The Corporate Social Responsibility is just a twist in a Möbius Strip

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Abstract

In recent years economics agents and systems have became more and more interacting and juxtaposed, therefore the social sciences need to rely on the studies of physical sciences to analyze this complexity in the relationships. According to this point of view we rely on the geometrical model of the Möbius strip used in the electromagnetism which analyzes the moves of the electrons that produce energy. We use a similar model in a Corporate Social Responsibility context to devise a new cost-benefit model in order to take into account of three positive crossed effects on the efficiency: 1) cooperation among stakeholders in the same sector; 2) cooperation among similar stakeholders in different sectors and 3) the stakeholders' loyalty towards the company. By applying this model to a firm's decisional problem we find that investing in Corporate Social Responsibility activities is ever convenient depending on the number of sectors, the stakeholders' sensitivity to these investments and the decay rate to alienation. Our work suggests a new method of analysis which should be developed not only at a theoretical but also at an empirical level.

Keywords: Corporate social responsibility, Econophysics, Firm Behavior.

JEL Classification Numbers: L13, D21, Z1

1 Introduction

In an even more globalized world very complex interactions characterize social and economic relationships. In Economics this interdependence among systems and among agents is just the core of the models of Corporate Social Responsibility (since now on CSR), which consider the global integration between firms and stakeholders, including workers, customers and the full environment (see Becchetti et al., 2014). The CSR implies a move from the maximization of the shareholders wealth to the satisfaction of a more complex objective function in which the interests of the other stakeholders are taken into account. On turn this creates also benefits for the business. For instance Becchetti et al.(2014) show that since more and more profit maximizing firms are adopting CSR practices there must be pecuniary benefits arising from them. The authors also document that the CSR has the potential to generate several values increasing effects by attracting better employees, enhancing their intrinsec motivation and loyalty, reducing turnover rates, improving the efficiency and by reducing operating costs. Nevertheless the CSR improves boosting sales revenues, increases rivals costs and attracts more ethical consumers, so that the firm can benefit from increases in her demand share.

All the above mentioned advantages can be seen as a sort of *ethical capital* accumulated trough the CSR practices, which also requires the payment of additional costs. By using a dynamic model, Becchetti et al. (2014) underline the conditions implying that such benefits overrun the costs. There is also a large literature on the benefits that in general arise by investing in CSR on stakeholders and in particular on workers. To this aim many analyses use the standard taxonomy of CSR criteria provided by Kinder, Lydenberg and Domini Research and Analytics, Inc. (KLD). They include the following eight wide-ranging categories into the Domini 400 index: i) community; ii) corporate governance; iii) diversity; iv) employee relations; v) environment; vi) human rights; vii) product quality; and viii) controversial business issues. Every category has its strengths and weaknesses identified and analyzed within the index, as well as the suggestion of corporate activities compliant with each specific category. For instance, by using the KLD index, Becchetti et al.(2016) show that the CSR firms which take into account the workers well-being are less exposed to business risks

and profit volatility. Rob et al.(200O) analyze the effects of increased productivity of the individual workers. The authors show how the specific investiments in Corporate Social Responsibility can be seen as the optimal incentives that foster the employees to allocate greater efforts to cooperative tasks because their derive utility from cooperation. In the meta-analysis devised by Harter et al (2003) positive workplace perceptions and feelings are associated with higher business-unit customer loyalty, higher profitability, higher productivity and lower rates of turnover. In Gond et al (2010) it is explained how employees' perceptions of CSR trigger attitudes and behavior in the workplace which affect organizational, social and environmental performance. In Degli Antoni et al (2011) it is analyzed the empirical relationship between CSR and social capital pointing out how the adoption of CSR good practices foster the creation of workers' social capital intended as cooperative social network, generalized trust, and relational skills. In addition for an analysis more specifically directed to the benefits of the cooperation between coworkers see Myers et al (2010) who discuss the effects of firm's values and workplace interaction on coworkers. Rast et al.(2012) also show, by using the data collected from employees of three private airline companies in Iran, that an important factor that have an impact on job satisfaction and productivity is the relationship with co-workers.

Therefore according to the CSR point of view firms and stakeholders can be depicted not as two distinct and unconnected systems, but they are a cross-system where transfers occur in a such a way that a business becomes a stakeholders' interest and conversely stakeholders well-being becomes part of the business. In this crossed-system the output of each part is transferred across them to become the others' input, so that these subsystems are strongly overloaded and linked inextricably together.

