Horizontal Mergers and Uncertainty*

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Abstract

This paper analyses the profitability of horizontal mergers and their impact on the welfare when there is uncertainty on the marginal cost of the newly merged firms. We consider that the merging firms decide their production strategy knowing the exact value of the production cost while outsiders are *a priori* uncertain about the exact amount of cost efficiency/inefficiency that will result from the merger. Nevertheless, the merged entity can signal its own cost to some rivals when it behaves as a leader. We depart from the literature dealing with the influence of uncertainty on incentive to merge by focusing on a market structure some firms are leaders and others followers. Within this sequential decision-making process, this paper studies whether the firms have private incentive to merge, whether the merged firm has interest or not to reveal the information about its own cost to competing firms. We distinguish four scenarios of bilateral mergers depending on the role of merging firm in the *pre-merger* game and on the behavior of the newly merged entity in the *post merger* game. We demonstrate that in such a context some scenarios of mergers generating efficiency losses can also be profitable. Concerning "merger control", this paper highlights the timing of regulation intervention (*ex ante* or *ex post*) and the different criterions (Consumer welfare standard and Aggregate welfare standard), and can potentially play a useful role in informing the merger policy.

Keywords: Merger; Competition Authorities; Uncertainty; Asymmetric information
JEL codes: D21; D80; L20; L40

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1 Introduction

In the line of the seminal paper of Salant, Switzer and Reynolds (1983), many authors take for granted the efficiency gains associated with horizontal mergers or consider that all competing firms have perfect knowledge about the future cost of the merged entity. In practice, outsiders and Competition Authorities[1] have a great deal of difficulty in knowing the exact future efficiency gains (or losses) when some competing firms decide to merge. Merged firms are not just larger firms but become more complex organizations. For instance, mergers create additional uncertainty for employees because of the potential clashes of cultures and management styles. This uncertainty can lead to such dysfunctional outcomes as stress, job dissatisfaction, low trust in the organization, and increased intentions to leave the organization. These dysfunctions can, in turn, diminish productivity and increase the production cost[2] (Morán and Panasian, 2005). On the other hand, improvement in productive efficiency could also result from a better allocation of resources within the merging firm. One can also imagine that the merger gives rise to efficiency gains by increasing the incentives of the merging parties to invest in cost-reducing R&D.

This paper analyzes the private incentive to merge, the *ex post* profitability of merger and the welfare effects of mergers in (quantity-setting) uncertain markets where output decision-making process is sequential. The merger in a Stackelberg game can give rise to either potential efficiency gains or potential efficiency losses, there is uncertainty on what will be the production cost of the merged entity. This framework is related to two strands of the merger literature. The first strand typically focuses on the relationship between sequentiality (leader and/or follower) and merger incentive in a context of deterministic markets. In the absence of uncertainty and when a merged firm changes its behavior from a Cournot-Nash player to a Stackelberg leader, Levin (1990) shows that the private incentive to merge is higher and the antagonism between the private and collective advantage of the merger disappears. In a game where asymmetric roles among the firms in the *pre-merger* situation (Stackelberg leader and follower compete in homogeneous good market) are introduced, mergers can also improve welfare and boost profit: if two followers decide to merge and the newly merged entity behaves as a leader, the social welfare and the merging firms' profits increase even without cost savings following the merger (Daughety, 1990). In Stackelberg markets with linear costs, two leaders rarely have an incentive to merge, nor do two followers when the new entity stays in the same category (Huck, Konrad and Mueller, 2001).

In a context of uncertainty, most of theoretical models are based on the key assumption that output (or price) decision-making process is simultaneous (Cournot or Bertrand). Amir et al. (2009) highlight the fact that the scope of profitable merger enlarges with uncertainty. The uncertain efficiency gains affect the *ex ante* beliefs on the merged firm's cost by outsiders and elicit the competitive advantage to the merged firm from strategic aspects. Hamada (2012) considers a Cournot oligopoly model with homogeneous in a context of cost uncertainty, but he does not introduce the distribution of roles in the industry. He shows that increased uncertainty itself can urge firms to merge. Our model can be viewed as a reassessment of the model developed by Hamada (2012) by investigating to what extent the introduction of asymmetric information among followers may alter the effect of uncertainty on the incentive to merge. Our results should also be compared to those of Cunha, Sarmento and Vasconcelos (2014). They consider the role of uncertainty in horizontal merger games where leaders compete with followers. But in contrast to our model, they consider that the uncertainty on the production cost of the merged entity affects all players, including insiders which remain uniformed on the true value of the production cost in the post merger game. Some authors also investigate how cost uncertainty and information structure affect the incentive to merge (Choné and Linnemer, 2008; Zhou, 2008a and 2008b). Stennek (2003) shows that merger with

[1]Merging firms in general have strong incentives to overestimate these gains in front of Competition Authorities.

[2]The failure of consolidation in the bio-technology and oil industries is a good illustration of this phenomenon and reflects the growing uncertainty caused by the rising R&D investments and raw-material costs.

cost uncertainty and private information may increase consumer welfare if consumers are sufficiently risk-averse. Banal-Estanol (2007) investigates merger incentive under cost uncertainty and concludes that uncertainty always enhances merger incentives if the signals are privately observed. In a context of merger under demand uncertainty, Gal-Or (1988) finds that asymmetric information may hinder mergers.

In this paper, we turn our attention to cost uncertainty on merger activity with sequential output decisions. Mergers not only create market power, but also yield efficiency gains (or losses) of random magnitude. The key assumption is that the insider benefits from the *first-to-know* advantage of its own actual cost since outsiders are unaware of this cost. Nevertheless, even if the merged entity's cost information is private, the insider may transmit its private information through its market conduct. When the insider behaves as a leader, outsider-followers can perfectly observe the output level of merged firm and infer the exact value of merged firm's cost. Consequently, this information structure is different from the one proposed by Amir et al. (2009) and Hamada (2012) where after merger all outsiders are uninformed about the merged firm's cost. We depart from their framework[3] by highlighting the role of sequentiality of output decision. Our model investigates mergers in a context of close relationship between the distribution of roles and the information structure. The behavior of merged entity alter the outsider firms' information configuration: leader strategy chosen by insider generates the asymmetric information amongst non-merged firms (the outsider-follower is aware of insider's cost, while the outsider-leader is not informed about it) and there will be the symmetric information amongst outsiders when the insider behaves as a follower. When the merged firm plays the follower role, the gap of information among outsiders disappears and all outsiders remain uninformed about the real cost of merged entity.

In order to capture the impact of role distribution and information configuration, we take into account all possible two-firm mergers, such as merger between leaders (or followers), merger between leader and follower, and merger between followers resulting in a leader[4]. We show that the incentive to merge grows with the enlargement of uncertainty. Till the extent of variance exceeds a certain threshold, the expected profit of the merged firm becomes larger than the sum of the *pre-merger* (participant) firms' profits (results in line with those of Hamada, 2012, Banal-Estanol, 2007 in a context of production rationalization or Zhou, 2008a in a game with information sharing). This finding highlights that even if there is neither efficiency gains nor informational advantage for merging firm, the cost uncertainty induces firms to merge. Our framework underlines that additional incentives are engendered by both role redistribution and lack of information. We also show that the two-follower merger aiming to a leader occurs more likely than the one choosing follower strategy. In the absence of role redistribution the merged firm has interests to pool the private signals to outsiders, however, in the presence of role redistribution, the concealment is more profitable from the viewpoint of insider.

Concerning "Merger Approval", we firstly study the case where Competition Authorities adopt the *ex ante* intervention, in other words, they must decide whether to approve or refuse the merger proposal without knowing[5] the actual cost of the merged entity. Under this assumption, the merger between leaders always enhances welfare, as long as the participants have incentives to merge. This generates the unanimity of private and collective incentives, and it provides support for *laisser-faire* policy. Furthermore, enforcement practice in most countries (including the US and the EU) is closest to a consumer welfare

[3]Hamada (2012) examines the effect of uncertainty on horizontal mergers, and focuses on the horizontal merger in a Cournot fashion. He uses a binomial distribution under which the merged firm will experience efficiency gains.

[4]The reason that we focus only on bilateral merger is explained by some illustrations in automotive domain, e.g. Daimler-Chrysler in 1998, Porsche-VW during 2004-2008, Chrysler-Fiat in 2009, etc. From the theoretical viewpoint, Zhou (2008a) demonstrates that two-firm mergers are far more frequent than three- or four-firm mergers.

[5]See US Merger Guidelines Section 4. Merging parties, arguably, know more about potential efficiency gains than Competition Authorities. Firms have strong incentives to dissemble about efficiency.

standard[6]. Thus, we carry on a separate analysis of consumer surplus in order to gain some insight into the relationship between distinct criterions of Competition Authorities and merger issue.

Without loss of generality, the *ex post* policy intervention is also used by Competition Authorities to judge the implemented merger. According to Ottaviani and Wickelgren (2011), the Competition Authorities can employ a "*wait and see*" approach by letting the merger go through in order to have more accurate information about it. This explains the recent renewed interest in *ex post* merger enforcement and why to introduce the *ex post* enforcement in this framework. By studying two alternative criterions under two different policy intervention timings[7], we find that the timing of policy intervention has important implication to the choice between two possible welfare standards: the consumer welfare standard is more rigorous than the aggregate welfare standard in the case of *ex ante* intervention, while the consumer welfare standard becomes more lenient under *ex post* enforcement. Since prudent Competition Authorities (using *ex ante* intervention) should take the restrictive policy, our framework gives a supplementary reason to explain why US Horizontal Merger Guidelines and EC Merger Regulation are biased in favor of the consumers' interests.

The reminder of the paper is organized as follows. Section 2 presents the model and specifies the sub-game perfect equilibria for different scenarios of mergers. Section 3 analyzes the "Ex ante and Ex post profitability of merger". Section 4 investigates the welfare implications of mergers and is also devoted to some research about Competition Authorities' distinct criterions (aggregate welfare standard or consumer welfare standard) and alternative intervention timings of CAs. Finally, section 5 discusses the main findings and concludes. All proofs and some detailed expressions are in the Appendix.

2 Model

The timing of this game is summarized in the sketch map (Figure 1) which shows both decision structure and information structure in a time axis. Benchmark competition is modelled as a standard Stackelberg game with complete information to all active firms. The merger may generate either efficiency gains or losses, and there is some uncertainty on what will be the exact value of the insider's marginal cost. Consequently, the merger not only gives rise to the productivity shock in newly merged entity at the time of merger, but also introduces a modification in the information structure of players, once the merger is implemented.

