Indirect Taxation of Monopolists: A Tax on Price.[1]

Henrik Vetter

Statsbiblioteket

Universitetsparken DK-8000 Aarhus C

Abstract: A digressive tax like a variable rate sales tax or a tax on price gives firms an incentive for expanding output. Thus, unlike unit and ad valorem taxes which amplify the harm from monopoly, a digressive tax lessens the harm. We analyse a tax on price with respect to efficiency and practical policy appeal. In particular, we show how tax reforms based only on observation of price and quantity can make use of a tax on price in order to improve welfare. That is, it is practical to use a tax on price. The argument extends to fixed number homogenous oligopoly.

Jel. No. H210, L130

Keywords: tax on price, ad valorem tax, tax incidence

[1] This work is not the result of a for-pay consulting relationship neither does any party have a financial interest in the results.

1. Introduction.

According to their widespread use in public finance unit and ad valorem taxes are popular taxes. For this reason there is a clear interest in analysing the workings of precisely these two taxes in spite of the fact that there are, as noted by Hamilton (2009), many other instruments. On the other hand, it is well known that both taxes are shifted into the price consumers pay. That is, when used in a monopolised market, or other imperfectly competitive markets, these taxes drive the price even further above the marginal cost. On this basis it is of interest to look at the efficiency of digressive taxes, and, moreover, if the application of such taxes are equally practical to unit and ad valorem taxes.

Digressive taxes provide the opposite incentive to unit and ad valorem taxes; hence, under such taxes firms expand output. When applied in imperfectly competitive markets the margin between price and marginal cost narrows under such taxes (Dalton, 1929[2] and Robinson, 1933). A tax on price is an example of a digressive tax. In this paper we study the workings of such a tax in monopoly, and we briefly discuss its extension to oligopoly. As a practical matter it is possible to implement a tax on price as variable rate sales tax (Hamilton, 1999). There are other ways to introduce digressive tax schemes. Assuming that the marginal cost is non-decreasing a tax based on the Lerner index will do. When the tax relates positively to the index the marginal tax is decreasing and, in turn, gives the monopolists incentive to expand output. A tax scheme based on the difference between price and average cost has similar effects.

[2] See Sumner (1993) for details.

The idea that a tax on price counteracts monopolistic behaviour and, at the same time, secures revenue is discussed in Shilling (1969). Subsequently Tam (1991) shows some results on the workings of a tax on price but, as argued by Sumner (1993) it is unclear how a tax on price relates to welfare.[3] It is also unclear whether it is practical to use a tax on price. The purposes of this paper are twofold. First, we want to see how far the efficiency of a tax on price goes. In the absence of lump sum taxes Ramsey pricing gives the most efficient allocation that can be reached subject to some restriction on tax revenue. That is, the Ramsey price is the price that maximises social welfare subject to a restriction on monopoly profits. Thus, the proper way to ask about efficiency is to analyse the relationship between the allocation under Ramsey pricing and under a combination of a tax on price and ad valorem taxation, respectively.[4]

Second, and more importantly for practical matters, an objection against digressive taxes is that they are impractical because of the information needed in order to use them. If information difficulties make it impractical to apply the theoretically ideal tax structure it is relevant to ask when a practical reform of existing taxes results in a gain. To demonstrate this way of reasoning, consider excise *versus* ad valorem taxes. A practical reform is a matched-pair tax reform; that is, an increase in the ad valorem tax rate that matches the decrease in the excise tax measured at before-reform price. This kind of reform does not require knowledge about demand and cost conditions. Based on first-round effects ad valorem taxes are better than unit taxes. What Suits and Musgrave (1953) show, is that inclusion of second-round effects will not change the dominance of ad valorem over excise

[3] Tam (1991) analyses a tax on price. However, as pointed out by Sumner (1993) the neglect of the restriction that profit is non-negative results in shortcomings and some results are (according to Sumner, 1993) plainly wrong. Neither of these papers related the tax on price to Ramsey pricing, and neither of the papers considers how to practically conduct tax reforms.

