ASSESSMENT FOR THE PAPER

"The BIP Trilogy (Bipolarization, Inequality and Polarization) One Saga but Three Different Stories"

The paper "The BIP Trilogy (Bipolarization, Inequality and Polarization) One Saga but Three Different Stories" presents a very interesting analysis of three different concepts (Bipolarization, Inequality and Polarization, BIP) that share some similarities but also have some sharp differences. I think that the discussion in the paper is very useful both to clarify some misconceptions on these issues and to highlight relevant features of these concepts. The authors also produce a practical example (using data from Luxemburg) that shows how relevant the differences between these three aspects can be and how they differ in terms of measurement and in terms of the economic mechanisms that generate them.

In my opinion the main contribution of the paper is the definition, clarification and illustration of these three different dimensions of (income) distribution. Therefore I believe that an even larger effort should be put forth to obtain clarity in the presentation of the issues and in the clarification of some misconceptions that may arise: this will be at the center of some of the detailed comments I describe below.

MAJOR REMARKS

- 1) I found interesting the distinction between polarization and bipolarization. In fact, I believe that some misconceptions may arise between these two concepts and I thus appreciated the effort to clarify the distinctions between the two concepts. The origin of misconceptions may also be due to the fact that, in physics (and physical) terminology, polarization means "an agglomeration towards two extremes" something that can lead to think economic polarization as the agglomeration on TWO poles. The latter concept may be inaccurate in economic terms and there is thus a difference in terminology between the economic term and the physics (and possibly widespread) term, generating a confusion that is worth to be clarified and that the paper could stress a bit more. From this point of view I have to admit that several past economic analyses highlight the existence of polarization whereas they are probably highlighting the existence of bi-polarization: the contribution in the present paper is thus even more useful within the economic discipline.
- 2) Partly related to what I said in the previous comment, I believe that there may be some widespread misconceptions in the three concepts (Inequality, Bipolarization and Polarization). Therefore I suggest that at the very beginning of the paper a clear and direct definition to the three terms should be immediately given. This would avoid possible confusion later on.

 From this point of view, I found a bit confusing the discussion on page 3 taken from the field of international relations: without a clear definition of the three terms, and possibly having in mind the "physics" definition of polarization, the discussion sounded misleading to me. I would suggest moving this discussion after a clear definition for the three terms is given, so that it appears as a possible application of the three concepts.
- 3) Given the concept of polarization that you are describing in the paper, is it correct to say that bipolarization is simply a special case of polarization? If so, the authors could explicitly say so; if not, the authors should stress more what makes them apart.
- 4) At the beginning of section 2.1, the authors claim that "the concept of bi-polarization is clearly related to that of middle class". I am not sure what the authors are hinting to. Actually I think that

- bi-polarization may be related to the LACK of middle-class as several studies argue how, with the emergence of (bi-)polarization of income, the middle-class is disappearing. If the authors were also referring to the absence of middle class, they should be clearer on it. If on the contrary they were referring to something else, I was not able to follow their reasoning.
- 5) Section 2.2 deals with the measurement of polarization. In the earlier parts of the paper, polarization had been described as a concept that could be applied to several aspects but, in section 2.2, measurement is strictly and directly related to income. The authors should signal clearly and motivate this change of approach and also discuss if and how other dimensions (for example education, skills, health, family background etc...) of polarization can be measured.
- 6) The results on the Shapley decomposition in section 4 are interesting and in my opinion they stir some further questions that could be discussed. In particular, some policy implications could be given: in fact, if the three different concepts are originated by different economic sources, this information can used to tailor specific policies that aim at mitigating each of these inequality dimensions. For example, the authors show that the main source of (bi-)polarization is income from labour, and thus policies that aim to mitigate this aspect should focus probably on the redistribution of labour income; on the contrary, the main component of the Gini inequality index appears to be income from capital, so that policies aiming at reducing this dimension of inequality, should focus probably on wealth redistribution.
- 7) The results of Shapley decomposition of the polarization index (Polarization DER) appear to be quite dependent on the value of α . However, very little is said about what α represents (I could not even find an exact definition for it) and in particular, more effort should be put forth to explain why, in economic terms, when α changes, the results also change. This would probably also explains while, overall, the authors suggest that the main contribution to polarization comes from income from wealth.
- 8) Section 2.2 argues how the measure of polarization should determine the number of poles endogenously and propose some procedures that actually do so. Given this premise, I could not understand whether, in section 4, the authors also determine endogenously the number of poles. This would be particularly interesting and if they actually identify endogenously several poles, they could also displays some characteristics of these poles, like the average education, age, family background of the individuals within each single pole.

MINOR REMARKS

- a) Throughout the paper I had the strong feeling that the authors were really interested in measuring the three dimensions of inequalities rather than explaining the economic rationale that generate them. This was also true when, in section 4, they actually obtain some insights on the economic mechanisms but the authors do not exploit them to describe those mechanisms. I would probably thus suggest to state from the beginning that the main focus of the paper is on the "definition"/"measurement" of these concepts (or as an alternative, to expand the discussion on the economic mechanisms).
- b) At the end of page 3 the authors argue how bi-polarization refers to the case where there is a large share of individuals that are poor and a relevant share of individuals that are rich. I was wondering whether bi-polarization can also refer to a case where most individuals belong to the rich and middle class (so that poor are negligible) or to a case where most individuals belong to the middle and poor class (so that rich are negligible). Would these cases still be considered a bi-polarization?

- c) The claim of the authors at the beginning on page 4 "The analysis of bi-polarization is hence linked to that of the importance of the middle class" sounded to me as taken for granted rather than explained and motivated.
- d) Partly related to point 5) in the main remarks: should the title of section five be changed to "On the measurement of polarization *of income*" or can it be generalized to other dimensions?
- e) I was a bit surprised in finding on page 10 a reference to an item listed as (1999, forthcoming): is this a misprint? If not, I may still have hope for some of my old working papers...