According to this point of view the best metaphor to approximate and represent this new conceptualization of links in economics systems and between agents is suggested by the physical sciences and it is the *Möbius strip*.

This is a topological enigma independently documented in 1858 by two mathematicians A. F. Möbius and J.B. Listing. It is a bend of paper given a 180 degree twist prior to having

its two ends connected.[1].

In this paper we want to explore in more details what mechanisms are at work to make CSR convenient for a Company. To this aim we extensively draw from the analogies with the behavior of fermions in a Möbius strip to show the kind of interactions among stakeholders are at work to realize improvements in the Company's performance (for a complete survey on the theoretical and empirical works on the forces driving CSR measures and the effects on the firms' performance see Crifo and Forget, 2015) The twist in the Mobius strip generates two important effects on the electrons' trajectories and the energy produced. First, unlike a cylinder, in a Möbius strip an electron move in the longitudinal direction along the ring encircles the system twice before returning to its initial position. This create flux periodicities generating more persistent current. Second, the Möbius strip cannot be pressed into a one dimensional structure and this implies the motion of electrons in the transverse direction. Therefore fermions can tunnel to their neighbours in more directions. Finally, thanks to the twist, the electrons in the last wire tunnel in the same wire on the corresponding replicated new element. Then it is possible to notice some very important analogies between fermions moving on a Möbius strip and the effects of CSR investments. In fact, these investments, just like the twist, should make stakeholders' relationships closer and persistent, so that one stakeholders' interest become also the others' interest. This is perfectly in agreement with the stakeholder theory which suggests simultaneous attention to the legitimate interests of all appropriate stakeholders, both in the establishment of organizational structures and general policies[2]. In our model these desired structures and practices are just measured

[1]The first use of the Möbius strip as a metaphor in the business relationships, on our knowledge, is that of Litz (2008), who discusses an alternative approach to business family and family business relationships

[2]Stakeholder theory was firstly introduced by Freeman (1984), who argues that a business organization must ensure a minimum benefit to all stakeholders (i.e. not only to the shareholders, but also to the customers, employee, suppliers, the community within which the organization interacts), which otherwise would leave the company, making it impossible to produce profits. This theory occupies an intermediate position between strategic management and political philosophy in that it presents a new form of sovereignty, i.e. it neutralises the sovereignty of the firm in favour of the stakeholders(Bonnafous-Boucher and Porcher, 2010) who "are persons or groups with legitimate interests in procedural and/or substantive aspects of corporate activity. Stakeholders are identified by their interests in the corporation, whether the corporation has any corresponding functional interest in them" (Donaldson and Preston, 1995). According to Jensen

in general by the investments in CSR for workers and sectors, corresponding to the energy dissipation for the fermion in a Möbius strip. Nevertheless, these costs represent the appropriate incentives for different stakeholders and in different sectors to become strightly interdependent, just as fermions tunneling to their neighbours.

These attention to the stakeholders' interests boost the consequent positive effects on companies' performance due to strength and weak ties characterized by "combination of the amount of time, the emotional intensity, the intimacy (mutual confiding), and the reciprocal services. Any given tie may be Contagious which enters social networks and is disseminated with increasing rapidity"(Granovetter,1973). In other words, trough those investments the business company creates the social capital, functionally defined as "a variety of entities with two elements in common: they all consist of some aspect of social structure, and they facilitate certain actions of actors within the structure, that is, social capital is anything that facilitates individual or collective action, generated by networks of relationships, reciprocity, trust, and social norms (Coleman, 1988). They may be seen as the incentives that boost prosocial behavior that combines heterogeneity in individual altruism and greed with concerns for social reputation or self-respect, making individuals' actions complements more than substitutes (Benabou and Tirole, 2006). In our model this prosocial behavior is measured in three crossed effects linking different stakeholders and in different sectors, but also enforcing stakeholder's attraction to the firm's mission, so that each of them can be viewed as a replicated one working both for his specific sector and for the firm's mission. Therefore, according to this point of view, it is possible to extensively draw from the topology of the interactions among fermions on Mobius strip the most appropriate analogies with the interactions among stakeholders in a context of Corporate Social Responsibility sectors(CSR). This is not just an exercise but it helps us to devise a new cost-benefit model, as we think those in the traditional Economics textbook are not ever appropriate for the CSR companies. In fact the traditional one does not take into account the crossed effects and the additional interactions among different stakeholders and different sectors, on which

(2001)"managers should make decisions so as to take account of the interests of all stakeholders in a firm (including not only financial claimants, but also employees, customers, communities, governmental officials, and under some interpretations the environment, terrorists, and blackmailers)".

the analogies with fermions shed lights.