[4] Sumner (1993) and Tam (1993) use some numerical calculations to discuss this.

taxes. By parallel reasoning, we ask about a practical welfare improving tax reform based on a tax on price to see if such reform in fact is stopped by information constraints.[5]

In Section 2 we make the point in general and discuss practically feasible tax reform. In Section 3 we analyse an example to demonstrate the more general results.[6] Section 4 concludes.

2. A Tax on Price

There are two ways to compare taxes: either to find a set of taxes which will result in the same final price and output and inspect the resulting revenues, or to find a set of taxes that produce equal revenue, and then to compare prices and outputs. We apply the latter comparison for the obvious reason that ad valorem taxation drives the price above the pure monopoly price and the tax on price drives the price below the pure monopoly price. Also, we analyse marginal tax changes. This follows the approach set forth by Delipalla and Keen (1992). The route taken originally in Suits and Musgrave (1953) is to assume that one tax tool fully replaces another tax tool. In our case, that a tax on price fully replaces ad valorem taxation. In the example in the next section we analyse the two taxes this way.

We consider a monopolist who acts in a market where demand is given by $p = p(x)$. The demand function is downward sloping so that $p_x(x)$ is negative (subscripts are used to denote partial

[5] Sumner (1993) discusses product quality as a source to information problems although he does not address the issue formally.

[6] In fact, the example used in Section 2 is the same as that used by Sumner (1993). However, we explicitly compare a tax on price to an ad valorem tax which allows us to demonstrate that there will not always exist a tax on price that can match the revenue produced by some ad valorem tax rates.

derivatives). Under a combination of an ad valorem tax with tax rate τ and a tax on price with tax rate s the monopolist's profit is $\pi = (1-\tau)p(x)x - c(x) - sp(x)$. The first- and second-order conditions are $(1-\tau)\big(p_x(x)x + p(x)\big) - c_x(x) - sp_x(x) = 0$, and $\theta = (1-\tau)(2p_x(x) + 2p_{xx}(x)x) - c_{xx}(x) - sp_{xx}(x) < 0$, respectively. It is easy to see that $x_\tau = \theta^{-1}\big(p_x(x)x + p(x)\big) < 0$ and $x_s = \theta^{-1}p_x(x) > 0$, showing that the monopolist's output decreases with an increase of the ad valorem tax rate whilst it increases as the rate of tax on price goes up. Plainly, an increase of the ad valorem tax rate reduces marginal revenue which explains why output goes down. Although an increase of the rate of tax on price shifts net demand inwards, marginal revenue increases, explaining why this tax expands output. It is easy to see how the positive effect on output is derived. Selling one more unit of output the monopolist must lower the price not just on the last unit sold but also on all other units sold. Under a tax on price there is a lessening of the tax burden since setting a lower price applies to all units sold. Hence marginal revenue under a tax on price is $p(x) + p_x(x)(x - s)$, which is why this tax has opposite effects from the ad valorem tax.

Consider combinations of tax rates with a fixed yield of $\hat{R} = \tau p(x)x + sp(x)$, where output and price are functions of the tax rates (but for brevity written without the arguments). Unsurprisingly, we restrict attention to ad valorem tax rates for which $0 \leq \tau \leq 1$. The tax on price must satisfy $(1-\tau)p(x)x - c(x) - sp(x) > 0$ since the monopolist closes down otherwise. Throughout we maintain the next assumption to characterise feasible tax plans.

Assumption 1. There is a pair of tax rates, $\{\tau_0, s_0\}$ so that $(1-\tau)p(x)x - c(x) - sp(x) > 0$ for $0 \leq \tau \leq \tau_0$ and $s \leq s_0$.

Unless Assumption 1 is satisfied it clearly does not make sense to compare the two taxes. When the monopolist's output is strictly positive in the absence of taxes it follows from continuity that Assumption 1 is satisfied for small tax rates. We maintain the following assumption about the marginal tax revenue of the ad valorem tax.

