory (which is more studied in economics). But these references are well known in the literature about reference effects. For instance, according to google scholar, of Kalyanaram and Winer (1995) and Mazumdar et al. (2005) have respectively 699 and 458 quotations. Note that contributions on (dynamic and behavioral) pricing come mainly from economics, marketing, and operations management. My references also come these three areas of research. Of course, some of theses references may be less known to economist, and I understand your "strange" feeling. So I will add other references from economics to better fit with an economic audience.

9) *I found the paper very hard to read because the authors used articles so sparingly.*

Reply: Thanks for this comment. I will use more articles in the text of the revised article so as to improve the readability of the article.

Refererences

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