The paper is divided into four sections (including introduction and conclusions). In the second section we describe the building of the geometrical model for the electrons travelling in a Möbius strip. In the third section we investigate how to apply this model to the behavior of firms and economics agents in a CSR context. We devise a new cost-benefit model that show the convenience to invest in social responsible activities thanks to three positive crossed effects on the efficiency: 1)cooperation among stakeholders in the same sector; 2)cooperation among similar stakeholders in different sectors and 3)the stakeholders' loyalty towards the company. We provide an example of a firm's decisional problem which decides whether to invest in social responsibility. Our analytical results show that this is ever the optimal choice depending on the number of sectors, the stakeholders' sensitivity to these investments and the decay rate to alienation, which occurs when the worker can only express individuality through a production system, but who derives very little satisfaction from the monotonous activity. Moreover the alienation of the worker does exists also from the other workers in a workplace which does not foster social relationships(for a better definition see also $https : //www.boundless.com/sociology/textbooks/boundless - sociology - textbook/economy - 16/work - 120/work - and - alienation - 678 - 7760$). Our results substantially confirm those of most of the theoretical and empirical literature on the positive effects of the CSR on the companies' performance (for a survey, see Crifo and Forget, 2015). Nevertheless, referring to the existing literature and in particular to the dynamical model in Becchetti at al (2014), trough the analogies with the Mobius strip it is possible to model what effects are at work to create what those authors called *ethical capital*. In fact this kind of capital accumulation is only assumed in the theoretical model of Becchetti et al. (2014), and in general in the CSR and stakeholders theory, while a broad range of empirical literature test the positive effects of several measures of CSR on the firms' performance. Our results shed more light on what particular forces are at work , to generate that ethical capital inside the organization. Consequently, they provide useful suggestions also about the variables which should be measured at an empirical level to produce more clear and appropriate results. In the fourth section we discuss our conclusions.

2 How to build a geometrical model for the electrons travelling in a Möbius strip

The Möbius strip is a bi-dimensional manifold with only one face. It can be built from a strip of paper by joining together its both ends after having twisted one of them a half turn (see Figure 1).

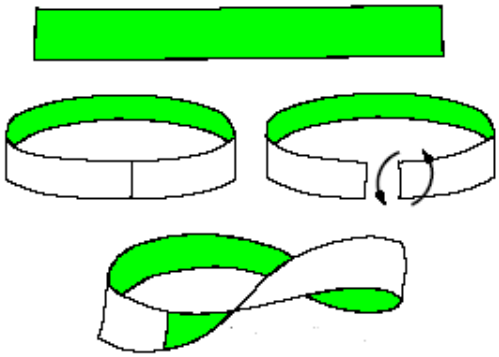

Figure 1: How to build a Möbius strip

The Möbius strip has one side and a single border and if we move along the centre line, the meridian, of the strip we need to go through the circle twice in order to return to the original position. This behavior is similar to that of the electrons generating a flux periodicity of persistent currents in a Möbius strip in Yacubo et al. (2003), who describe it by using the Hubbard model (1963). This last is the simplest model of interacting particles (electrons) in a lattice and consists of a Hamiltonian with only two terms: a *kinetic term* which represents the kinetic energy of electrons hopping between atoms and a *potential term* consisting of an on-site interaction which represents the potential energy arising from the charges on the electrons. Therefore the Hamiltonian is a sum of potential and kinetic energy and is applied to describe how the one kind of energy repeatedly changes into the other one over time.

If we assume that there are N sites then we'll say that if an electron tunnels from lattice site j to site l, its energy changes by an amount $-t_{jl}$. This tunneling effect is equivalent of annihilating the electron at site j and creating it again at site l, so the portion of the Hamiltonian, the kinetic term, dealing with tunneling can be written as

$$-\sum_{j,l=1}^{N} t_{jl} a_l^\dagger a_j$$

where $a_l^\dagger, a_j$ are the fermion (since electrons are fermions) creation and annihilation operators. For many practical purposes it suffices to assume that t_{jl} is none-zero, only when j and l are the nearest neighbors in which case it is usually approximated by a constant t. Because of the electron may tunnel also from lattice site l to site j, the Hamiltonian becomes

$$-t\sum_{j,l=1}^{N} a_l^\dagger a_j + a_j^\dagger a_l$$

where $-t\sum_{j,l=1}^{N} a_j^\dagger a_l$ is defined Hermitian conjugate and denoted by $h.c.$
The potential term is

$$\sum_{k=1}^{N} \varepsilon_k a_k^\dagger a_k$$

where ε_k represents the site energy and $a_k^\dagger, a_k$ are the fermion creation and annihilation operators at the site k.