[6] In merger control, the emphasis is now firmly on consumer surplus. It is worth reflecting on the rationale put forward in support of a consumer welfare policy standard in these areas (as opposed to a total welfare standard). In principle, economists advocate a total welfare standard that encompasses a balancing of rents to producers and consumers. Nevertheless, there are several arguments in support of entrusting a competition agency with a consumer surplus standard. These are based on the following considerations: (1) informational advantages, (2) merger selection bias, and (3) lobbying activities. In addition a consumer standard is considered to be easier to implement.

[7] See ex ante versus *ex post* merger control in Ottaviani and Wickelgren (2009).

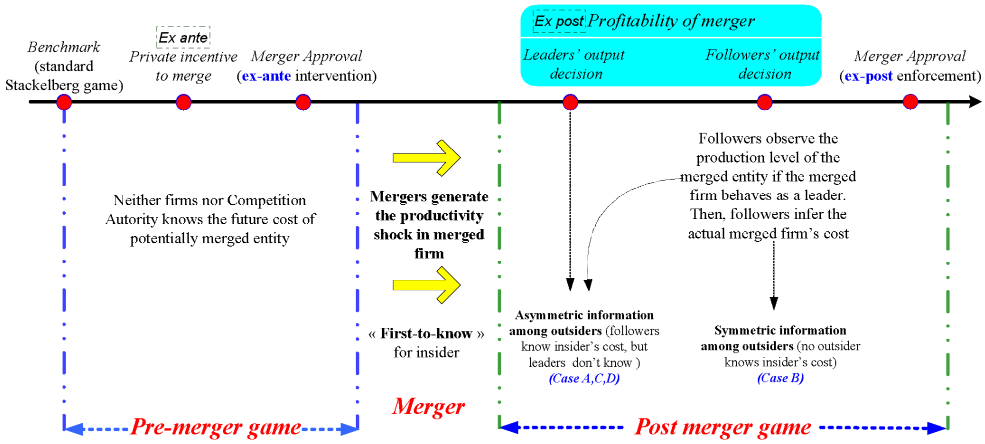

Figure 1: Game structure

At the point of "Private incentive to merge", all firms (including the merging firms) in industry face uncertainty as to the efficiency gains, in terms of variable marginal cost, that the merged firm could achieve. Thus at this point, any merging firms must evaluate the profitability of the merger without knowing their true cost in the *pre-merger* game.

Once mergers are authorized, we turn to the *post merger* game where insider *first-to-knows* its own exact cost, and part of outsiders (outsider-followers) could be aware of the actual cost of insider. The strategic behavior of merged entity can alter the outsider firms' information configuration: leader strategy chosen by insider generates the asymmetric information amongst non-merged firms (because the outsider-follower firms are aware of insider's cost, but the outsider-leader firms are not informed about it) and there will be the symmetric information amongst outsiders when insider behaves as a follower. We follow a brand of literature which identifies different types of mergers that may occur in Stackelberg competition. This literature (Daughety, 1990, Feltovich, 2001, Huck et al., 2001, Escrihuela-Villaraud and Fauli-Oller, 2008, Heywood and McGinty, 2008, Brito and Catalão-Lopes, 2011) has addressed four possible merger cases. This is why we consider a merger between two leaders (case A), a merger between two followers (case B), a merger between two followers resulting in a newly merged leader (case C) and a merger between one leader and one follower resulting in a newly merged leader (case D).

Without loss of generality, we assume that there are two alternative timings of antitrust intervention. *Ex ante intervention* refers to the situation where the Competition Authorities must decide the approval of the merger facing cost uncertainty. *Ex post enforcement* corresponds to the situation where the insider signals its private information through its market conduct and thus, Competition Authorities know[8] the production cost of the insider.

[8]According to timing, CAs with *ex post* enforcement interfere after the "Output decision", it is logical that CAs are aware of the actual production cost of insider. We suppose that there is no cost for acquiring the information. For instance, if the insider behaves as a leader, followers and CAs have the complete information on actual production cost of merged entity at no cost.

2.1 The benchmark situation

We consider an industry composed of n initially active firms producing homogenous products, who compete by setting quantity schedules. In the first stage, $m < n$ firms act as Stackelberg leaders and independently decide on their individual supply. In the second stage, $n-m$ Stackelberg followers make their quantity decisions after learning about the total quantity supplied by the leaders. Initially, we assume $m > 2$ and $n-m > 2$, the strict inequalities ensure that in every case the outsiders gather both leader and follower in the *post merger* situation.[9] All firms face the same constant average cost normalized to c. The market price is determined by the linear inverse demand curve $p = a - Q$ where $a > c$. The aggregate industry output is given by $Q = Q^l + Q^f$ with $Q^l = \sum_{i=1}^{m} q_i^l$ and $Q^f = \sum_{i=m+1}^{n} q_i^f$, q_i denotes the firm i's individual quantity. The superscript "l" stands for a leader and "f" represents a follower.

The equilibria are obtained by backward induction. At the second (follower output decision) stage, each follower maximizes its profit (π_i^f) considering as given the production level of leader (Q^l). The best response function (q_i^f) of a follower firm results from:

$$\max_{q_i^f} \pi_i^f = (a - Q^l - Q^f - c)q_i^f$$

At the first (leader output decision) stage, a leader selects its profit-maximizing output (q_i^l) anticipating the best response function of each follower:

$$\max_{q_i^l} \pi_i^l = \left[a - c - Q^l - Q^f(Q^l)\right] q_i^l$$

In the benchmark situation, the corresponding individual outputs and profits are:

$$q_i^l(m) = \frac{a-c}{m+1} \qquad \pi^l(n,m) = \frac{(a-c)^2}{(m+1)^2(n-m+1)}$$

$$q_i^f(n,m) = \frac{a-c}{(n-m+1)(m+1)} = \frac{1}{n-m+1} q_i^l$$
$$\pi^f(n,m) = \frac{(a-c)^2}{(m+1)^2(n-m+1)^2} = \frac{1}{n-m+1}\pi^l$$

Obviously, the distribution of roles among firms exhibits the first mover advantage[10]: each leader benefits from higher market share and earns higher profit in benchmark game.

2.2 The different merger scenarios

In this subsection, we focus on a bilateral (two-firm) merger. When two firms make the decision whether to merge, all firms including the merging firms in the market are uncertain over the marginal cost of the

[9] The particular cases: both $m = 0$ and $m = n$ correspond to a Cournot industry, the firms are in the simultaneous game. The Stackelberg and Cournot models are similar because in both competition is on quantity. However, as seen, the first move gives the leader in Stackelberg a crucial advantage. There is also the important assumption of perfect information in the Stackelberg game: the follower must observe the quantity chosen by the leader, otherwise the game reduces to Cournot.

[10] The leader's profit under the sequential-game equilibrium will be higher than under Cournot equilibrium. Since follower firm reacts in a "Nash fashion", leader firm could just choose to produce the Cournot output level. In this case, leader firm would earn exactly the Cournot profit. However, since in the sequential game leader firm chooses to produce a different output level, it must be increasing its profit compared with the Cournot profit level. The kind of reasoning is called a *revealed profitability* argument.

newly merged entity. Thus, any two merging firms must decide whether or not to merge without knowing the actual cost of the merged firm in the future. We suppose that the expected marginal cost of the merged firm is equal to the non-merged firm's cost "c" which is the same as the benchmark firm's one[11]. The exact value of newly merged entity's cost "c_i" is uncertain, it could be either higher or lower than this critical value c. Hence, we assume that $a > \max\{c, c_i\}$ and the variance of this uncertain cost c_i is independently drawn from an identical distribution with $Var(c_i) = \sigma^2$. The variance σ^2 represents the degree of the uncertainty and captures marginal cost fluctuation. The merging firms can generate efficiency gains if $c_i - c < 0$. This situation corresponds to the usual argument which puts forward to the increase in productive efficiency generated by the merger itself. Conversely, when $c_i - c > 0$ the merger is assumed to cause efficiency losses (*i.e.* due to the clash of company culture).

Case A: Merger between two leaders

In this case, the industry is composed of $m-1$ leaders but still $n-m$ followers since the newly merged entity behaves as a leader. Let $q_t^{j,i}$ represent firm's output, the superscript $j = \{l, f\}$ stands for the firm's role (leader or follower), and the superscript $i = \{A, B, C, D\}$ corresponds to one of the four possible cases; the subscript $t = \{I, O\}$ signifies the firm's status (Insider or Outsider). For instance, consider $q_I^{l,A}$ as the merged firm's quantity and $q_O^{l,A}$ as outsider-leader firm's output and $q_O^{f,A}$ as outsider-follower's output. From the standpoint of information structure, since insider *first-to-knows* its production cost (or productivity), its output level will depend on the actual cost (c_i), namely $q_I^{l,A}(c_i)$; outsider-followers observe the output level of insider and then perfectly infer the merged entity's cost, accordingly $q_O^{f,A}(c_i)$; since all leaders simultaneously decide the quantity level, outsider-leaders have no chance to observe the insider production. Consequently, the outsider-leaders regard c as the insider's productivity, we have $q_O^{l,A}(c)$.

By backward induction, we begin with the follower production stage. The optimizing question is

$$\max_{q_O^{f,A}} \pi_O^{f,A} = (p^A - c)q_O^{f,A} = [a - c - Q_O^{-f,A} - q_O^{f,A} - Q_O^{l,A}(c) - q_I^{l,A}(c_i)]q_O^{f,A}$$

From the first-order-condition, we derive the best response function of followers (See detail in **Appendix A**):

$$(n-m+1)q_O^{f,A} = a - c - Q_O^{l,A}(c) - q_I^{l,A}(c_i) \tag{1}$$

In the first (leader production) stage, outsider-leaders are not aware of the actual cost of insider, thereby, they consider the insider's cost as the expected value c and maximize the following profit function:

$$\max_{q_O^{l,A}} \pi_O^{l,A} = (p^A - c)q_O^{l,A} = [a - c - Q_O^{f,A} - Q_O^{-l,A}(c) - q_O^{l,A} - q_I^{l,A}(c)]q_O^{l,A}(c)$$

For the insider, since it knows the real cost c_i

$$\max_{q_I^{l,A}} \pi_I^{l,A} = (p^A - c_i)q_I^{l,A} = [a - c_i - Q_O^{l,A}(c) - Q_O^{f,A} - q_I^{l,A}(c_i)]q_I^{l,A}(c_i)$$

We then obtain the following expressions for the equilibrium output (See detail in **Appendix B**):

[11] This assumption allows us to focus on the effect of uncertainty on mergers even without any uncertain efficiency gains.

$$
\begin{aligned}
q_I^{l,A}(c_i) &= \frac{2(a-c)-m(n-m+1)(c_i-c)}{2m} \\
q_I^{l,A}(c) &= \frac{(a-c)}{m} \\
q_O^{l,A}(c) &= \frac{(a-c)}{m} \\
q_O^{f,A}(c_i) &= \frac{2(a-c)+m(n-m+1)(c_i-c)}{2m(n-m+1)}
\end{aligned} \tag{2}
$$

The aggregate quantity is expressed as

$$
Q^A = q_I^{l,A}(c_i) + (m-2)q_O^{l,A}(c) + (n-m)q_O^{f,A}(c_i)
$$

Both the equilibrium profits and the expected equilibrium profits of firms are given as follows (See detail in **Appendix C**).

