Reply to the referees' comments 1 and 2

The authors would like to thank the referees for their valuable comments.

Remind some critical prices.

$\hat{p}$: price at which the profit of each firm in duopoly is zero.

$\tilde{p}$: price at which the profit of monopolist is zero.

$\bar{p}$: price at which the profit of each firm in duopoly is equal to the profit of monopolist.

In Dastidar(1995) it was shown that if the cost functions are convex, $\hat{p} < \tilde{p} < \bar{p}$.

In the cases of absolute profit maximization and relative profit maximization the range of the equilibrium prices is $[\hat{p}, p^*]$. p^* is the price at which the value of the objective function of a firm in duopoly is equal to the profit of monopolist. Note that in monopoly the absolute profit and the relative profit are equal.

1. Pure absolute profit maximization

   Since $p^* = \bar{p}$, the range of the equilibrium prices is $[\hat{p}, \bar{p}]$.

2. Pure relative profit maximization

   If the firms have the same cost function in duopoly, that is, the model of duopoly is symmetric, the relative profit of each firm in duopoly at the equilibrium is zero. Then, p^* is equal to $\tilde{p}$, and the range of the equilibrium prices is $[\hat{p}, \tilde{p}]$. Since $\tilde{p} < \bar{p}$, the range of the equilibrium prices under pure relative profit maximization is lower and narrower than that under pure absolute profit maximization.

3. Maximization of weighted average of absolute and relative profits

   In this case p^* satisfies the relation $\tilde{p} < p^* < \bar{p}$. So, the range of the equilibrium prices in this case is lower and narrower than that under pure absolute profit maximization, but higher and wider than that under pure relative profit maximization. The larger the weight on the relative profit, the lower and narrower the range of the equilibrium prices

4. Public firms (Ogawa and Kato(2006), Dastidar and Sinha(2011))

   If both firms in duopoly are public firms, at the equilibrium

the value of the objective function of a firm in duopoly
$= (1-\beta)\times$profit of duopolist$+\beta\times$(joint profits+consumers' surplus)
$= (1+\beta)\times$profit of duopolist$+\beta\times$consumers' surplus,

where $0 < \beta < 1$, and

the value of the objective function of monopolist
$= (1-\beta)\times$profit of monopolist$+\beta\times$(profit of monopolist+consumers' surplus)
$=$profit of monopolist$+\beta\times$consumers' surplus.

The condition that they are equal is equivalent to the following condition.

$(1+\beta)\times$profit of duopolist$=$profit of monopolist.

This condition corresponds to a case where the weight on the relative profit is negative ($-\beta$) in the case of maximization of weighted average, or a case of $\alpha = -\beta$ in the model of this paper although we assume $0 < \alpha < 1$ in the paper. If the weight on the relative profit is zero, $p^* = \bar{p}$. Since p^* is decreasing in α, in the case of public firms $p^* > \bar{p}$, and the upper bound of the range of the equilibrium prices is higher than that under absolute profit maximization.

The lower bound of the range in the cases of pure absolute profit maximization, pure relative profit maximization, and weighted average maximization is the price at which the absolute profit a firm in duopoly is zero. But in the case of public firms the lower bound is the price at which ``$(1-\beta)\times$profit of duopolist$+\beta\times$(joint profits+consumers' surplus)" is zero. This lower bound seems to be lower than $\hat{p}$. So, the range of the equilibrium prices in the case of public firms expands both directions.

We depict comparison of the ranges of the equilibrium prices in Figure 1. π^M is the profit of monopolist, π^D is the profit of a firm in duopoly, and CS denotes consumer' surplus.

Sincerely

The authors

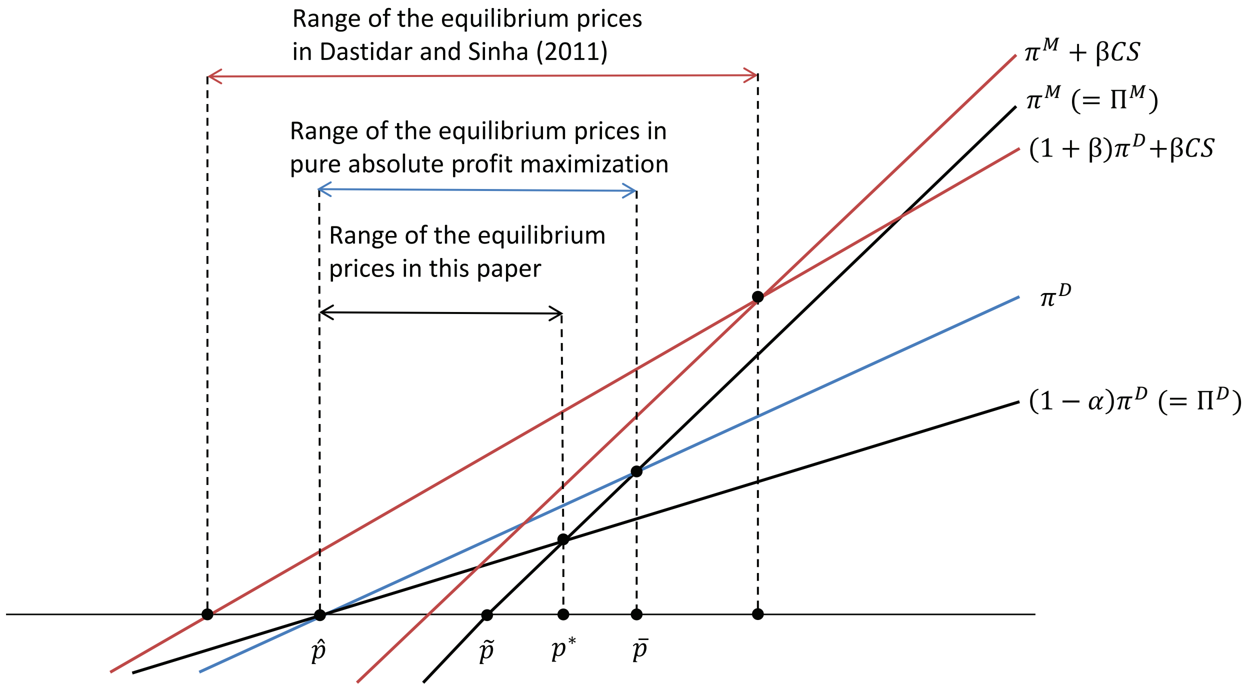

Figure 1: Comparison of ranges