Yacubo et al.(2003) consider electrons moving on a Möbius strip in the longitudinal directions on $2M$ wires and transverse directions on N wires. Specifically, starting from a rectangular lattice including $N \times 2M$ sites (see Figure 2), the rectangle is then twisted by 180 degrees and its two sides are connected, such that longitudinal wire 1 is attached to wire $2M$, wire 2 is attached to wire $2M-1$ and so on (see Figure 3). The Möbius strip so constructed includes M longitudinal wires with $2N$ sites on each one.

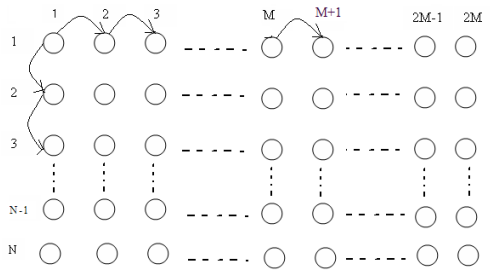

Figure 2: The electrons moving in a lattice $N \times 2M$.

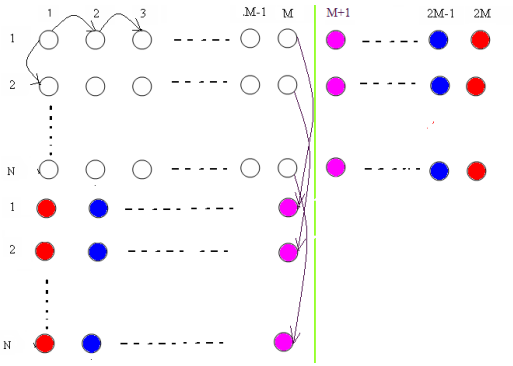

Figure 3: The electrons moving in a Möbius strip.The previous lattice has became a lattice $2N \times M$. The area behind the green line,after the twist,shifted in the bottom on the left.The electrons in the column M that tunneled in the $M+1$ column,now tunnel in the same column M on the corresponding replicated new element.

According to the Hubbard model (1963) the Hamiltonian is then

$$H_{M\ddot{o}bius} = \sum_{n=1}^{2N}\sum_{m=1}^{M}[\varepsilon_{nm}a^{\dagger}_{nm}a_{nm} - t_1 e^{-2i\pi\Phi/N}a^{\dagger}_{nm}a_{n+1m}] \tag{1}$$

$$-t_2\sum_{n=1}^{2N}\sum_{m=1}^{M-1}a^{\dagger}_{nm+1}a_{nm} - \frac{t_2}{2}\sum_{n=1}^{2N}a^{\dagger}_{nM}a_{n+NM} + h.c.$$

where a_{nm} is the fermion operator at the site (n, m) with $n = 1, 2, ..., 2N$ and $m = 1, 2, ..., M$).

The quantity ε_{nm} is the site energy so that

$$\sum_{n=1}^{2N} \sum_{m=1}^{M} \varepsilon_{nm} a_{nm}^{\dagger} a_{nm}$$

represents the potential term.

The kinetic term is made up of three parts:

1. $-t_1 \sum_{n=1}^{2N} \sum_{m=1}^{M} e^{-2i\pi\Phi/N} a_{nm}^{\dagger} a_{n+1m}$ measures the longitudinal hopping, where $e^{-2i\pi\Phi/N}$ measures the effect of the magnetic field accumulated along the longitudinal direction on each link and t_1 is the longitudinal hopping amplitude;

2. $-t_2 \sum_{n=1}^{2N} \sum_{m=1}^{M-1} a_{nm}^{\dagger} a_{nm}$ measures the transverse hopping on $M-1$ longitudinal wires and t_2 is the transverse hopping amplitude;

3. the transverse hopping on the last wire M is measured by $-\frac{t_2}{2} \sum_{n=1}^{2N} a_{nM}^{\dagger} a_{n+NM}$. Without the twist the electron would tunnel from the site (n, M) to the site $(n, M+1)$. But, because of the twist, now the wire $M+1$ is attached to the wire M becoming the same longitudinal wire with $2N$ sites on it. Therefore the site $(n, M+1)$ is now the site $(n+N, M)$ (see Figure 3).Obviously the sum is divided by two because the electrons tunnel only from (towards) the original N sites.