Insider:

$$
\pi_I^{l,A} = \frac{[2(a-c)-m(n-m+1)(c_i-c)]^2}{4m^2(n-m+1)} \tag{3}
$$

$$
\mathbb{E}[\pi_I^{l,A}] = \frac{(a-c)^2}{m^2(n-m+1)} + \frac{n-m+1}{4}\sigma^2 \tag{4}
$$

Since the marginal cost of outsiders is unchangeable and the merged entity learns its own production cost after merger, the merged entity possesses complete information at the moment of "Production decision". $\pi_I^{l,A}$ represents the exact value of merged firm's profit which will be used to analyze the *ex post* profitability of merger. In addition, the expected profit of merged firm is determined at the moment of "Private incentive to merge" where the actual cost of merged firm is concealed from all firms including merging parties, and this expected term is used to analyze the incentive to merge in the following section.

Outsider-leader:

$$
\pi_O^{l,A} = \frac{(a-c)[2(a-c)+m(n-m+1)(c_i-c)]}{2m^2(n-m+1)} \tag{5}
$$

$$
\mathbb{E}[\pi_O^{l,A}] = \frac{(a-c)^2}{m^2(n-m+1)} \tag{6}
$$

Outsider-leader firms commit to quantities before the uncertainty is resolved, therefore, only the expected value of the cost is relevant to them, they have *zero information* on merged entity's cost. A larger uncertainty, in the sense of an increased variance in the cost distribution with the same expected value, will not change the profit of outsider-leader firms. Consequently, uncertainty has no effect on them, and each outsider-leader's expected profit is the same as the one when merged firm's cost is deterministic ($c_i = c$).

Outsider-follower:

$$
\pi_O^{f,A} = \frac{[2(a-c)+m(n-m+1)(c_i-c)]^2}{4m^2(n-m+1)^2} \tag{7}
$$

$$
\mathbb{E}[\pi_O^{f,A}] = \frac{(a-c)^2}{m^2(n-m+1)^2} + \frac{1}{4}\sigma^2 \tag{8}
$$

It is worthwhile to note that, since both the merged firm and the outsider-follower firms know the exact marginal cost of merged entity, in addition, outsider-leader firms recognize no change in merged firm's cost after merger, the asymmetric information about the merged entity's cost not only does work in favor of the merged firm, but also is propitious to outsider-follower firms. This is because firms of both categories can adjust their production accordingly. In expected terms, the sensibility of firms' gains to the uncertainty is not the same. The cost uncertainty effect affects more strongly the merged entity than the outsider (followers) group.

The consumer surplus (**CS**) and the social welfare (**W**) are easily found to be:

$$CS^A = \frac{\{2[1-m(n-m+1)](a-c)+m(n-m+1)(c_i-c)\}^2}{8m^2(n-m+1)^2} \tag{9}$$

$$W^A = CS^A + \pi_I^{l,A}(c_i) + (m-2)\pi_O^{l,A}(c) + (n-m)\pi_O^{f,A}(c_i) \tag{10}$$

By simple calculation, we obtain the following expected values of CS and W.

$$\mathbb{E}[CS^A] = \frac{(a-c)^2[1-m(n-m+1)]^2}{2m^2(n-m+1)^2} + \frac{1}{8}\sigma^2 \tag{11}$$

$$\begin{aligned}\mathbb{E}[W^A] = &\quad \mathbb{E}[CS^A] + \mathbb{E}[\pi_I^{l,A}] + (m-2)\mathbb{E}[\pi_O^{l,A}] + (n-m)\mathbb{E}[\pi_O^{f,A}] \\ = &\quad \tfrac{(a-c)^2}{2}\left[\tfrac{m^2(n-m+1)^2-1}{m^2(n-m+1)^2}\right] + \left(\tfrac{n-m}{2} + \tfrac{3}{8}\right)\sigma^2\end{aligned} \tag{12}$$

Note that both consumer surplus and social welfare are increasing functions with respect to the variance σ^2. Concretely, we have $\frac{\partial \mathbb{E}[CS^A]}{\partial \sigma^2} = \frac{1}{8}$ and $\frac{\partial \mathbb{E}[W^A]}{\partial \sigma^2} = \frac{n-m}{2} + \frac{3}{8}$. The extent of the uncertainty effect on welfare evidently depends on the role distribution. Precisely, the more leader firms, the lower impact of uncertainty on welfare.

Case B: Merger between two followers

In this case, we consider that two followers take part in the merger. The distribution of roles in the industry is assumed not to be altered by the merger decision in the way that merged entity behaves as a follower. The industry contains $n-1$ firms with m leaders. Concerning informational structure, neither outsider-leader firms nor outsider-follower firms can infer the exact marginal cost of the merged firm, because this new second-mover entity and the non-merged followers simultaneously make the output decisions. Therefore, there is the informational symmetry between the outsider-leaders and the outsider-followers which are both unaware of the merged firm's actual cost. The relevant equilibrium values are shown in Table 1. (See brief demonstration in **Appendix D**)

Table 1: Equilibrium values in case B

	Case B	
Equilibrium	Actual terms[a]	Expected terms[b]
Output	$q_I^{f,B}(c_i) = \frac{2(a-c)-(m+1)(n-m)(c_i-c)}{2(m+1)(n-m)}$ $q_O^{l,B}(c) = \frac{(a-c)}{(m+1)}$ $q_O^{f,B}(c) = \frac{(a-c)}{(m+1)(n-m)}$	$q_I^{f,B}(c) = \frac{(a-c)}{(m+1)(n-m)}$
Profit	$\pi_I^{f,B} = \frac{[2(a-c)-(m+1)(n-m)(c_i-c)]^2}{4(m+1)^2(n-m)^2}$ $\pi_O^{l,B} = \frac{(a-c)[2(a-c)-(m+1)(n-m)(2-c-c_i)]}{2(m+1)^2(n-m)}$ $\pi_O^{f,B} = \frac{(a-c)[2(a-c)-(m+1)(n-m)(2-c-c_i)]}{2(m+1)^2(n-m)^2}$	$\mathbb{E}[\pi_I^{f,B}] = \frac{(a-c)^2}{(m+1)^2(n-m)^2} + \frac{1}{4}\sigma^2$ $\mathbb{E}[\pi_O^{l,B}] = \frac{(a-c)^2}{(m+1)^2(n-m)} - \frac{a-c}{m+1}$ $\mathbb{E}[\pi_O^{f,B}] = \frac{(a-c)^2}{(m+1)^2(n-m)^2} - \frac{a-c}{(m+1)(n-m)}$
Consumer surplus	$CS^B = \frac{\{2(a-c)[(m+1)(n-m)-1]-(m+1)(n-m)(c_i-c)\}^2}{8(m+1)^2(n-m)^2}$	$\mathbb{E}[CS^B] = \frac{(a-c)^2[(m+1)(n-m)-1]^2}{2(m+1)^2(n-m)^2} + \frac{1}{8}\sigma^2$
Social welfare	$W^B = CS^B + \pi_I^{f,B} + m\pi_O^{l,B} + (n-m-2)\pi_O^{f,B}$	$\mathbb{E}[W^B] = \mathbb{E}[CS^B] + \mathbb{E}[\pi_I^{f,B}] + m\mathbb{E}[\pi_O^{l,B}]$ $+(n-m-2)\mathbb{E}[\pi_O^{f,B}]$ $\frac{\partial \mathbb{E}[W^B]}{\partial \sigma^2} = \frac{3}{8}$

[a] **Actual terms** refer to the post merger game where the insider learns its own cost level. The merger profitability and *ex post* merger assessment are analyzed based on these values.

[b] **Expected terms** refer to the pre-merger game where the (merger) participants do not know the future productivity level. The private incentive to merge and *ex ante* enforcement merger control are studied by means of these expected values.

Case C: Merger between two followers resulting in a leader

Consider a special type of merger wherein two followers merge and result in a firm behaving as leader. As a result, there are $m+1$ leaders and $n-m-2$ followers. This case was examined by Daughety (1990) who found that the horizontal merger was potentially profitable for the merged firm and this merger might be advantageous from the viewpoint of social welfare in the absence of cost variation. We restudy this scenario by introducing two elements: cost uncertainty and information structure, to proceed the in-depth analysis. Of course, the outcome found by Daughety (1990) corresponds to our result in the extreme situation where there is no uncertainty and the information is perfect and complete. The equilibrium values are displayed in Table 2.

