

Dear Editor,

I thank the reviewers for their comments and discuss the raised issues separately.

Review for Economics 1073 R2:

1. The reviewer suggested that the case of the distribution of firm sizes is similar to the discussed bank size distribution and it may be useful to discuss some of this work more and explain how the dynamics of the banking system differs from that of other firms.

I completely agree with the reviewer's statement. As a matter of fact the presented paper of the bank size distribution is based on an equivalent approach on firm sizes (see reference Kaldasch 2012, Evolutionary model of the growth and size of firms). However, there is a key difference between firms and banks. Banks can create money, while firms can just collect money in the reproduction process. Therefore the size of firms should be counted in terms of a flow quantity (usually sales in units or revenue<sup>1</sup>) while the size of banks can be given by a stock quantity (deposits). The key idea of the derivation of the size distribution of firms as well as banks is competition. While banks compete for permanent deposits, firms compete for sales. It is in both cases the origin of Gibrat's law. Both suffer from a size dependent contribution of the growth process (economies of scale) and generate therefore a similar size distribution. The reviewer mentioned the paper by Gabaix (Power Laws in Economics and Finance). Here other samples of power law distributions are discussed. So for instance the city size distribution can be modelled with similar relations on an evolutionary basis. While the competition between banks is for deposits, the competition between cities is for migrants (not further outlined here). As recognized by the reviewer the relations established in the presented model for the bank size distribution are most general. It can be expected that equivalent distributions found in other social systems can be explained analogous to the presented model by a competition and a small mean growth component. I have discussed the importance of competition in the conclusions including the suggested references.

2. The second critic concerns the velocities of the processes, similar to the first critic of reviewer R1.

As outlined above the velocities of the processes are in fact not relevant. The key condition for the derivation of the size distribution is that the mean growth rate is small compared to fluctuations. The paper is completely rewritten as a result of this critic (see also reviewer 1).

3. The reviewer is correct. Since  $\tilde{\sigma} \neq 1$  the mean fitness have to be scaled by  $\tilde{\sigma}$ .
4. The reviewer has correctly emphasized that fitness fluctuations are not necessarily independent.

Fitness fluctuations are approximated in this model as independent events. Indeed a variation of the fitness of a large bank has, via a change of the mean fitness, also an impact on the fitness of all other banks. This effect comes in particular into play due to the uneven size distribution. However, while correlations between banks may occur within short time intervals they are considered to disappear sufficiently fast on the considered time scale, such that interactions between banks can be neglected.

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<sup>1</sup> Note that the number of workers is also often used. But this number is a function of the sales since they have to be paid from the revenue.

The model is confined to a polypoly banking system. If the banking system approaches a monopoly market, however, this approximation may be invalid. A discussion of this issue goes beyond the ability of the presented model but is a subject of further research.

5. and 6. What is the meaning of  $M$  and why is  $s_i < 1$ ?

In order to give  $M$  a meaning we introduce a long time interval over which the evolution of the sized distribution is studied. Further we set  $M$  equivalent to the total deposits at the end of this time interval. So the  $s_i$  are simply the deposit shares at the last time step  $t_1$ , such that the constraint  $\tilde{s}(t) \leq 1$  is always fulfilled. Since the total deposits are time dependent, it is not useful to scale by  $\tilde{s}$ . The scaling by  $M$  is used to establish a continuous theory.

7. What is the time evolution of  $n(t)$ ?

As long as restructuring processes preserve the total amount of deposit money, the actual number of banks is irrelevant for the bank size distribution at a given time step<sup>2</sup>. This is because a probability density function determines just the relative abundance of a quantity, not its absolute value. However, for the evolution of the size distribution we have to take restructuring processes ( e.g. mergers) into account, since they change the deposits of a bank. This is done in this statistical approach by the growth rate  $\beta(t)$ . This growth rate can be separated into a mean growth rate and growth rate fluctuations  $\delta\beta(t)$ . Since a restructuring is usually determined by distinct events with large changes of the total deposits of a bank, the fluctuating part can be expected to dominate the time evolution of  $\beta(t)$ . The presented model applies under the condition that restructuring events are not correlated. i.e. variations of the number of banks occur at random time steps. The magnitude of the corresponding growth rate fluctuation determines the noise amplitude  $D$ . The case of a collapse of the banking system due to couplings between banks is excluded. So the exact evolution of  $n(t)$  is not known, but the evolution of the number of banks is implicitly contained in the evolution of the deposits.

8. The reviewer suggested that  $\langle \beta \rangle \tilde{s} = 0$ ,  $\langle \eta \rangle \tilde{s} = 0$  and  $\sum_{i=1}^n y_i(t) = 0$ .

The point is that the processes indicated in the text by ii),iii), iv) are exchange processes. They keep the total amount of deposits constant. Therefore only the relation Eq.(9) is always valid, suggesting:

$$\sum_{i=1}^n (\beta_i(t)s_i(t) + \eta_i(t)s_i(t) + y_i(t)) = \langle \beta \rangle \tilde{s} + \langle \eta \rangle \tilde{s} + \sum_{i=1}^n y_i(t) = 0$$

An additional assumption of the model concerns the fast exchange of money caused by economic activities (e.g. due to the purchase of goods, receiving wages etc.) . Under the condition that this activity is independent of the bank<sup>3</sup> and the exchange of money due to this process is random we obtain that the inflow and outflow of money cancel out, except of fluctuations  $\zeta(t)$ . Taking the time

<sup>2</sup> There must be of course enough banks to allow the application of statistical methods.

<sup>3</sup> That means that the banks are not too specialized.

average the fluctuations disappear leading indeed to  $\langle y_i \rangle_t = 0$ . However, this is a consequence of the assumption to treat the economic activity as random<sup>4</sup>.

The paper is adjusted with respect to the issues raised by the reviewers. Best regards,

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<sup>4</sup> Suppose economic activity is not random, but could be governed by a bank. In this case, the fast in- and outflow of money do not cancel out and economic activity becomes a contribution in the bank fitness. However, because banks can hardly influence money exchange by economic activities (e.g. where goods are purchased) , this contribution can be separately investigated as performed in this model.