3 The Economics of the CSR-Möbius strip

3.1 How to build a CSR-Möbius strip economics model

In this section we aim to investigate whether what we have seen in the previous one can be applied to firms and economics agents in a CSR context. Are there some similarities between their activities and contributions to production and the move of electrons in the strip that produces energy? At a first sight we notice that $-H_{Möbius}$ strongly approaches a

benefits-costs function. In fact, the energy dissipation measured by ε can be assimilated to the production costs unrecovered trough the sell of the added value of the final consumption good.

Similarly, the terms with t_1 and t_2 may represent the benefits associated to the joint contributions of N stakeholders or type of stakeholeders operating in M sectors.

For instance in the generalized Leontief production function analyzed in Diewert (1971) the interindustrial relations of an economy are conventionally represented by a matrix in which each column lists the monetary value of an industry's inputs and each row lists the value of the industry's outputs. Each cell of this matrix might correspond to the site (n, m) of the electrons in the strip (for instance see Iyetomi et al. 2010).

Nevertheless we think that in a context of CSR this function does not take into account all the crossed effects that social responsible activities can generate in terms of productivity and costs saving (see Becchetti et al. 2014). In particular some of these effects concern the externalities due to the CSR benefits on the stakeholders, which on turn are transferred into positive returns on the firm's traditional activities. According to this point of view, we consider a SR company with $n = 1, 2, .., N$ stakeholders or cluster of stakeholders and $m = 1, 2, ..., 2M$ activities, where $m = 1, 2, ..., M$ represents the traditional sectors of production of intermediate goods, necessary to produce the final good M, while $m = M+1, ..., 2M$ are the specific activities devoted to the CSR. We denote by $0 \leq a_{nm} < 1$ the contribution of the stakeholder n in the sector m measured as percentage per unit of a product. For instance if $a_{11} = \frac{1}{5}$ we say the stakeholder 1 is able to produce the 20 per cent of a unit in a working hour. Like in a Möbius strip also in a social responsible firm the effects of a twist may be considered as the returns due to the CSR activities on the stakeholders and firm production, which therefore amplify the crossed contributions of different stakeholders also operating in different sectors of the company (see Figure 4).

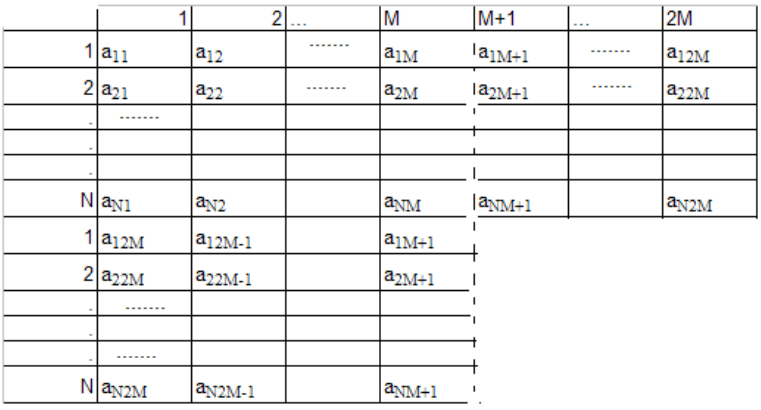

Figure 4: The matrix of stakeholders'contributions in a CSR context.

The stakeolder 1 contributes with a_{11} to the production of the sector 1 and with a_{12} to the production of the sector 2 and so on. The stakeolder 2 contributes with a_{21} to the production of the sector 1 and with a_{22} to the production of the sector 2 and so on. The same for all the other stakeholders. The value of a_{12M} measures the expected additional contribution that the stakeholders 1 would give thanks to the social responsible activity $2M$. The same for the other social responsible activities which are ordered in such a way that $2M$ is more relevant for the sector 1, $2M-1$ is more relevant for the sector 2, etc (for instance $2M$ could be seen as the social responsible activities dedicated to assure safety work condition in sector 1, $2M-1$ those to assure safety work condition in sector 2 and so on). Therefore in this work we propose the use of a new cost function for CSR companies suggested by (1), that in our case becomes:

$$H_{CSR} = -\sum_{n=1}^{2N}\sum_{m=1}^{M}[c_{nm} - t_1(1-\delta)a_{nm}a_{n+1m}] + t_2\sum_{n=1}^{2N}\sum_{m=1}^{M-1}a_{nm+1}a_{nm} + \frac{t_2}{2}\sum_{n=1}^{2N}a_{nM}a_{n+NM} \tag{2}$$

where

1. $-\sum_{n=1}^{2N}\sum_{m=1}^{M} c_{nm}$ represents the sum of the costs supported by a company for social responsible activities devoted to each n in the sector m. The company can decide to give a prize also for the stakeholder's social responsible engagement and his increased productivity in the traditional sectors, so that the cost can be different from zero for the $n = N+1, ..., 2N$ replicated stakeholders.