Table 2: Equilibrium values in case C

	Case C	
Equilibrium	Actual terms	Expected terms
Output	$q_I^{l,C}(c_i) = \frac{2(a-c)-(m+2)(n-m-1)(c_i-c)}{2(m+2)}$ $q_O^{l,C}(c) = \frac{a-c}{m+2}$ $q_O^{f,C}(c_i) = \frac{2a-c[(m+2)(n-m)-m]+(m+2)(n-m-1)c_i}{2(m+2)(n-m-1)}$	$q_I^{l,C}(c) = \frac{(a-c)}{(m+2)}$
Profit	$\pi_I^{l,C} = \frac{[2(a-c)-(m+2)(n-m-1)(c_i-c)]^2}{4(m+2)^2(n-m-1)}$ $\pi_O^{l,C} = \frac{(a-c)[2(a-c)+(m+2)(n-m-1)(c_i-c)]}{2(m+2)^2(n-m-1)}$ $\pi_O^{f,C} = \frac{[2(a-c)+(m+2)(n-m-1)(c_i-c)]^2}{4(m+2)^2(n-m-1)^2}$	$\mathbb{E}[\pi_I^{l,C}] = \frac{(a-c)^2}{(m+2)^2(n-m-1)} + \frac{(n-m-1)}{4}\sigma^2$ $\mathbb{E}[\pi_O^{l,C}] = \frac{(a-c)^2}{(m+2)^2(n-m-1)}$ $\mathbb{E}[\pi_O^{f,C}] = \frac{(a-c)^2}{(m+2)^2(n-m-1)^2} + \frac{1}{4}\sigma^2$
Consumer surplus	$CS^C = \frac{\{2(a-c)[(m+2)(n-m-1)-1]-(m+2)(n-m-1)(c_i-c)\}^2}{8(m+2)^2(n-m-1)^2}$	$\mathbb{E}[CS^C] = \frac{(a-c)^2[(m+2)(n-m-1)-1]^2}{2(m+2)^2(n-m-1)^2} + \frac{1}{8}\sigma^2$
Social welfare	$W^C = CS^C + \pi_I^{l,C} + m\pi_O^{l,C} + (n-m-2)\pi_O^{f,C}$	$\mathbb{E}[W^C] = \mathbb{E}[CS^C] + \mathbb{E}[\pi_I^{l,C}] + m\mathbb{E}[\pi_O^{l,C}]$ $+(n-m-2)\mathbb{E}[\pi_O^{f,C}]$ $\frac{\partial \mathbb{E}[W^C]}{\partial \sigma^2} = \frac{n-m}{2} - \frac{5}{8}$

Case D: Merger between one leader and one follower

Finally, we focus on the merger between one leader and one follower (the merged entity behaves as a leader). In the *post merger* market, the number of leaders is the same as that in the case B, and the number of leaders outside of merger equals to $m-1$. This case without taking into account the issue of information sharing and uncertainty, was studied by Huck, Konard and Muller (2001), who were the first to observe that the merger between two firms from different categories increased the joint profits of firms. They compared the profitability of two-follower merger with that of leader-follower merger, and showed that mergers between a leader and a follower were unambiguously profitable. Their outstanding result is verified in this framework, and corresponds to the extreme situation where the merged firm's cost is unaltered and equals to c. The equilibrium values are shown in Table 3.

Table 3: Equilibrium values in case D

	Case D	
Equilibrium	Actual terms	Expected terms
Output	$q_I^{l,D(c_i)} = \frac{2(a-c)-(m+1)(n-m)(c_i-c)}{2(m+1)}$ $q_O^{l,D}(c) = \frac{a-c}{m+1}$ $q_O^{f,D}(c_i) = \frac{2(a-c)+(m+1)(n-m)(c_i-c)}{2(m+1)(n-m)}$	$q_I^{l,D}(c) = \frac{(a-c)}{(m+1)}$
Profit	$\pi_I^{l,D} = \frac{[2(a-c)-(m+1)(n-m)(c_i-c)]^2}{4(m+1)^2(n-m)}$ $\pi_O^{l,D} = \frac{(a-c)[2(a-c)+(m+1)(n-m)(c_i-c)]}{2(m+1)^2(n-m)}$ $\pi_O^{f,D} = \frac{[2(a-c)+(m+1)(n-m)(c_i-c)]^2}{4(m+1)^2(n-m)^2}$	$\mathbb{E}[\pi_I^{l,D}] = \frac{(a-c)^2}{(m+1)^2(n-m)} + \frac{n-m}{4}\sigma^2$ $\mathbb{E}[\pi_O^{l,D}] = \frac{(a-c)^2}{(m+1)^2(n-m)}$ $\mathbb{E}[\pi_O^{f,D}] = \frac{(a-c)^2}{(m+1)^2(n-m)^2} + \frac{1}{4}\sigma^2$
Consumer surplus	$CS^D = \frac{\{2(a-c)[(m+1)(n-m)-1]-(m+1)(n-m)(c_i-c)\}^2}{8(m+1)^2(n-m)^2}$	$\mathbb{E}[CS^D] = \frac{(a-c)^2[(m+1)(n-m)-1]^2}{2(m+1)^2(n-m)^2} + \frac{1}{8}\sigma^2$
Social welfare	$W^D = CS^D + \pi_I^{l,D} + (m-1)\pi_O^{l,D}$ $+(n-m-1)\pi_O^{f,D}$	$\mathbb{E}[W^D] = \mathbb{E}[CS^D] + \mathbb{E}[\pi_I^{l,D}] + (m-1)\mathbb{E}[\pi_O^{l,D}]$ $+(n-m-1)\mathbb{E}[\pi_O^{f,D}]$ $\frac{\partial \mathbb{E}[W^D]}{\partial \sigma^2} = \frac{n-m}{2} - \frac{1}{8}$

It is worth noting that the merged firm's profit, the levels of consumer surplus and social welfare (prior to the merger consummation) are increasing functions with respect to the variance. Thus, the merged firm's expected profit and the expected surpluses grow, as the uncertainty increases. By comparing the four aforementioned cases, we have the following remarks:

Remark 1 *i). In private view, the cost uncertainty has the strongest impact on the merged firm's expected profit, when this entity is composed of two leaders. By contraries, it generates the weakest effect on expected profit when two followers merge without role redistribution. More precisely,* $\frac{\partial \mathbb{E}[\pi_I^A]}{\partial \sigma^2} > \frac{\partial \mathbb{E}[\pi_I^D]}{\partial \sigma^2} > \frac{\partial \mathbb{E}[\pi_I^C]}{\partial \sigma^2} > \frac{\partial \mathbb{E}[\pi_I^B]}{\partial \sigma^2}$. *ii). In public view, the same ranking is found* $\frac{\partial \mathbb{E}[W^A]}{\partial \sigma^2} > \frac{\partial \mathbb{E}[W^D]}{\partial \sigma^2} > \frac{\partial \mathbb{E}[W^C]}{\partial \sigma^2} > \frac{\partial \mathbb{E}[W^B]}{\partial \sigma^2}$. *iii). Furthermore, the intensity of uncertainty impact on merged firm's profit and on the social welfare depends upon the distribution of roles* (n,m) *except for case B.*

In the cases A, C and D, the newly merged firm behaves as a leader, there is asymmetric information between outsider-leaders and outsider-followers. The greater the number of followers $(n-m)$ in *pre-merger* market, the larger the intensity of uncertainty (on merged firm's profit and welfare). By contrast, when there is symmetric information between outsiders, the extent of uncertainty effect on merged firm's profit and on welfare are constant, irrespective of the number of followers.

It is worthwhile to note when the merged firm behaves as a leader, the cost uncertainty effect for outsider-follower firms is always positive and the extent of this effect will be the same. Namely, $\frac{\partial \mathbb{E}[\pi_O^{f,A}]}{\partial \sigma^2} = \frac{\partial \mathbb{E}[\pi_O^{f,C}]}{\partial \sigma^2} = \frac{\partial \mathbb{E}[\pi_O^{f,D}]}{\partial \sigma^2} = \frac{1}{4}$

Remark 2 *Compared to consumer surplus, welfare is more sensitive to the cost uncertainty. Concretely,* $\frac{\partial \mathbb{E}(W^i)}{\partial \sigma^2} > \frac{\partial \mathbb{E}(\pi_I^{j,i})}{\partial \sigma^2} > \frac{\partial \mathbb{E}(CS^i)}{\partial \sigma^2}$ *(*$i = \{A,B,C,D\}$ *and* $j = \{l,f\}$*)*

This remark is important, and it is able to help us to explain why the government prefers consumer welfare standard to aggregate welfare standard when under *ex ante* intervention. The in-depth analysis will be carried on in the welfare section. In the following section, we provide a detailed account of the consequences of the merger on profits. By dealing with the effects of uncertainty, information structure and role redistribution, we analyze the firms' incentives to merge and the *ex post* profitability of merger.

3 Merger analysis

We assume that the "private incentive to merge" results from the comparison between the *ex ante* expected profit of the merged firm and the sum of merging parties' profits in the *pre-merger* game. The "*ex post* profitability of the merger" is determined by the difference between the actual profit earned by the newly merged entity and the sum of profits of merging firms in benchmark case.

3.1 Private incentive to merge: *ex ante* profitability of merger

Let $\Delta^i_{\mathbb{E}[\pi]}$ ($i = \{A,B,C,D\}$) represent the private incentive to merge. The firms have incentive to merge when $\Delta^i_{\mathbb{E}[\pi]} \geq 0$. The relationship between merger incentive and cost uncertainty under different scenarios is shown in Table 4.

Table 4: Merger incentive and cost uncertainty

Scenarios	$n \geq 6$ and $3 \leq m \leq n-3$
Case A $\left(\Delta^A_{\mathbb{E}[\pi]} = \mathbb{E}[\pi_I^{l,A}] - 2\pi^l\right)$	$\Delta^A_{\mathbb{E}[\pi]} \geq 0$ when $\sigma^2 \geq \sigma^2_{\pi_A}$
Case B $\left(\Delta^B_{\mathbb{E}[\pi]} = \mathbb{E}[\pi_I^{f,B}] - 2\pi^f\right)$	$\Delta^B_{\mathbb{E}[\pi]} \geq 0$ when $\sigma^2 \geq \sigma^2_{\pi_B}$
Case C $\left(\Delta^C_{\mathbb{E}[\pi]} = \mathbb{E}[\pi_I^{l,C}] - 2\pi^f\right)$	$\Delta^C_{\mathbb{E}[\pi]} \geq 0$ always holds true
Case D $\left(\Delta^D_{\mathbb{E}[\pi]} = \mathbb{E}[\pi_I^{l,D}] - (\pi^l + \pi^f)\right)$	$\Delta^D_{\mathbb{E}[\pi]} \geq 0$ always holds true

With

$$\sigma^2_{\pi_A} = \frac{4(a-c)^2(m^2-2m-1)}{m^2(m+1)^2(n-m+1)^2} > 0$$
$$\sigma^2_{\pi_B} = \frac{4(a-c)^2[(n-m)^2-2(n-m)-1]}{(m+1)^2(n-m)^2(n-m+1)^2} > 0$$

This table shows that for mergers of types C and D firms always have incentive to merge, irrespective of the cost uncertainty. This finding is consistent with theoretical models of stackelberg mergers in deterministic environment. For instance, according to Daughety (1990), when two followers decide to merge and the newly merged entity behaves as a leader on the product market, the firms have incentives to merge even without cost-saving (or efficiency gains). In addition, Huck, Konard and Muller (2001) show that the merger between one leader and one follower is profitable, in the absence of information issue and cost

fluctuation. Based on the Table 4, we derive the following proposition:
:

Proposition 1 *i). When there is role redistribution, the merging firms always have incentives to merge, irrespective of cost uncertainty. ii). A merger without role redistribution is ex ante profitable if the cost uncertainty is sufficiently large, i.e., if* $\sigma^2 \geq \min(\sigma^2_{\pi_A}, \sigma^2_{\pi_B})$.