Assumption 2. There is a pair of tax rates, $\{\tau_1, s_1\}$ so that R_τ is positive and decreasing with τ for $\tau \leq \tau_1$ and $s \leq s_1$.

With respect to Assumption 2 we can express the condition that R_τ is positive in terms of conditions on the demand function and, additionally, show that R_τ is positive with certainty under mild conditions.[7] Since $R_\tau = p(x)x + \big(\tau p_x(x)x + sp_x(x) + \tau p(x)\big)x_\tau$ it follows from the first order condition for profit maximisation that $R_\tau = p(x)x + \big(p_x(x)x + p(x) - c_x(x)\big)x_\tau$. In turn, using $x_\tau = \theta^{-1}\big(p_x(x)x + p(x)\big)$ we have:

$$R_\tau = p(x)x + \big(p_x(x)x + p(x) - c_x(x)\big)\theta^{-1}\big(p_x(x)x + p(x)\big). \tag{1}$$

Now, define $E \equiv xp_x(x)^{-1}p_{xx}(x)$ and $k \equiv 1 - p_x(x)^{-1}c_{xx}(x)$. Seade (1980) demonstrates that entry into an oligopolistic industry reduces output of incumbent oligopolists when $E + n + k > 0$, where $n \geq 1$ is the number of firms. In our context $n = 1$ and the term θ is rewritten as:

$$\theta = (1-\tau)p_x(x)\big(E + 1 + k + (1-\tau)^{-1}(1-k)\big). \tag{2}$$

[7] I owe this idea to a referee.

The condition set forth by Seade (1980, equation (6)) is thus that $E + 1 + k$ is strictly positive. Now, using equation (2) in equation (1) the condition that R_τ is positive can be written as:

$$\eta_p (1 + \eta_p)^{-1} < -(E + 1 + k)^{-1} (1 + \eta_p - c_x(x) p(x)^{-1}), \tag{3}$$

where $\eta_p = dp/dx \cdot x/p$. Now, when $\tau \to 0$ and $s \to 0$ it follows that $1 + \eta_p \to c_x(x)p(x)^{-1}$ because the monopolist is maximising profit. Since the denominator on the right hand side is strictly positive the right hand side is vanishing so that the condition is met. Lemma 1 summarises this.

Lemma 1. When the tax rates are sufficiently small R_τ is positive by invoking the stability condition of Seade (1980).

Assumptions 1 and 2 together with Lemma 1 make clear that $0 \le \tau \le \bar{\tau}$ where $\bar{\tau} = min(\tau_0, \tau_1)$ and $s \le \bar{s}$ where $\bar{s} = min(s_0, s_1)$ is a feasible tax policy. That is, the monopolist will not close down under the tax scheme, and, moreover, the ad valorem tax's marginal revenue is positive.

As noted, we compare tax changes that end up with the unchanged tax revenue. From $\hat{R} = \tau p(x)x + sp(x)$, denoting by $\Delta = \tau(p_x(x)x + p(x)) + sp_x(x)$, the change in tax rates satisfy

$$(px + \Delta x_\tau)d\tau + (\Delta x_s + s)ds = 0. \tag{4}$$

Clearly, a tax scheme characterised by tax rates $\tau = \bar{\tau}$ and $s = 0$ is feasible. Moreover, output is below unregulated monopoly output in the absence of a tax on price and for a positive ad valorem tax rate. This implies $\bar{\tau}\,(p_x(x)x + p(x)) > 0$ why the term Δ is strictly positive. In turn, it follows from equation (4) that a revenue-neutral reform implies more tax on price when ad valorem taxation is downsized. In fact, the revenue neutral trade-off between the two tax rates is negative whenever $\Delta > 0$, that is $\tau p_x(x)x + \tau p(x) > -sp_x(x)$.