2. $t_1 \sum_{n=1}^{2N}\sum_{m=1}^{M} (1-\delta) a_{nm} a_{n+1m}$, that we call *the neighbouroud efficiency term*, measures the gains associated to the crossed contributions of n in the sector m with the nearest $n+1$ in the same sector. For instance if $a_{11} = \frac{1}{5}$ and $a_{21} = \frac{1}{7}$, when the SR stakeholder 1 supports the stakeholder 2 helping him to produce his share $\frac{1}{7}$, the stakeholder 1 contributes with his ability of $\frac{1}{5}$ to the production of $1+\frac{1}{7}$ units of the good. Therefore his total contribution is now $\frac{1}{5}\left(1+\frac{1}{7}\right)$. Obviously also the stakeholder 2 can support the stakeholder 1 and this would correspond to Hermitian conjugate of this term. In the rest of the paper, to avoid excessive complexity, we don't consider the hermitian conjugate of (2) because this doesn't affect our analysis. Moreover we assume that $0 < \delta < 1$ is the decay rate due to the possible effect of alienation (caused for instance by satiety, low free time, etc.). Finally t_1 represents the sensitivity of the stakeholders' contributions to the SR activities devoted to them;

3. $t_2 \sum_{n=1}^{2N}\sum_{m=1}^{M-1} a_{nm+1} a_{nm}$, that we call *sector cooperation efficiency term*, measures the gains associated to the crossed contributions of n in the sector m with the others type n in the nearest sector $m+1$. Moreover t_2 (which can be equal or different from t_1) measures the sensitivity of the stakeholders contributions to the SR activities devoted to their and to other nearest sector.

4. $\frac{t_2}{2}\sum_{n=1}^{2N} a_{nM} a_{n+NM}$, that we call *loyalty efficiency term*, measures the gains associated to the increased productivity of each n which contributes to the production of the

final good M twice: directly trough his own task and indirectly trough the increased efficiency and cooperative attitudes.

Clearly all the above mentioned crossed effects could run among more distant stakeholders and sectors. Nevertheless it is reasonable to assume that this would imply not negligeable transaction costs, necessary to raise useful and continuous connections among them. Moreover the associated benefits should be netted from the intermediate effects running among the nearest ones. Therefore, all this things considered, it is possible to assume, in our model, that those effects are very low and less important for the company when she decides her investment in CSR.

Moreover, we think that the main point is that SR firms make specific investments (the sectors from $M+1$ to $2M$) to foster stakeholders' socially responsible contributions and productivity(which for examples are empirically measured by some index as in the KLD metrics, see Becchetti et al. 2016) so to reverse the upper side of our matrix in the lower bound on the left just as if we have two replicated stakeholders. The traditional one making is own task, and the second is a sort of replicated socially responsible stakeholders adding new contributions to the firm.

Therefore the order matters as investments and return are specific into the firm. Obviously we can imagine there are also externalities requiring no specific orders, but they are difficult to measure and not related to specific company's activities and investments while CSR measures are specific for sectors and stakeholders so implying specific returns. In particular the three above mentioned effects depend on the extremely strick and precise conditions of how CSR investments operate so that the twist is just a Mobius strip twist rather than some less well-ordered reshuffling of cross-cutting effects across the stakeholders.

In that follows we aim to apply this function to a general decisional problem of a company which wants to minimize the costs taking into account these crossed benefits due to the SR activities.

3.2 An application to a firm decisional problem with constant contributions and costs

In this section we consider only one type of stakeholders and specifically we assume that there are N workers in $m = 1, 2, ..., M$ traditional sectors. We assume that the total production is equal to the sum of the contributions of these workers, which could be measured in term of pieces produced by worker in that sector in a working hour, which is constant for each worker and sector, $a_{nm} = a$, with $a \in \mathbb{R}$ and $0 \leq a < 1$ for all $n = 1, 2, ..., N$ and $m = 1, 2, ..., M$. Therefore if we denote by p the price of the final good and by w the wages paid to workers, the firm's profit function is:

$$\pi = \sum_{n=1}^{N} \sum_{m=1}^{M} (p - w) a_{nm} = NMa(p - w).$$

We also assume that the company finances the social responsible activities with an expense $c \geq 0$ equal for each sector and worker and proportional to their contributions, that is $c_{nm} = ca$ for all $n = 1, 2, ..., N$ and $m = 1, 2, ..., M$. Notice that this assumptions constant expense c is not trivial and unrealistic. In fact, if we consider the same type of stakeholders, in order to avoid any discrimination the firm should invest, for each them, the same amount which is proportional only to the own contribution (meritocracy). Otherwise it might have counterproductive effects (like envy, frustration due to inequality, etc) instead of stimulating cooperation and efficiency. In addition we suppose that the worker's sensitivities t_1 and t_2 are equal and are related to the investment in CSR through the function

$$t_1 = t_2 = k(ca)^{\beta}$$

where k is a positive constant and $\beta \in \mathbb{R}$.

Under these assumptions, the company, for given values p and w, wants to maximize the benefits associated to the investment in CSR measured by the function (2) that in this case is

$$H_{CSR}(c) = -\sum_{n=1}^{2N}\sum_{m=1}^{M}[ca - t_1(1-\delta)a^2] + t_2\sum_{n=1}^{2N}\sum_{m=1}^{M-1}a^2 + \frac{t_2}{2}\sum_{n=1}^{2N}a^2 \tag{3}$$

subjected to

$$NMa[(p-w)-c] \geq 0 \tag{4}$$

Obviously the constraint (4) implies that the firm can't expend in CSR more than what she would earn without social responsible activities.[3]

Simplifying (3) we get

$$H_{CSR}(c) = -ca2NM + 2kc^{\beta}NM(1-\delta)a^{2+\beta} + 2kc^{\beta}N(M-1)a^{2+\beta} + kc^{\beta}Na^{4+\beta} \tag{5}$$

Therefore the company chooses the value of c that solves

$$\frac{dH_{CSR}}{dc} = 0$$

under (4), that is

$$\frac{dH_{CSR}}{dc} = -a2NM + 2\beta kc^{\beta-1}NM(1-\delta)a^{2+\beta} + 2\beta kc^{\beta-1}N(M-1)a^{2+\beta} + k\beta c^{\beta-1}Na^{4+\beta} = 0$$

$$c^{\beta-1}\beta k[2M(1-\delta)a^{1+\beta} + 2(M-1)a^{1+\beta} + a^{3+\beta}] = 2M.$$

We can distinguish three cases:

[3] Obviously they can exist several constraints, not always binding, and it could be very interesting to develop a more complex analysis taking into account all these possibilities in a future research. Nevertheless, for sake of simplicity, we focus only on the most essential constraint according to the objectives of this paper.

1. for $\beta > 1$

$$c_1^* = \sqrt[\beta-1]{\frac{2M}{\beta k a^{1+\beta}[2M(2-\delta) - 2 + a^2]}}$$

which is a feasible solution only if $c_1^* < p - w$. We can see that c_1^* increases for high values of δ. In fact, being convenient to enforce workers' sensitivity to SR to earn the high benefits due to $\beta > 1$, the company should invest more c to counteract the negative effect of δ. Instead the optimal c decreases for high values of β because no huge investments are necessary to stimulate workers' sensitivity and the firm can save costs getting the same great benefits. Finally, given the budget constraint, if there are many sectors M the company must invest a little amount c for each of them, therefore c decreases for high values of M.

2. for $\beta < 1$

$$c_2^* = \sqrt[1-\beta]{\frac{\beta k a^{1+\beta}[2M(2-\delta) - 2 + a^2]}{2M}}.$$

Obviously the above mentioned effects of δ, β and M on the optimal value of c are reversed when the workers are low sensitive to SR activities.

3. for $\beta = 1$

$$\frac{dH_{CSR}}{dc} = k a^2[2M(2-\delta) - 2 + a^2] - 2M$$

which is constant. Therefore, if

$$k a^2[2M(2-\delta) - 2 + a^2] - 2M > 0$$

it is ever convenient to invest in CSR and the company chooses the optimal value of c satisfying (4), as she can easily recover the costs from the proportional increase in t for $k \geq 1$. This condition is more probably satisfied for high values of k and a.

Our findings reveal that the convenience to invest in CSR, and therefore to take care of the stakeholders' interests, produces the effects of strength ties among different workers and

in different sectors, boosting social capital and their intrinsec motivation both towards the others' and towards the firm mission. Nevertheless these effects on turn are mediated by the other three following factors: 1) the workers' sensitivity; 2) the alienation; 3) the number of sectors.