Proof: See detail in **Appendix H.1**

Proposition 1 shows that as the cost uncertainty grows larger, firms have more incentives to merge (the expected profit of the merged firm grows with the enlargement of variance). When uncertainty is sufficiently large (the extent of the variance exceeds a certain threshold, such as $\sigma^2_{\pi_A}$ and $\sigma^2_{\pi_B}$), the expected profit of the merged firm becomes larger and firms facing cost uncertainty choose to merge. This outcome is in line with Hamada (2012). Banal-Estanol (2007) finds that cost uncertainty always enhances the incentives to merge and argues that an extra incentive to merge is driven by information sharing. Zhou (2008a) shows under cost uncertainty the merger incentives are reinforced by production rationalization. In contrast with these, our model stresses that additional incentives are engendered by both role redistribution and informational asymmetry. When the variance of merged entity's cost is close to zero, the firms without role redistribution have no incentive to merge. By opposite, as the variance grows, the expected profit for merged firms also increases since the gain of the optimal quantity adjustment for the insider enlarges.

3.2 Ex post profitability of merger

The *ex post* profitability of the merger is evaluated by considering the variation in actual profits (Δ^i_π). The difference between the merged firm's exact cost (c_i) for the merger type i ($i = \{A, B, C, D\}$) and expected firm's costs (c) is defined as "δ^i". For instance, $\Delta^A_\pi = \pi^{l,A}_I(\delta^A, n, m) - 2\pi^l(n, m)$ in the case A. We define δ^A_{sup} the threshold value of δ^A which separates profitable from unprofitable mergers. When $\delta^A < \delta^A_{sup}$ (respectively $\delta^A > \delta^A_{sup}$) we have $\Delta^A_\pi > 0$ (respectively $\Delta^A_\pi < 0$). In addition, in order to avoid boundary problems in which some firms are inactive, we also define δ^A_{inf} as the value of δ^A below which outsiders are ruled out of the market. It is given by the conditions : $q^{l,A}_O = 0$ and $q^{f,A}_O = 0$. Note that when we have $\delta^A_{inf} < \delta^A < \delta^A_{sup}$, the merger is profitable and two categories of outsiders remain on the market.

3.2.1 Incomplete information

Under incomplete information, the merged firm knows its own marginal cost, whereas not all outsider firms are aware of the actual cost of merged entity. In the cases A, C and D, outsider-leader firms are uninformed about the exact value[12] c_i, however, the timing of the game implies that outsider-follower firms are aware of c_i. In Table 5, we summarize the ranges of cost variation (δ^i) in different scenarios wherein the merger is profitable.

[12]In the case B where two followers take part in the merger, all outsider firms are uninformed about the exact value c_i.

Table 5: Merger profitability and potential efficiency gains (or losses)

Scenarios	$n \geq 6$ and $3 \leq m \leq n-3$
Case A $\left(\Delta_\pi^A = \pi_I^{l,A} - 2\pi^l\right)$	$\delta_{inf}^A < \delta \leq \delta_{sup}^A$
Case B $\left(\Delta_\pi^B = \pi_I^{f,B} - 2\pi^f\right)$	$\delta \leq \delta_{sup}^B$
Case C $\left(\Delta_\pi^C = \pi_I^{l,C} - 2\pi^f\right)$	$\delta_{inf}^C < \delta \leq \delta_{sup}^C$
Case D $\left(\Delta_\pi^D = \pi_I^{l,D} - (\pi^l + \pi^f)\right)$	$\delta_{inf}^D < \delta \leq \delta_{sup}^D$

With

$$\delta_{inf}^A = -\frac{2(a-c)}{m(n-m+1)} < 0 \qquad \delta_{sup}^A = \frac{2(a-c)}{m(n-m+1)} - 2\sqrt{2}\frac{a-c}{(n-m+1)(m+1)} < 0$$

$$\delta_{sup}^B = -2\sqrt{2}\frac{(a-c)}{(m+1)(n-m+1)} + \frac{2(a-c)}{(n-m)(m+1)} < 0$$

$$\delta_{inf}^C = -\frac{2(a-c)}{(m+2)(n-m-1)} < 0 \qquad \delta_{sup}^C = -2\sqrt{2}\frac{(a-c)}{(m+1)(n-m-1)\sqrt{n-m-1}} + \frac{2(a-c)}{(m+2)(n-m-1)} > 0$$

$$\delta_{inf}^D = -\frac{2(a-c)}{(m+1)(n-m)} < 0 \qquad \delta_{sup}^D = 2[\frac{a-c}{(m+1)(n-m)} - \frac{(a-c)}{(m+1)(n-m+1)}\sqrt{\frac{n-m+2}{n-m}}] > 0$$

To ensure that none of outsider firms exit the market and the merger is profitable, the potential cost change in different scenarios should satisfy the condition that δ^i lies in the interval $(\delta_{inf}^i, \delta_{sup}^i]$. Note that there is no constraint on the exit of outsider in case B.

Remark 3 *By comparing δ_{sup}^i, we obtain:*

$$\delta_{sup}^C > \delta_{sup}^D > 0 > \begin{cases} \delta_{sup}^A > \delta_{sup}^B & \text{if } m \text{ belongs to } [3, \frac{n}{2}) \\ \delta_{sup}^B > \delta_{sup}^A & \text{if } m \text{ belongs to } (\frac{n}{2}, n\text{-}3] \end{cases}$$

Since the values of upper bound δ_{sup} in the case C and in the case D are greater than zero, a merger with anticompetitive effects can also lead to efficiency losses. If there are more leaders in the *pre-merger* market (*i.e., $m \in (\frac{n}{2}, n-3]$*), a profitable merger between two leaders requires more marginal cost reduction in comparison with a profitable merger between two followers. In other words, the conditions on efficiency gains, under which the two-follower merger is profitable, are less restrictive. By contrast, if there are more follower firms in the *pre-merger* market, two-follower merger needs more efficiency gains to be profitable.

The higher δ_{sup}^i, the greater the allowed potential efficiency losses, the more likely mergers occur. Since the merger composed of two followers to form a leader (case C) generates potential efficiency losses higher than the merger between one leader and one follower (case D), to some extent that the condition on profitable merger in the case C is less restrictive, and the merger of this type takes place more likely. The profitable merger in the case C is more likely to be inefficient than the one in case D since the former brings more advantages to the merged entity by changing the roles of both merging parties from a follower to a leader in the market.

Note that the ceiling of δ^i depends upon the redistribution of roles. For instance, if we focus on the two-follower mergers (case B and C), it is found that the condition on (profitable) resulting leader is less restrictive than the one on resulting follower. This means even the merger leads to efficiency losses, the resulting leader can be profitable due to the effect of role redistribution; however, the resulting follower is unprofitable without ambiguity. It is clear that the two-follower merger aiming to the leader strategy takes place more likely than the one choosing the follower strategy.

3.2.2 Incomplete *Vs.* complete information for all outsiders

Under complete information[13], the information about merged firm's real cost is no longer private, not only the merged firm is aware of its own marginal cost c_i, but also all outsider firms are informed about it. Using the deterministic case as a criterion, we study whether the merged firm has interests to reveal its own cost to competing firms[14].

Consider $\hat{\pi}_I^{j,i}$ ($i = \{A,B,C,D\}$ and $j = \{l, f\}$) the merged firm's profit in the situation where there is complete and perfect information (see expressions of $\hat{\pi}_I^{j,i}$ in **Appendix E**). It will be interesting to compare the profit of the insider under incomplete information scenario to that under complete information situation.

Proposition 2 *Within the range of* $\delta^i \in (\delta^i_{inf}, \delta^i_{sup}]$, *i). the merged firm's profit will be greater under complete information than under incomplete information, when there is no role redistribution. ii). the merged firm's profit will be greater under incomplete information, when there exists the role redistribution.*

Proof: See detail in **Appendix H.2**

The acquisition of market power is usually the first motive for horizontal mergers. The argument is that horizontal mergers increase market concentration, which, by increasing market power, increases profitability. In the absence of the redistribution of roles (cases A and B), the equilibrium price is higher under complete information than incomplete information, the higher price gives rise to higher market power, in addition, the merged firm produces more under complete information. Because of these two above-mentioned reasons, the merged firm will be more profitable under complete information, and it has interests to reveal information about its own cost to competing firms. This outcome is consistent with the well-known conclusion in the information sharing literature[15], that, concentrates on a firm's incentives to share its private information with competing firms. In particular, it shows that firms competing in quantities are willing to reveal their private information about production costs, but are not willing to reveal their private information about market demand.

By contrast, in the presence of role redistribution (cases C and D), the strengthening of market power under incomplete information leads to more profitable merger compared to the one under complete information. This finding is in line with the conclusion of Zhou[16] who delineates that "firms are less likely to merge when they possess more information" (Zhou, 2008a, p.68).

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[13] The framework under complete information is studied in the working paper Le Pape and Zhao (2010).

[14] Under some circumstances (case A, C and D), outsider-follower firms can observe the insider's output level, and then infer the exact value of its marginal cost.

[15] There are some important contributions to this information sharing literature without merger issue, such as, Novshek and Sonnenschein (1982), Clark (1983), Vives (1984), Gal-Or (1985), Li (1985), Shapiro (1986) and Raith (1996).

[16] The reason for Zhou (2008a) is that mergers are driven by production rationalization under cost uncertainty. When firms have more information, they are able to rationalize their production even without a merger, thus having less incentive to merge.

We demonstrate that a "*first-to-know disadvantage*" could appear if the merged firm adopts the same strategic behavior as *ex ante* merging firms. In this case, the informational asymmetry created by merger is detrimental to the merged entity.

Let $\hat{\delta}^i_{sup}$, $\hat{\delta}^i_{inf}$ denote respectively the upper bound and the lower bound under complete information (see **Appendix F**). By comparison with the boundary under incomplete information, we derive the following lemma.

Lemma 1 *i). Without role redistribution, profitable mergers necessarily generate efficiency gains. Moreover, the ceiling of this potential efficiency gains under incomplete information δ^i_{sup} (with $i = A, B$) is lower than that under complete information. ii). With role redistribution (cases C and D), profitable mergers can generate efficiency losses.*

Proof: See detail in **Appendix H.3**

4 Welfare analysis and potential policy guidance

We investigate the welfare implications of mergers, and discuss the possible antitrust policy toward horizontal mergers. The consumer welfare (CS) and social welfare (W) in benchmark are given as follows:

$$CS = \frac{(a-c)^2(n+mn-m^2)^2}{2(m+1)^2(n-m+1)^2}$$

$$W = \frac{(a-c)^2[(m+1)(n-m+1)+1](n+mn-m^2)}{2(m+1)^2(n-m+1)^2}$$

4.1 Ex ante intervention of Antitrust Authorities

When Competition Authorities intervene *ex ante*, we assume that they are uninformed about the merged firm's cost, so that decisions are based on the expected welfare.

