## Report on manuscript 968 for Economics

## "Endogenous Ranking in a Two-Sector Urn-Ball Matching Process"

by Giuseppe Rose
The main idea behind this paper is that the matching function used in search models of the labor market should be endogenous rather than exogenous. In particular, the author studies an environment in which the ranking of applicants according to their individual characteristics is not an assumption but rather an endogenous outcome of firms' strategic behavior. Both the market tightness and hence the availability of workers, and the composition of the labor force in terms of workers' abilities, are relevant determinants of whether or not applicant ranking will turn out to be an endogenous outcome.

Overall evaluation:
The question this paper addresses is certainly a significant one. There has not been enough research on this question and the assumption of an exogenous matching process is all too common in the search and matching literature. I would encourage the author to keep working on this research question as I believe it is a very important one with significant policy implications.

On the positive side, this paper is at least a starting point for a contribution to this important question. Unfortunately, however, while the idea is promising the execution of it is not successful or convincing. The paper is not well written, the authors' arguments are unclear and lacking in precision, and there are some clear errors. I don't believe it is publishable in its current form and it would require some fairly extensive improvements to bring it to a publishable standard.

## Specific comments:

The author first introduces the urn-ball matching function and then says that this has been "enriched in two main directions". He then cites Albrecht, Gautier, and Vroman's 2003 paper on matching with multiple applications. The author does not however cite Albrecht, Gautier and Vroman's 2006 ReStud paper on directed search with multiple applications. This is quite puzzling for two reasons. First, why not include a discussion of AGV's 2006 paper? Second, there is an entire literature (discussed in AGV's 2006 paper) on the basic idea behind directed search that the urn-ball matching function can be seen as endogenous outcome of the strategic behavior of either workers or firms about how to direct their search behavior. The author should discuss this prior literature. I would have thought that papers like Burdett, Shi and Wright (JPE 2001), and Julien, Kennes and King (RED 2000), in particular, as well as other directed search papers, should be mentioned here.

I am doubtful that it is correct to use a continuous time setting here as directed search does not readily carry over into the continuous time environment - perhaps the author should use discrete time?

I believe that Equation (12) on the top of page 11 is incorrect. It should be multiplied by $\lambda$.

I believe that the proof in the Appendix is incorrect. Here the author wishes to prove that the right term in (34) is less than the right term in (35). The author tries to prove this by contradiction. However, the use of a first-order Taylor series approximation can be problematic when proving an inequality unless one is sure of whether the first term in the Taylor series is less than or greater than the entire sum. Unless I am missing something (which is possible since it is very unclear), the step the author makes in (37) would only be valid if we had $\log (1-y) \geq-y$, which is not true. So the jump to (37) is not valid as far as I can see, and hence neither is the conclusion.

It is best to avoid Taylor series and proof by contradiction altogether here. One can attempt to prove the desired inequality directly by considering the function $f(\chi)=1-e^{-\lambda(1-\chi)}-\lambda e^{-\lambda}$. The desired inequality is equivalent to $f(\chi) \geq 0$. It can be shown that $f(0)>0$ for all $\lambda$ but $f(1)<0$. Since $f^{\prime}(\chi)<0$, there exists a unique critical value $\chi_{c}$ such that $f(\chi) \geq 0$ whenever $\chi \leq \chi_{c}$ and $f(\chi)<0$ otherwise. I am not sure how this change in the result would affect Proposition 2.

In general, the English language and the clarity of the writing, as well as the degree of precision in the use of terminology, need to be greatly improved. Some examples are found below.

Minor comments:
It might be better to say "skilled and unskilled" rather than "graduate and undergraduate". I think this language would be nicer stylistically as well as potentially fitting in better with the literature on skilled/unskilled labor (and Gavrel's paper).
$\theta$ is a poor notational choice for the ability of a worker, since it is often used to represent the labor market tightness ratio in the search literature. It might be better to just use $x$ for worker ability.

The production function taken from Acemoglu (1997) is simply stated but no justification is given.

I don't find the talk about "distance to the technological frontier" useful and I am not sure that it has been precisely defined - it may be better to omit mention of this.

There is continual reference to the "probability" when it is strictly speaking the density function.

Abstract. Debate about micro-foundations of matching functions. Which debate? I think the author needs to be more specific. What exactly is being debated? What exactly is the question?
"MF" abbreviation for matching function is not standard or appropriate
p1. First paragraph. "a flourishing of studies" but only mentions one. Which studies exactly? My guess is rather that there has been a neglect of the question of micro-founding the matching function relative to its importance, not a "flourishing of studies". I'd like to see the references.
p1. First paragraph. Why mention the Lucas critique here? It is distracting. It's enough to state simply that the endogeneity of the matching function could have a big impact on policy prescriptions.
p1. Second paragraph. "according to Gavrel and Moen". Should this be "following"?
p3. Firms maximize their "expected actual value"? There is continual reference to "actual expected values" (?) throughout the paper. This is non-standard and imprecise language.
p4. "evaluates the equilibria discussing the endogeneity"?
p5. The discussion seems to suggest that Blanchard and Diamond (1994) involves ranking applications by productivity, when in fact they are ranked by duration of unemployment.

P5. The "choice between eqs. (1) and (3) should be solved endogenously"? How can we choose between equations? Choice by whom?

P6 onwards. Why is the word "continuum" always italicized?

P6. Workers and firms are simply stated to be "matching in the labor market following the lines set out by Diamond-Mortensen-Pissarides". This does not make sense; it needs to be spelled out - especially given that the matching environment is actually a non-standard one and not just random bilateral matching in any case.

P6. Reference to "these agents" when previous sentence mentioned both firms and workers. Is this meant to be workers? Again, it needs to be clear.

P6. Should this be "following" Moen (1999), not "according to Moen?

P 7. It is okay to write $T=g$, but not $T=\{g, u g\}$. The choices $e$ and $T$ are elements of the set $\{g, u g\}$.

P8-9. The " $\forall T$ " quantifier notation is unnecessary and doesn't fit with the style of the paper. Just say "a vacancy of either type".

P 13. "In order to set the equilibrium of the model"?
P 13-14. Expected value of vacancy given $\theta$ ? - isn't adequately explained
Conclusion. "Debate concerning endogeneity of matching functions". Again, it is not really a debate, but rather an attempt to provide foundations for the matching function by endogenizing it. As far as I know, there is no single feature that is being "debated" in particular, so I don't think the word "debate" is the best one to describe the current situation regarding this research area. Rather, I think there has been a lack of research or interest in this particular topic relative to its importance. The author might be better off emphasizing that instead as a motivation.

P25. Appendix. It doesn't make sense to say that "eq. (34) is always less than eq. (35)". An equation can't be less than another equation. A term in an equation can be less than another term.