1. Being favorable to CSR influences productivity, wages of efficiency and intrinsic motivation (see Becchetti et al.,2012). In our decisional problem we find that, other factors being equal, high workers' sensitivity to CSR practices makes absolutely convenient for the firm to support the related expenses because the workers' productivity increases. Nevertheless these expenses decrease as the sensitivity raises because the workers' marginal productivity for units of investment is higher and the firm can get the same effort even with lower costs.Clearly this raises the problems of the right incentives not to reduce the workers' sensitivity, because as in Benabou and Tirole (2003) the wrong incentives, as donors or wages, could produce the counterproductive effect of crowding out the intrinsec motivations reducing the value of β.

2. The alienation effect, measured by delta, implies an higher workers' aversion to the task and the company or a greater preference for other activities, leisure or family. If the workers show a high sensitivity to CSR then the company can enhance their motivations trough appropriate investments. In such a way the company can balance costs and benefits for the workers to be engaged in those activities. In other words delta measures the typical effect of the depreciation rate in dynamical systems. Therefore as in the traditional models of investment in physical capital also in this case it is necessary to invest in more ethical capital to counterbalance the negative effect of the depreciation rate.

3. The number of sectors affects the investments in CSR in two ways. First, given the budget constraint, if there are many sectors the company can invest a limited amount for each of them. Second, the social capital and workers relationships are better in smaller sized firms (Tamm et al., 2010) so probably fewer additional responsible investments are necessary with respect to larger sized firms. According to our results this is the case when workers' sensitivity is not significatively high. On the contrary

the larger sized firm with many sectors can reduce the expenses for each of them getting the same benefits because no huge investments are necessary to stimulate workers' greater efforts. Clearly being now more convenient investing in CSR, being the budget constraints equal, the smaller sized firms will be favorable to support more expenses for each sector instead of the largest firms. Other works on the problem of the mediator effect of firm size on the adoption of CSR measures substantially confirm that there is not a clear effect just in agreement to our results where they strongly depend on β, that is on workers sensitivity. In particular, Udayasankar(2008) arguments that in terms of visibility, resource access and operating scale, very small and very large firms are equally motivated to participate in CSR. However, the motivational bases for CSR participation are likely to be different, so that the author suggests cautions against the broad categorization of firms, without adequate attention to the firm's size. Blomback and Wigren (2009) from examples of far-reaching CSR activities in the small business community and local initiatives by large firms, find that the distinctions suggested in the current discourse do not appear in practice are shown. The local embeddedness, corporate governance, and individual motivation are examples of issues that appear to explain a firm's CSR activities and characteristics, regardless of firm size. Other works approach to the question from an empirical point of view. For instance, Youn et al.(2015), performing a two-way fixed-effects model by firm restaurant context, find that firm size moderates the effect of positive CSR on Corporate Financial Performance(CFP), while it does not moderate the effect of negative CSR on CFP.

4 Conclusions

In the ongoing times characterized by an even more globalized world, the reduction of distances thank to technologies make people and systems (economic, social, cultural, etc) strongly interrelated and juxtaposed. Therefore what happens somewhere influences things happening elsewhere. From a theoretical point of view to study these more interacting systems the traditional economic models are improved also relying on the discoveries of the

physical sciences to take into account the several crossed effects among the agents' actions. In particular in a CSR context her related activities generate a sort of interlinked effects which should be adequately analyzed. In this work we extensively draw from the physical science and specifically from the geometrical model of the Möbius strip where the electrons move in several directions to produce energy.

Similarly in a CSR context the social responsible activities have the effects going in several directions which can increase the stakeholders' productivity and efficiency so reducing production costs. Therefore we devise a new cost-benefit model where three crossed effects are at work: 1) increases in the efficiency in virtue of the augmented cooperation among the nearest stakeholders in the same sector; 2) increases in efficiency in virtue of the augmented cooperation among stakeholders in the nearest sectors; 3) increases in the efficiency due to the augmented stakeholders loyalty towards the vision of the company (and also the management and the shareholders)and so towards her final production.

We show how the benefits of the CSR in terms of those three effects incentive the investment in CSR activities and we also provide an example on how this new cost-function can be used to analyze a simple SR firm's decisional problem. Our results show that investing in CSR activities can ever be convenient depending on the number of sectors, the stakeholders' sensitivity to these investments and the decay rate to alienation.

We think that this approach could make light on effects in productivity which not have been adequately taken into account and need to be more analyzed both at a theoretical and empirical level. In particular proceeding from our theoretical model new empirical measures on these crossed effects should be produced to translate our model into reality.

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