The output effect of a revenue-neutral tax reform is $x_\tau d\tau + x_s ds$, and using equation (4):

$$x_\tau d\tau + x_s ds = (\Delta s + x_s)^{-1}(s x_\tau - px)d\tau. \tag{5}$$

Thus, $\Delta > 0$ is a sufficient condition for the right hand side of equation (5) to be positive under a tax reform involving less ad valorem taxation and more tax on price. Next, define $x_m = Argmax_x \; xp(x) - c(x)$. We have Lemma 2.[8]

Lemma 2. $\Delta \geq 0$ for $x \leq x_m$.

It follows from Lemma 2 that a tax reform introducing more tax on price and less ad valorem taxation can be revenue neutral and increase output whenever the monopolist's output under existing tax rates falls short of monopoly output in the absence of taxes. However, the tax change defined by equation (4) is feasible only if the monopolist continues production after the tax change, that is, the profit restriction $\big((1-\tau)x - s\big)p(x) - c(x) \geq 0$ is satisfied under the new tax rates. We can show Proposition 1.

[8] From the first order condition $(1-\tau)\big(p_x(x)x + p(x)\big) - c_x(x) - sp_x(x) = 0$ we have $\tau\big(p_x(x)x + p(x)\big) + sp_x(x) = p_x(x)x + p(x) - c_x(x)$ which is non-negative for $x \leq x_m$.

Proposition 1. When a pair of tax rates satisfies Assumptions 1 and 2, and if output under the pair of tax rates is less than the unregulated monopoly output, a tax reform can increase output to the unregulated monopoly output without violating the profit restriction.

Proof.

Lemma 2 makes sure that the tax revenue is unchanged and that output goes up. We have to show feasibility. That is, the profit restriction is satisfied after the tax reform. To see this write the monopolists profit as $\pi = xp(x) - c(x) - R$ where $R = (\tau x + s)p(x)$. Start out at $\tau = \bar{\tau}$ and $s = 0$. The monopolists output is clearly below x_m so that $xp(x) - c(x)$ is increasing in x. Thus, around $\tau = \bar{\tau}$ and $s = 0$ profit cannot fall if output increases and, simultaneously, the tax revenue is unchanged after the tax change. At $\tau = \bar{\tau}$ and $s = 0$ an increase in the tax on price and a corresponding decrease in the ad valorem tax rate defined by equation (4) leaves the tax revenue unchanged. Equation (5) shows that output increases. Hence, the monopolists after tax profit cannot go down.

When the monopolist's output after the first round of tax changes is less than x_m we can apply the above argument once more in as far as $\Delta \geq 0$, and when the revenue neutral trade-off between the tax rates is negative, cf. equation (4). Lemma 2 affirms that Δ is non-negative when output is less than x_m. When $\Delta \geq 0$ equation (5) shows that output goes up as the ad valorem tax rate goes down. Under Assumption 2 the marginal tax revenue of the ad valorem tax, $p(x)x + \Delta x_\tau$, is positive. Assumption 2 and $\Delta \geq 0$, together, confirm that the trade-off between the tax rates is negative.

End of proof.

Proposition 1 shows that a series of tax reforms bring output to the level the monopolist chooses in the absence of taxation (x_m). At this point the effect of further tax changes is uncertain because the monopolist's net of tax profits goes down. Trivially, shifting taxation further away from ad valorem taxation and towards unit taxation comes to a stop when $xp(x) - c(x) = R$. We summarize it as Proposition 2.

Proposition 2. When output under a pair of taxes equals output in unregulated monopoly, further tax reforms can reduce price without harming tax revenue. The possibility for tax reform stops when $\big((1-\tau)x - s\big)p(x) - c(x) = 0$.

Proof. When output under a pair of taxes equals the output produced by the unregulated monopoly we know that $\Delta = 0$. It follows from equation (4) that an increase in the tax on price and a reduction of the ad valorem tax rate is revenue neutral when $p(x)xd\tau + sds = 0$. From equation (5) output changes by $x_\tau d\tau + x_s ds = {x_s}^{-1}(sx_\tau - px)d\tau$ which is positive. Hence the price goes down (since we are on the demand curve).

