Authors' Response to Referee Report 1

We would like to thank the Referee for providing us with an opportunity to clarify a number of points in our manuscript. We address the Referee's comments in detail below. The numbering of comments is ours, for convenience.

> 1. The contribution of the paper is not sufficient to be published in the journal. The paper only focuses on numerical points and ignores discussions and insights in terms of economics.

We would like to point out that the paper's main focus is both conceptual and numerical. The main conceptual idea is that an *explicit* (*analytical*) Taylor-like rule can be developed in the presence of a ZLB on the interest rate, using a *parametric programming* approach. This approach has never been proposed before, it is independent of the specific model used for development of Taylor-like rules, and is valid regardless of the specific numerical values of the key variables involved (output gap and inflation). Moreover, as our calculations presented in the paper indicate, the proposed approach provides qualitative conclusions as to when clipping a negative interest rate produced by the standard Taylor-rule is optimal and when it is not. Finally, the approach we propose (namely using a combination of *parametric programming* and *model predictive control*) can certainly be the basis for many additional studies relying on different models or control objectives of varying complexities. Therefore, we believe that insight in terms of economics can be gained both from the work presented in this paper and from future work that may rely on the approach/framework proposed.

We admit that the paper is lengthy, and key insight may have been difficult to single out. We will make an effort to indicate such insight more clearly in the revised version.

> 2. Moreover, some conclusions are misleading. For example, the authors need to think of role of Taylor principle in back-ward looking model rather than in forward-looking model. I think that Taylor principle works well in forward-looking model. But I am not sure whether it works well in back-ward looking model. The authors at least should clarify this point.

The Taylor principle is a feedback rule. Any feedback rule relies on information about the controlled variables (output gap and inflation) available up to the point the decision is made and makes a decision for adjustment of the manipulated variable (interest rate) based on what the effects of such adjustment would be. The particular calculation for adjustment of the manipulated variable is performed at each time point; this is what is called the feedback law. A feedback law can range from a very simple calculation (e.g., adjustment proportional to the deviation of the controlled variable from its desired value) to elaborate real-time optimization. Taylor's rule is a simple and powerful feedback law, which simply uses the current values of the controlled variables (output gap and inflation) to determine the

value of the manipulated variable (interest rate). It turns out that this feedback law is equivalent to (the result of) real-time optimization of a form of a quadratic objective using a linear model for the effect of the manipulated variable on the controlled variables, <u>in the absence of inequality constraints (such as ZLB)</u>. Variants of the Taylor rule (e.g. with inertia) can be easily derived if the objective function of the real-time optimization is modified accordingly. To a certain extent, as we point out in the paper, these facts were discussed in previous publications. However, <u>no explicit Taylor-like rule had been presented in the presence of ZLB</u> (some numerical studies that addressed the issue are cited in our paper).

We presume that the Referee refers to forward and backward looking models in association with rational expectations and how these affect current decisions. But rational expectations into the future inevitably rely on information <u>up to the present point.</u> A model that substitutes such expectations in terms of information available up to the present point will be a <u>standard causal model</u>. It is such a model that we have used in our work. While models can be written that show how future expectations affect current decisions, <u>such expectations should not be confused with future reality.</u> The causality principle, cornerstone of all science, is that the future depends on the past; what one believes the future will be depends on what one knows for the past and present. Of course, the accuracy of any model is not infinite, and the interplay between model uncertainty and a feedback law is always worth studying.

3. Also, I cannot understand why new penalty terms are necessary to think of the optimal rule under ZLB of the nominal interest rate in the class of Taylor like rules. The authors at least should provide reasons for it.

The penalty terms are used to indicate that the proposed approach is general. Furthermore, penalty terms are well known to produce feedback rules that are stabilizing (a sine qua non of any feedback rule). Finally, there is some numerical evidence, presented in literature as well as in our paper, that recent policies followed by the US Fed follow a Taylor-like rule with inertia, which is equivalent to the feedback law emerging out of the real-time optimization with the penalty terms included.

4. Moreover, we know that rules with inertia work well in forward-looking model since the inertia plays a role of commitment device. But I am not sure whether this still works in back-ward looking model or not. The authors at least should explain this point.

Please see response to comment #2.