In a first time, we consider that the decision of the Competition Authorities is based in the following principle: a merger is approved whenever the expected change of social welfare is positive. In a second time, we will compare mergers approvals based on social welfare to those based on consumer surplus.

Consider $\Delta^i_{\mathbb{E}[W]} = \mathbb{E}[W^i] - W$ as the yardstick which judges whether the merger improves the social welfare. In the case of $\Delta^i_{\mathbb{E}[W]} > 0$, the merger enhances the welfare, and it will damage the welfare if $\Delta^i_{\mathbb{E}[W]} < 0$. Table 6 enumerates the thresholds $\sigma^2_{W_i}$ beyond which the merger always gives rise to welfare improvement.

Table 6: Private Vs. social incentive to merge

	Welfare-enhancing	
Scenarios	Threshold $\sigma^2_{W_i}$	Comparison with $\sigma^2_{\pi_i}$
Case A	$\sigma^2_{W_A} = \frac{4(a-c)^2(2m+1)}{m^2(m+1)^2(n-m+1)^2[4(n-m)+3]}$	$\sigma^2_{\pi_A} > \sigma^2_{W_A} > 0$
Case B	$\sigma^2_{W_B} = \frac{4(a-c)\{(a-c)[2(n-m)+1]+2(m+1)(n-m)[(m+1)(n-m)-2](n-m+1)^2\}}{3(m+1)^2(n-m+1)^2(n-m)^2}$	1).$\sigma^2_{\pi_B} > \sigma^2_{W_B} > 0$ when $n > 6, m \in [3, n-3), a > \Phi + c$ 2).$\sigma^2_{W_B} > \sigma^2_{\pi_B} > 0$ when $n = 6, m = 3$ or $n > 6, m \in [3, n-3), a < \Phi + c$
Case C	$\sigma^2_{W_C} = \frac{4(a-c)^2[(2m+1)(n-m-1)+2n](n-3m-3)}{(m+1)^2(m+2)^2[(n-m)^2-1]^2[5-4(n-m)]}$	$\sigma^2_{W_C} > 0$ ($\nexists\ \sigma^2_{\pi_C}$)
Case D	$\sigma^2_{W_D} = \frac{4(a-c)^2[2(n-m)+1]}{[4(n-m)-1](m+1)^2(n-m+1)^2(n-m)^2}$	$\sigma^2_{W_D} > 0$ ($\nexists\ \sigma^2_{\pi_D}$)

$$\text{with} \quad \Phi = \frac{2(m+1)(n-m)(n-m+1)^2[(n-m)(m+1)-2]}{3(n-m)^2 - 4[2(n-m)+1]}$$

Proposition 3 *i). Profitable mergers between leaders always constitute a welfare-enhancing merger. ii). In the presence of role redistribution, when the uncertainty is sufficiently high, a profitable merger between two followers are also welfare-enhancing.*

Proof: See detail in **Appendix H.4**

Mergers between leaders always generate unanimity between private and social incentives. By contrast, when the merger implies two-follower without role redistribution, profitable mergers may decrease expected social welfare.

Antitrust criterions may refer either to social welfare or consumer welfare. So an analysis of the effect of merger types on expected consumer surplus is proposed. The new criterion is given by the sign of the expression $\Delta^i_{\mathbb{E}[CS]} = \mathbb{E}[CS^i] - CS$, so that we evaluate the thresholds $\sigma^2_{CS_i}$ beyond which the merger improves the expected consumer surplus.

Corollary 1 **i).** *Profitable merger between leaders enhances the ex ante total welfare, but it can hurt ex ante consumer surplus.*

ii). *In the absence of role redistribution, all mergers between followers that improve expected consumer surplus, are welfare-enhancing and profitable without ambiguity.*

iii). *In the presence of role redistribution, when there are few active firms in the market ($n \leqslant 12$), or the large market ($n > 12$) contains a great deal of follower, a merger between followers which improves the consumer surplus is absolutely welfare-enhancing and profitable.*

iiii). *When the merger is composed of one leader and one follower, the consumer-surplus-improving merger will be unambiguously welfare-enhancing and profitable.*

Proof: See detail in **Appendix H.5**

As the antitrust decision on the basis of consumer surplus effectively guarantees both the welfare enhancement and the private intention of firms, to some extent, the severity of consumer surplus criterion can be regarded as the precision feature. This precision stems from the fact that the consumer surplus is less sensitive to uncertainty (see Remark 2). On the other hand, because of cost uncertainty, the agencies should not adopt the default assumption that a merger would enhance the producer surplus portion of total welfare simply because the firms have proposed it. Nor should the agencies put much stock in the existence or magnitude of efficiencies claimed by merging parties in their negotiations with the agencies. As Porter (2005) summarizes, "we cannot assume that a merger will be efficient and profitable just because companies propose it." And this leads us to the conclusion that if the analysis of the impact of a merger on competition is implemented under (efficiency or merged firm's productivity) uncertainty, consumer surplus is what agencies and courts do best.

4.2 Ex post enforcement of Antitrust Authorities

When regulating the behavior of a private party which proposes a merger plan, the Competition Authorities are often uncertain about the sign and extent of the externality due to the shock caused by mergers. However, uncertainty will be disclosed, and information on the magnitude of the externality typically becomes available, once the merger is consummated. Clearly, the advantage of *ex post* merger enforcement is that it can focus more on (certain) history than on (uncertain) predictions. Assume Δ_W^i the variation in social welfare (before and after the merger): $\Delta_W^i = W^i - W$. We find the ranges of δ_W^i wherein the merger improves the social welfare (see **Appendix G**). Furthermore, by comparing the upper bound of δ_W^i with the critical value δ_{sup}^i demonstrated in merger analysis section, we shed light on the following proposition.

Proposition 4 *i). If the merger is composed of two leaders, a welfare-enhancing merger is not always profitable, but a profitable merger improves social welfare without ambiguity. ii). When two followers take part in the merger and the newly merged entity behaves as a leader, a welfare-enhancing merger is always profitable, however, a profitable merger can damage the aggregate surplus. iii). If the merger stems from firms of different types, a welfare-enhancing merger is always profitable.*

Proof: See detail in **Appendix H.6**

Proposition 4 shows that under this circumstance, as long as the merger between leaders is profitable, it is always welfare-enhancing. This point is also consistent with Zhou (2008a) who has found that if a merger (with efficiency gains) is profitable to the merging firms, it will also be welfare-improving. This result provides support for a "*laisser-faire*" policy if the criterion rests on social welfare. Nevertheless, in other cases, profitable mergers can damage the *ex post* social welfare. Therefore Competition Authorities must supervise more closely bilateral mergers which are consisted of either one or two followers.

Suppose now that Competition Authorities adopt the *ex post* consumer surplus criterion, we find the ranges of δ_{CS}^i wherein the merger improves the consumer surplus, and then we compare the upper bound of δ_{CS}^i (namely $\delta_{CS_{sup}}^i$), with both δ_{sup}^i and $\delta_{W_{sup}}^i$ to achieve the following corollary.

Corollary 2 *The ex post consumer welfare standard is more lenient than the ex post social welfare standard.*

Proof: See detail in **Appendix H.7**

Concerning the impact of the merger scenario, our model shows that in the case of two-leader merger, when there are more than four leaders in the market, the profitable merger is unambiguously welfare-enhancing and consumer-surplus-improving. In the case of two-follower merger with role redistribution, when there are sufficiently less leader firms in the market, the profitable merger generating efficiency losses can improve both consumer and aggregate welfare, and the welfare-enhancing merger ensures the improvement of consumer surplus. Finally, when the merger is composed of one leader and one follower, the merger which improves consumer surplus can damage the social welfare.

The model gains some insight into the relationship between the distinct criterions of Competition Authorities and the timing of policy intervention. When Competition Authorities adopt *ex ante* intervention, the consumer welfare standard is more restrictive than the aggregate welfare standard. By contrast, when Competition Authorities choose *ex post* enforcement, they are aware of the actual cost of merged firm, the consumer welfare standard is more lenient than the aggregate welfare standard.

5 Concluding remarks

This paper extends the strand of literature on horizontal mergers in a homogeneous oligopoly where there are leaders and followers. We emphasize the cost uncertainty and the efficiency gains (or losses) within sequential output decisions. We find that the expected profit of merged firms grows with uncertainty and when uncertainty is sufficiently high firms choose to merge. Moreover, if there is role redistribution due to the merger, even in the absence of uncertainty effect, firms have incentives to merge. Concerning the *ex post* profitability of merger when the merged firm cost is private information, we show that the two-follower merger aiming to a leader strategy occurs more likely than the one satisfying the *status quo*. Furthermore, the merged firm has interests to pool the private signals to outsiders, in the absence of role redistribution while the concealment is more profitable (for the insider) if we assume role redistribution.

Concerning the social desirability of mergers, it is found that a merger between leaders always enhances welfare if participants have incentives to merge leading to unanimity between private and collective intentions. Nevertheless, the merger with role redistribution leads to the private-collective conflict. From the standpoint of Competition Authorities, after separately studying the two possible criterions (aggregate welfare standard and consumer welfare standard), we find that the latter is more restrictive and more accurate than the former.

This framework has a number of limitations, which constitute a fruitful area that we leave for future research. 1). we have restricted our analysis to a bilateral merger. A generalization would be to consider the merger composed of more than two firms, in order to relax the assumption and check the robustness of this framework. 2). we would verify whether these findings hold if there was some noise on outsider-followers' knowledge about the merger's cost impact after observing the merged entity's quantity choice. There are many reasons which might create such type of noise on outsider-followers' information (e.g., any form of demand uncertainty will do that). 3). the possible issue with respect to the policy implications is the following: In practice, how an antitrust authority can identify which firms were leaders or followers before the merger, and whether the merging parties have changed their roles from being a follower to a leader. This is not a concern specific to the current paper, but a general concern for the literature of horizontal mergers assuming sequential firm competition. 4). the Endogenous Stackelberg issue in the context of cost uncertainty would be also taken into account.