When Δ is sufficiently negative $px + \Delta x_\tau$ is positive and $\Delta x_s + s$ is negative. It follows from equation (4) that an increase in the tax on price must be matched by an increase of the ad valorem tax rate according to $(px + \Delta x_\tau)d\tau + (\Delta x_s + s)ds = 0$. Using equation (5) we see that the overall effect on output, $x_\tau d\tau + x_s ds = (\Delta s + x_s)^{-1}(sx_\tau - px)d\tau$, is positive because $\Delta s + x_s$ is now negative. Hence, the price goes down because output goes up.

The tax changes are by construction tax revenue neutral. Since we push the monopolists output further and further above x_m profits exclusive of the taxes go down. Hence, profits inclusive of the taxes go down. At $\left((1-\tau)x-s\right)p(x)-c(x)=0$ or $xp(x)-c(x)=R$ the monopolist is at the point of closing down.

End of proof.

When output under taxation exceeds that of unregulated monopoly output, the rate of tax on price is driven into a region where marginal revenue is negative. This is seen immediately since $\Delta = \tau\left(p_x(x)x+p(x)\right)+sp_x(x)$ is negative from the first order condition. The reason that such reforms make sense is of course that increasing the tax on price benefits price more than the adverse price effect of the higher ad valorem taxation that is needed to keep revenue unchanged. What Propositions 1 and 2 show is that maximisation of consumers' surplus, subject to a revenue constraint, will never involve output falling below unregulated monopoly output. As a matter of fact, subject to satisfaction of the profit restriction, the tax policy sustains output exceeding unregulated monopoly output.

It is an immediate upshot of Proposition 2 that ad valorem taxes and a tax on price combine to achieve the Ramsey price. To see this suppose a regulator picks a point on the demand curve under the restriction that the price is no less than the sum of average cost and average revenue, that is, $p(x) \geq c(x)/x + R/x$. Now, if the sum of average cost and average revenue exceeds the price for all prices, there is clearly no solution: there is never enough revenue to cover production costs and simultaneously meet the revenue restriction. Figuratively, the demand curve lies below the average cost curve adjusted with average tax revenue. If the monopoly price precisely satisfies $p(x) = c(x)/x + R/x$, then the restriction on the mix of ad valorem and price tax is that output under the taxes must be equal to unregulated monopoly output, that is, the tax rates must satisfy $\Delta = 0$, evaluated at the price in unregulated monopoly. Finally, suppose the unregulated monopoly price satisfies $p(x) > c(x)/x + R/x$. In terms of tax reforms, suppose that a pair of tax rates secure the needed revenue at an output that equals unregulated monopoly output. Increasing output by increasing the tax on price (and lowering the ad valorem tax rate) reduces the price. If, the price after the reform is characterised by $p(x) \geq c(x)/x + R/x$, there is room for another round of tax reform. This goes on until the price satisfies $p(x) = c(x)/x + R/x$ which is the Ramsey price. We summarise this as a Corollary.

Corollary 1. When ad valorem taxes and a tax on price are combined to minimise price subject to a revenue restriction, the resulting price is the Ramsey price.

The Corollary is unsurprising because of the effects of the two tax instrument. One tax rotates demand whilst the other tax shifts demand.[9] Combinations of ad valorem and unit taxes can do the same only if tax rates are allowed to be negative (Myles, 1996).

Digressive taxes are only rarely discussed in the literature. They are dismissed by Dalton (1929) and Robinson (1933). Also, Glaister (1987) suggests that it is impractical to use such taxes because of

[9] I owe this observation to a referee.

the information required to design them. Hence, it is relevant to ask whether there are circumstances when an ad valorem tax can be partly replaced by a tax on price without the need for a lot of information. It is clear from equation (4) that specification of a revenue-neutral tax reform requires some detailed knowledge of demand and cost conditions, including information about the monopolist's second-order condition. This is not practical. To the contrary, the tax reform $xd\tau = -ds,\, d\tau < 0$ and $ds > 0$, is obviously practical. Tax reforms defined this way are what Suits and Musgrave (1953) call matched-pair reforms. Such reforms do not call for knowledge about demand and cost relations.

Suppose that tax revenue derives from an ad valorem tax alone and consider the effects of a series of matched-pair reforms. First, consumers benefit from a series of reforms because the price goes down ($x_\tau d\tau + x_s ds$ is positive). Second, the reforms are feasible because the monopolist's profit is not harmed. To see this, observe that the profit change is $\Delta\pi = -p(x)xd\tau - p(x)ds + \pi_x(x_\tau d\tau + x_s ds)$. By specification of the tax reform $-p(x)xd\tau - p(x)ds = 0$. The term $\pi_x(x_\tau d\tau + x_s ds)$ also cancels because of the first-order condition. Thus, the monopolist's profit is unchanged. This implies that the matched-pair tax reforms are not stopped by feasibility. Third, consider the effect on tax revenue. The change in tax revenue is:

$$\Delta R = p(x)[xd\tau + ds + \tau(x_\tau d\tau + x_s ds)] + [\tau x + s]p_x(x)(x_\tau d\tau + x_s ds). \qquad 6)$$

This reduces to $\Delta R = \left(\tau\big(p_x(x)x + p(x)\big) + sp_x(x)\right)(p(x)/\theta)d\tau$. Once again, using the first-order condition, $\tau\big(p_x(x)x + p(x)\big) + sp_x(x) = p_x(x)x + p(x) - c_x(x)$ is positive whenever output is less than unregulated monopoly output. The term $(p(x)/\theta)d\tau$ is positive when the reform involves less ad valorem taxation ($d\tau$ is negative) since the second-order condition implies that θ is negative.

That is, starting out with a combination of positive ad valorem taxation and no tax on price, the first round of reform is positive. After a series of reforms, tax revenue starts to fall. This occurs for tax rates giving the monopolist incentives to produce as he would do without taxation. We summarise this in Proposition 3.[10]

Proposition 3. Starting with pure ad valorem taxation, a series of matched-pair reforms continuing until tax revenue begins to fall brings output to that chosen by the monopolist in the absence of taxes.

Proposition 3 is interesting because it demonstrates that there are practical reforms which are welfare-improving without harmful revenue effects. In particular, there is no need for extraordinary information to avoid the negative output changes that go along with ad valorem (and unit) taxes. In fact, the result in Proposition 3 applies to homogenous oligopoly with a fixed number of firms. [11] Of course, see for example Delipalla and Keen (1992), when oligopolistic firms are subject to ad valorem taxation the price increases and is above the price in the absence of taxation. Replacing some ad valorem taxation by a tax on price has the effects described for the case of monopoly. Too see this follow Seade (1980) and write the first order condition of a profit maximising oligopolist as $(1-\tau)\big(p_X(X)\cdot dX/dx\cdot x+p(X)\big)-c_x(x)-sp_X(X)\cdot dX/dx=0$, where aggregate output is $X=nx$ and n the number of firms and x output per firm. The term $dX/dx=1$ in Cournot oligopoly but we need not impose restrictions on the conjectural variations parameter, see Seade

[10] Proposition 3 applies to the case where the tax on price is initially zero. In this case the ad valorem tax rate must satisfy $\tau\leq -x/x_\tau\cdot\left(1+\eta_p\right)^{-1}$ for R_τ to be positive.

[11] The referee suggested this extension as well as extensions to the case where the monopolist is subject also to rate-of-return regulation.