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Appendix:

A Best response function of followers

In the follower production stage. The optimizing question is:

$$\max_{q_O^{f,A}} \pi_O^{f,A} = (p^A - c)q_O^{f,A} = [a - c - Q_O^{-f,A} - q_O^{f,A} - Q_O^{l,A}(c) - q_I^{l,A}(c_i)]q_O^{f,A}(c_i) \tag{13}$$

From the standpoint of information structure,

- $Q_O^l(c)$: outsider-leaders consider that the cost level of insider is equal to c

- $q_I^l(c_i)$: first-to-know
- $q_O^f(c_i)$: outsider-followers observe the production level and perfectly infer the cost level of merged entity c_i

the *FOC* (first-order-condition) is

$$2q_O^{f,A} = a - c - Q_O^{-f,A} - Q_O^{l,A}(c) - q_I^{l,A}(c_i)$$

perfect symmetry for outsider-followers:

$$Q_O^{-f,A} = (n-m-1)q_O^{f,A}$$

reaction function of outsider-follower is

$$(n-m+1)q_O^{f,A} = a - c - Q_O^{l,A}(c) - q_I^{l,A}(c_i) \tag{14}$$

and note the sum

$$\begin{aligned} Q_O^{f,A} &= (n-m)q_O^{f,A} \\ &= (\frac{n-m}{n-m+1})(a-c) - (\frac{n-m}{n-m+1})\big(Q_O^{l,A}(c) + q_I^{l,A}(c_i)\big) \end{aligned} \tag{15}$$

B Best response function of leaders and equilibrium output

In the (first) leader production stage, outsider-leaders are not aware of the actual cost of insider, thereby they take into account the expected value c

$$\max_{q_O^{l,A}} \pi_O^{l,A} = (p^A - c)q_O^{l,A} = [a - c - Q_O^{f,A} - Q_O^{-l,A}(c) - q_O^{l,A} - q_I^{l,A}(c)]q_O^{l,A}(c) \tag{16}$$

plug the sum of follower quantity *Eq.* (15) into *Eq.* (16), the maximization problem becomes

$$\max_{q_O^{l,A}} \pi_O^{l,A} = \frac{1}{n-m+1}[(a-c) - Q_O^{-l,A}(c) - q_O^{l,A}(c) - q_I^{l,A}(c)]q_O^{l,A}(c) \tag{17}$$

FOC:

$$2q_O^{l,A}(c) = (a-c) - Q_O^{-l,A}(c) - q_I^{l,A}(c)$$

perfect symmetry for outsider-leaders:

$$Q_O^{-l,A}(c) = (m-3)q_O^{l,A}(c)$$

reaction function of outsider-leader is

$$(m-1)q_O^{l,A}(c) = a - c - q_I^{l,A}(c) \tag{18}$$

and note the sum

$$Q_O^{l,A}(c) = (m-2)q_O^{l,A}(c) = \frac{m-2}{m-1}(a - c - q_I^{l,A}(c))$$

For insider (merged entity), when insider knows the real cost c_i, the optimizing question is

$$\max_{q_I^{l,A}} \pi_I^{l,A} = (p^A - c_i)q_I^{l,A} = [a - c_i - Q_O^{l,A}(c) - Q_O^{f,A} - q_I^{l,A}(c_i)]q_I^{l,A}(c_i)$$
$$= \frac{1}{n-m+1}[(a-c) + (n-m+1)(c-c_i) - Q_O^{l,A}(c) - q_I^{l,A}(c_i)]q_I^{l,A}(c_i)$$

FOC:

$$2q_I^{l,A}(c_i) = (a-c) + (n-m+1)(c-c_i) - Q_O^{l,A}(c) \tag{19}$$

when insider is not informed about the exact cost $E(c_i) = c$

$$\max_{q_I^{l,A}} \pi_I^{l,A} = (p^A - c)q_I^{l,A} = [a - c - Q_O^{l,A}(c) - Q_O^{f,A} - q_I^{l,A}(c)]q_I^{l,A}(c)$$
$$= \frac{1}{n-m+1}[(a-c) - Q_O^{l,A}(c) - q_I^{l,A}(c)]q_I^{l,A}(c)$$

FOC with respect to expected value c is

$$2q_I^{l,A}(c) = (a-c) - Q_O^{l,A}(c) \tag{20}$$

then yield

$$q_I^l(c) + \frac{1}{2}(n-m+1)(c-c_i) = q_I^l(c_i)$$

It is straightforward that in case of $c_i < c$, we obtain $q_I^l(c_i) > q_I^l(c)$; otherwise, $q_I^l(c_i) < q_I^l(c)$.

Based on *Eqs.* (18), (19) and (20), it is possible to derive leaders' equilibrium outputs:

$$q_I^{l,A}(c_i) = \frac{2(a-c) - m(n-m+1)(c_i - c)}{2m}$$
$$q_I^{l,A}(c) = \frac{(a-c)}{m}$$
$$q_O^{l,A}(c) = \frac{(a-c)}{m}$$

plugging them into follower's reaction function *Eq.* (14), it yields

$$q_O^{f,A}(c_i) = \frac{2(a-c) + m(n-m+1)(c_i - c)}{2m(n-m+1)}$$

and then, we derive the aggregate output

$$Q = q_I^{l,A}(c_i) + (m-2)q_O^{l,A}(c) + (n-m)q_O^{f,A}(c_i)$$
$$= a - \frac{a}{m(n-m+1)} - [\frac{1}{2} - \frac{1}{m(n-m+1)}]c - \frac{c_i}{2}$$

C Real and expected profits

The profit of *insider*:

$$\begin{aligned}
\pi_I^{l,A} &= (a-Q-c_i)q_I^{l,A}(c_i) \\
&= \frac{a^2}{m^2(n-m+1)} + \frac{[m^2+2-m(n+1)]^2(c_i-c)^2}{4m^2(n-m+1)} - \frac{2ac_i}{m^2(n-m+1)} \\
&+ \frac{c_i^2}{m^2(n-m+1)} + \frac{a(c_i-c)(\frac{2}{n-m+1}-m)}{m^2} + \frac{c_i(c_i-c)(m-\frac{2}{n-m+1})}{m^2} \\
&= \frac{[2(a-c)-m(n-m+1)(c_i-c)]^2}{4m^2(n-m+1)}
\end{aligned}$$

Knowing that $\mathbb{E}[(c_i-c)^2]=\sigma^2$, $\mathbb{E}[c_i]=c$, $\mathbb{E}[c_i^2]=c^2+\sigma^2$, $\mathbb{E}[c_i-c]=0$, $\mathbb{E}[(c_i-c)c_i]=\sigma^2$, the expected profit of *insider*:

$$\begin{aligned}
\mathbb{E}[\pi_I^{l,A}] &= \frac{(n-m+1)\sigma^2}{4} + \frac{c^2}{m^2(n-m+1)} - \frac{2ac}{m^2(n-m+1)} + \frac{a^2}{m^2(n-m+1)} \\
&= \frac{(a-c)^2}{m^2(n-m+1)} + \frac{n-m+1}{4}\sigma^2
\end{aligned}$$

The profit of *outsider-leader*:

$$\begin{aligned}
\pi_O^{l,A} &= (a-Q-c)q_O^{l,A}(c) \\
&= \frac{(a-c)[2(a-c)+m(n-m+1)(c_i-c)]}{2m^2(n-m+1)}
\end{aligned}$$

and then the expected profit of *outsider-leader* is

$$\mathbb{E}[\pi_O^{l,A}] = \frac{(a-c)^2}{m^2(n-m+1)}$$

The profit of *outsider-follower*:

$$\begin{aligned}
\pi_O^{f,A} &= (a-Q-c_i)q_O^{f,A}(c_i) \\
&= \frac{[2(a-c)+m(n-m+1)(c_i-c)]^2}{4m^2(n-m+1)^2}
\end{aligned}$$

the expected value is

$$\mathbb{E}[\pi_O^f] = \frac{(a-c)^2}{m^2(n-m+1)^2} + \frac{1}{4}\sigma^2$$

D Merger between two followers

Using the similar method (See **Appendix A and B**), the equilibrium outputs for followers are resolved on the basis of the following equations:

- $a-(n-m-2)q_O^{f,B}(c) - Q_O^{l,B}(c) - q_I^{f,B}(c) - c - q_O^{f,B}(c) = 0$ (outsider-followers do not realize the insider's real cost)

- $a-(n-m-2)q_O^{f,B}(c)-Q_O^{l,B}(c)-q_I^{f,B}(c_i)-c_i-q_I^{f,B}(c_i)=0$ (insider know his own cost level)
- $a-(n-m-2)q_O^{f,B}(c)-Q_O^{l,B}(c)-q_I^{f,B}(c)-c-q_I^{f,B}(c)=0$ (insider does not know his own cost level)

The expression of followers' outputs can be found

$$q_O^{f,B}(c)=\frac{(a-c)-Q^{l,B}(c)}{(n-m)}$$
$$q_I^{f,B}(c_i)=\frac{2(a-c)-(n-m)(c_i-c)+2Q^{l,B}(c)}{2(n-m)}$$
$$q_I^{f,B}(c)=\frac{(a-c)-Q^{l,B}(c)}{(n-m)}$$

and then, plugging them into leader's profit function:

$$\max_{q_O^{l,B}}\pi_O^{l,B}=(p^B-c)q_O^{l,B}=[a-c-(n-m-2)q_O^{f,B}(c)-q_I^{f,B}(c)-Q_O^{l,B}(c)]q_O^{l,B}(c)$$

It is easy to calculate the leader output level:

$$q_O^{l,B}(c)=\frac{a-c}{m+1}$$

Put the expression of q^l into the output for followers, we obtain

$$q_O^{f,B}(c)=\frac{(a-c)}{(m+1)(n-m)}$$
$$q_I^{f,B}(c_i)=\frac{2(a-c)-(m+1)(n-m)(c_i-c)}{2(m+1)(n-m)}$$
$$q_I^{f,B}(c)=\frac{(a-c)}{(m+1)(n-m)}$$

The equilibrium values in terms of price, profit, consumer surplus and social welfare, are displayed in Table 1. The other cases (case C and case D) can be resolved by the similar method.