(1980) for details. Apart from the fact that one must notice, in Lemma 1, that the number of firms is not one but n the analysis is qualitatively unchanged. In particular, equation (5) and the subsequent analyses are unaffected except that price is a function of aggregate output rather than the output of the monopolist. However, locally (around some aggregate output) the equation still applies. We summarise this as Corollary 2.

Corollary 2. For fixed number homogenous oligopoly, matched pairs of ad valorem taxes and taxes on price can maintain the unregulated oligopoly output and extract revenue.

3. Example

In this section we analyse an example assuming that cost and demand are linear functions of output, that is $c(x) = cx$ and $p(x) = a - bx$. Suits and Musgrave (1953) showed that ad valorem taxes are superior to excise taxes by asking what happens when one type of tax replaces to other. In our example we follow this approach. Also, we include a unit tax in the analysis so that we have two sets of comparisons; ad valorem versus unit taxes, and ad valorem taxes versus a tax on price.

Revenue under an ad valorem tax is $R^{\tau} = \tau p^{\tau} x^{\tau}$, where the ad valorem tax rate is τ. Output and price under ad valorem taxation is $x^{\tau} = ½b^{-1}(a - (1-\tau)^{-1}c)$ and $p^{\tau} = ½(a + (1-\tau)^{-1}c)$, respectively. Ad valorem taxation harms consumers through changing the final price. The usual alternative to ad valorem taxes is the unit tax. Under a unit tax the monopolist's output is $x^t = ½b^{-1}(a - c - t)$ where t is the tax rate. The price is $p^t = ½(a + c + t)$ showing that consumers are harmed by the excise tax. Comparing tax rates with the same final price, $p^{\tau} = p^t$, gives

$\tau(1-\tau)^{-1}c = t$ and revenue generated by the ad valorem tax exceeds that generated by the excise tax when $\tau p^\tau x^\tau > tx^t$ or $\tau p^\tau > t$. Using $\tau(1-\tau)^{-1}c = t$, the condition $\tau p^\tau > t$ comes down to $a > (1-\tau)^{-1}c$ after some rewriting.[12] This result is what is shown more generally in Suits and Musgrave (1953).

Consider the tax on price. When the tax rate is s, the monopolist's profit is $(a-bx)x - cx - s(a-bx)$. Output and price are $x^s = ½b^{-1}(a-c+sb)$ and $p^s = ½(a+c-sb)$, in that order. Plainly, a tax on price drives the price below the price in an unregulated monopoly and, contrary to ad valorem taxes, the price change is a benefit to consumers. For positive tax rates output is positive and the price is well-defined when $a+c > bs$. The tax revenue is $R^s = s½(a+c-sb)$and we want to ask whether it is possible that $R^s = R^\tau$ for some given R^τ, say $\hat{R}^\tau$, whilst $p^s < p^\tau$. Clearly, tax revenue is the same under the two taxes when $s = ½b^{-1}\left(a+c \pm \sqrt{(a+c)^2 - 8b\hat{R}^\tau}\right)$. Suppose that$(a+c)^2 > 8b\hat{R}^\tau$ and use the positive radical so that s is positive. It is easy to see that the solution satisfies $a+c > bs$. Hence, when the solution to $R^s = R^\tau$ for some R^τ, say $\hat{R}^\tau$, is well-defined, we can find a tax on price that is of equal yield to the ad valorem tax but with lower consumer price.

The restriction $(a+c)^2 > 8b\hat{R}^\tau$ gives a restriction on the possibility of replacing the ad valorem tax with a tax on price when one uses either one or the other of the two taxes. Under ad valorem taxation revenue is $4b\hat{R}^\tau = \tau(a^2 - (c/(1-\tau))^2)$. Hence, the restriction $(a+c)^2 > 8b\hat{R}^\tau$ comes down to $(a+c)^2 > 2\tau(a^2 - (c/(1-\tau))^2)$. For ad valorem tax rates less than half, the inequality

[12] The condition $a > (1-\tau)^{-1}c$ holds since otherwise there would be no production under the ad valorem tax.

is easily seen to be satisfied. For ad valorem tax rates higher than a half, the inequality is satisfied dependent on parameters. An example where the inequality fails to be satisfied is in the case of vanishing marginal costs and ad valorem tax rates higher than one half. This is unsurprising. The monopolist maximises revenue when marginal costs are imperceptible and can be ignored. Thus, tax revenue is equal or close to $\tau\, a^2/4b$. A tax on price drives the price down, thus limiting the revenue that can be generated.

The example shows that it is not in general possible to find a tax on price which can fully replace the ad valorem tax and simultaneously match the revenue from ad valorem taxation. Of course, this does not suggest that a tax on price is irrelevant. Rather, it illustrates why it is more proper to ask about the effect of a tax reform where a price on tax gradually replaces the ad valorem tax.

4. Conclusions.

In this paper we have re-examined a tax on price. The appeal of such a tax is that it simultaneously provides revenue and incentives for firms to reduce price (Tam 1991 and 1993, and Sumner 1993). Taking the profit constraint explicitly into consideration we have worked out how far the efficiency of a tax on price goes (Propositions 1 and 2). Moreover, we have shown that a combination of ad valorem tax and a tax on price produces the allocation corresponding to Ramsey pricing (Corollary 1). In this way, the combination of the two taxes is an efficient tax policy in the sense that the unavoidable deadweight loss that goes with taxing a monopolist (given non-availability of lump-sum taxes) is minimised.

Unsurprisingly, identifying fixed revenue combinations of a tax on price and an ad valorem tax calls for knowledge about demand and cost conditions. On this account Glaister (1987) suggests that the tax is of limited practical value. Similarly, Dalton (1929) and Robinson (1933) discuss output-based subsidies but dismisses them as a practical possibility. This is surely the case when the tax scheme is to be constructed so as to induce the Ramsey optimum. Nevertheless, it is possible to design a practical and beneficial tax reform that combines ad valorem taxation with a tax on price. First, the tax reform is practical since it is based on matched-pair tax reforms, i.e., it is based on observation of price and output. Second, it is beneficial because it goes some way in minimising the deadweight loss that is unavoidable when taxing a monopolist.

References

Dalton, H. (1929) "*Public Finance*," Routledge, London.

Delipalla, S. and Keen, M. "The Comparison between Ad-valorem and Specific Taxation under Imperfect Competition," *Journal of Public Economics*,1992, 49(3), pp. 351-367.

Glaister, S. (1987) "Regulation Through Output Related Profits Tax," *Journal of Industrial Economics,* 35(3), pp. 281-296.

Hamilton, S.F. (2009) "Excise Taxes with Multiproduct Transactions," *American Economic Review*, 99(1), pp. 458-471.

Hamilton, S.F. (1999) "Tax Incidence Under Oligopoly: A Comparison of Policy Approaches," Journal of Public Economics 71(2), pp. 233-245.

Myles, G. (1996) "Imperfect Competition and the Combination of ad valorem and Specific Taxation," *International Tax and Public Finance*, 3(1), pp. 29-44.

Robinson, J. (1933) "*The Economics of Imperfect Competition*," Macmillan, London.

Shilling, N. (1969) "*Excise Taxation of Monopoly*," Columbia University Press, New York.

Seade, J. (1980) "On the Effects of Entry," *Econometrica*, 48(2), pp. 479-489.

Suits, D.B. and Musgrave, R.A. (1953) "Ad Valorem and Unit Taxes Compared," *Quarterly Journal of Economics*, 67, pp. 598-604.

Sumner, M-Y. S. (1993) "Tax on Price: A Comment," *Public Finance*, 48, pp. 297-302.

Tam, M-Y. S. (1991) "Tax on Price: A New Commodity Tax Structure For Efficiency," *Public Finance*, 46, pp. 233-133.

Tam, M-Y. S. (1993) "Tax on Price: A Response," *Public Finance*, 48, pp. 303-309.