E Merged firm's profit under complete and perfect information ($\hat{\pi}_I^{j,i}$)

$$\hat{\pi}_I^{l,A}=\frac{\Big[(a-2c+c_i)+(c-c_i)\big[(m-1)n-(m-2)m\big]\Big]^2}{m^2(n-m+1)}$$
$$\hat{\pi}_I^{f,B}=\frac{\big[a-2c+c_i+(c_i-c)(n-m)(m+1)\big]^2}{(n-m)^2(m+1)^2}$$
$$\hat{\pi}_I^{l,C}=\frac{\Big[(a-2c+c_i)+(c-c_i)\big[m(n-m)+(n-2m)\big]\Big]^2}{(m+2)^2(n-m-1)}$$
$$\hat{\pi}_I^{l,D}=\frac{\big[a-c+m(c-c_i)\big]\big[(a-2c+c_i)+(c-c_i)(n-m)(m+1)\big]}{(n-m)(m+1)^2}$$

See also in Lepape and Zhao (2010)

F $\hat{\delta}^i_{sup}$ and $\hat{\delta}^i_{inf}$

$$\hat{\delta}^A_{inf} = -\frac{a-c}{n-m+1} \qquad \hat{\delta}^A_{sup} = \frac{(a-c)[1-m(\sqrt{2}-1)]}{(m^2-1)(n-m+1)}$$

$$\hat{\delta}^B_{inf} = -(a-c) \qquad \hat{\delta}^B_{sup} = \frac{a-c}{(n-m)(m+1)-1} - \frac{\sqrt{2}(a-c)(n-m)}{m^3-m^2n+mn(n-1)+n^2-1}$$

$$\hat{\delta}^C_{inf} = -\frac{a-c}{n-m-1} \qquad \hat{\delta}^C_{sup} = \frac{a-c}{(m+1)(n-m-1)} - \frac{(a-c)(m+2)}{(m+1)^2(n-m+1)}\frac{1}{\sqrt{n-m-1}}$$

$$\hat{\delta}^D_{inf} = -\frac{a-c}{n-m} \qquad \hat{\delta}^D_{sup} = \frac{a-c}{m(n-m)} - \frac{(a-c)}{m(n-m+1)}\sqrt{\frac{n-m+2}{n-m}}$$

G δ^i_{Wsup}

$$\delta^A_{Wsup} = -\frac{2\left(a(-3+2m-2n)+c(3-2m+2n)+m\left(3+4m^2+7n+4n^2-m(7+8n)\right)\sqrt{\frac{(a-c)^2\left(4m^4-4m^3(1+2n)+8m\left(1+3n+n^2\right)+4\left(3+4n+n^2\right)+m^2\left(-19-4n+4n^2\right)\right)}{m^2(1+m)^2\left(3+4m^2+7n+4n^2-m(7+8n)\right)^2}}\right)}{m\left(3+4m^2+7n+4n^2-m(7+8n)\right)}$$

$$\delta^C_{Wsup} = 2\left(\frac{(a-c)(2n-2m-1)}{(2+m)\left(5+4m^2+m(9-8n)-9n+4n^2\right)}\right) - 2\sqrt{\frac{(a-c)^2\left(4m^5-12m^4n+m^3\left(17-8n+12n^2\right)+m\left(60-68n+4n^2-8n^3\right)+m^2\left(65-17n+16n^2-4n^3\right)+4\left(4-12n+n^2-n^3\right)\right)}{\left(2+3m+m^2\right)^2(1+m-n)\left(5-m-4m^2+n+8mn-4n^2\right)^2}}$$

$$\delta^D_{Wsup} = 2\left(\frac{c(-1+2m-2n)}{(1+m)\left(m+4m^2-8mn+n(-1+4n)\right)} + \frac{a(1-2m+2n)}{(1+m)\left(m+4m^2-8mn+n(-1+4n)\right)} - \sqrt{\frac{(a-c)^2\left(-8+4m^3-21n-12n^2-4n^3-12m^2(1+n)+3m\left(7+8n+4n^2\right)\right)}{(1+m)^2(m-n)\left(1-4m^2-3n-4n^2+m(3+8n)\right)^2}}\right)$$

H Proofs

H.1 The proof of Proposition 1

$$\begin{cases} \sigma^2_{\pi A} > \sigma^2_{\pi B} > 0, & \text{when } \frac{n}{2} < m \leq n-3; \\ \sigma^2_{\pi B} > \sigma^2_{\pi A} > 0, & \text{when } 3 \leq m < \frac{n}{2}. \end{cases} \qquad \blacksquare$$

H.2 The proof of Proposition 2

- $\pi^{l,A}_I < \hat{\pi}^{l,A}_I$ and $\pi^{f,B}_I < \hat{\pi}^{f,B}_I$
- $\pi^{l,C}_I > \hat{\pi}^{l,C}_I$ and $\pi^{l,D}_I > \hat{\pi}^{l,D}_I$ $\blacksquare$

H.3 The proof of Lemma 1

Case A:	$\delta^A_{sup} < \hat{\delta}^A_{sup} < 0$	$0 > \delta^A_{inf} > \hat{\delta}^A_{inf}$
Case B:	$\delta^B_{sup} < \hat{\delta}^B_{sup} < 0$	$\nexists$
Case C:	$\delta^C_{sup} > \hat{\delta}^C_{sup} > 0$	$0 > \delta^C_{inf} > \hat{\delta}^C_{inf}$
Case D:	$\delta^D_{sup} > \hat{\delta}^D_{sup} > 0$	$0 > \delta^D_{inf} > \hat{\delta}^D_{inf}$ ■

H.4 The proof of Proposition 3

(a). *In the case of merger between leaders, the magnitude of variance guaranteeing the incentives to merge ensures the enhancement of social welfare without ambiguity.* $\sigma^2_{\pi_A} > \sigma^2_{W_A} > 0$.

(b). If the market size is sufficiently large ($a > c + \Phi$), the magnitude of variance guaranteeing the private incentive ensures the welfare enhancement. $\sigma^2_{\pi_B} > \sigma^2_{W_B} > 0$ when $n > 6, m \in [3, n-3), a > \Phi + c$; otherwise, $\sigma^2_{W_B} > \sigma^2_{\pi_B} > 0$.

(c). *When two followers result in a newly merged firm behaving as leader (case C), or when the merger is composed of one leader and one follower (case D), uncertainty should be greater than the critical value ($\sigma^2_{W_C}$ or $\sigma^2_{W_D}$) to guarantee the enhancement of welfare respectively.* $\sigma^2_{W_C} > 0$ $(\nexists\ \sigma^2_{\pi_C})$ or $\sigma^2_{W_D} > 0$ $(\nexists\ \sigma^2_{\pi_D})$.

See also Table 6. ■.

H.5 The proof of Corollary 1

(a). *Profitable merger between leaders requires more uncertainty to guarantee the enhancement of consumer surplus compared to the welfare criterion, i.e.*
$\sigma^2_{CS_A} > \sigma^2_{\pi_A} > \sigma^2_{W_A}$ with $\sigma^2_{CS_A} = \frac{4(a-c)^2(2mn+2m^2n-2m^3-1)}{m^2(m+1)^2(n-m+1)^2}$.

(b). *In the case of the merger between followers without role redistribution, the variance guaranteeing the consumer surplus enhancement ensures the welfare improvement and the private incentive to merge, when the market size is sufficiently large, i.e.*
$\sigma^2_{CS_B} > \max\{\sigma^2_{\pi_B}, \sigma^2_{W_B}\}$ if $a > \Phi + c$ *with* $\sigma^2_{CS_B} = \frac{4(a-c)^2\{2(n-m)[n(m+1)-m^2]-1\}}{(m+1)^2(n-m)^2(n-m+1)^2}$.

(c). *In the case of merged leader firm composed of two followers, when there are enough active firms in market where the proportion of leaders is smaller than followers, the required uncertainty guaranteeing welfare enhancement covers with the one guaranteeing consumer surplus; otherwise, the reverse outcome appears, i.e.*

$$\begin{cases} \sigma^2_{W_C} > \sigma^2_{CS_C} & \text{if } n > 12,\ 3 \le m < \frac{n}{3} - 1 \\ \sigma^2_{CS_C} > \sigma^2_{W_C} & \text{otherwise} \end{cases}$$

with $\sigma^2_{CS_C} = \frac{4(a-c)^2(3m-n+3)\{2(m+1)(m+2)n^2-2mn[2m(m+3)+5]+m[2m(m+1)(m+2)-3]-3(n+1)\}}{(m+1)^2(m+2)^2[(n-m)^2-1]^2}$

(d). *When the merger is composed of one leader and one follower, the uncertainty guaranteeing consumer surplus improvement ensures the one guaranteeing welfare enhancement without ambiguity, i.e.*
$\sigma^2_{CS_D} > \sigma^2_{W_D}$ with $\sigma^2_{CS_D} = \frac{4(a-c)^2\{2(n-m)[n(m+1)-m^2]-1\}}{(m+1)^2(n-m)^2(n-m+1)^2}$. ■

H.6 The proof of Proposition 4

Case A: $\delta^A_{sup} < \delta^A_{W_{sup}} < 0$

Case B: Complicated (depending upon numerous parameters such as the market size "a", the marginal cost "c", the numbers of leaders and followers "n" and "m", *etc.*)

Case C: $0 < \delta^C_{W_{sup}} < \delta^C_{sup}$, if $n > 12$ and $m \in [3, \frac{n}{3} - 1)$

$\delta^C_{W_{sup}} < 0 < \delta^C_{sup}$, otherwise

Case D: $\delta^D_{W_{sup}} < 0 < \delta^D_{sup}$ ■

H.7 The proof of Corollary 2

(a). In the case of two-leader merger, (1).when there are three or four leaders in the pre-merger market, the profitable merger always improves the consumer surplus, but possibly damages the social welfare; (2). when there are more than four leaders in the market, the profitable merger is unambiguously welfare-enhancing and consumer-surplus-improving.

$$\begin{cases} \delta^A_{CS_{sup}} < \delta^A_{sup} < \delta^A_{W_{sup}} < 0 & \text{if } m = 3 \textit{ or } 4 \\ \delta^A_{sup} < \delta^A_{CS_{sup}} < \delta^A_{W_{sup}} < 0 & \text{if } m \geq 5 \end{cases}$$

$$\textit{with } \delta^A_{CS_{sup}} = \frac{-2(a-c)}{m(m+1)(n-m+1)}.$$

(b). In the case of two-follower merger with role redistribution, when there are sufficiently less leader firms in the market, the profitable merger generating efficiency losses can improve both consumer and aggregate welfare, and the welfare-enhancing merger ensures the improvement of consumer surplus.

$$\begin{cases} 0 < \delta^C_{W_{sup}} < \delta^C_{CS_{sup}} < \delta^C_{sup} & \text{if } n > 12,\ m \in [3, \frac{n}{3} - 1) \\ \delta^C_{CS_{sup}} < \delta^C_{W_{sup}} < 0 < \delta^C_{sup} & \text{otherwise} \end{cases}$$

$$\textit{with } \delta^C_{CS_{sup}} = \frac{2(a-c)(n-3m-3)}{(m+1)(m+2)[(n-m)^2-1]}.$$

(c). When the merger is composed of one leader and one follower, the merger which improves consumer surplus can damage the social welfare. $\delta^D_{CS_{sup}} < \delta^D_{W_{sup}} < 0 < \delta^D_{sup}$ *with* $\delta^D_{CS_{sup}} = \frac{-2(a-c)}{(m+1)(n-m)(n-m+1)}$
. ■