## "The BIP Trilogy (Bipolarization, Inequality and Polarization): one saga but three different stories"

## Summary

In this paper it is posed that inequality, bipolarization and polarization are three different concepts, with the main argument being that the impact of different income sources on these concepts differs. To this end the authors first briefly review the theoretical literature related to these concepts, their measures and propose the Shapley decomposition as an approach to analyze the particular role played by each income component on the concepts above. And second, the authors propose an application using 2008 income data from Luxembourg. It is found that the impact of income from work, capital and transfer differs depending on whether one considers inequality, polarization and bipolarization. The authors conclude then that these concepts are indeed different.

## Comments

The paper analyze an important issue, inequality and other related measures of wellbeing, that is becoming more and more popular and thus attracting more interest from economist and other social scientist since the beginning of the actual financial crisis. In addition it seems to provide interesting results. However, I do have some serious concerns with the analysis proposed here.

1.- The authors argue that inequality, bipolarization and polarization are three different concepts. However, they do not analyze indeed these three concepts in the paper but rather focus on the role played by the various (income) sources that generate them in some measures of inequality, bipolarization and polarization. As they find (in the empirical application) that these sources play a different role in these measures they conclude that these concepts differs. First, this should be more clearly stated in the introduction. And second, I find this is not a strong enough argument for such a conclusion.

2.- An important part the work consists on the presentation of the three concepts: inequality, bipolarization and polarization and the Shapley decomposition (Sections 2 and 3). I believe that this part would benefit by presenting the different concepts in a more systematic fashion. That is, for each case concept the authors should provide firstly a formal definition (rather than just similarities with other ideas and/or contexts), secondly a formal description of the properties of the corresponding proposed measure, and finally a formal description of the consequences of applying the Shapley decomposition on these measures (rather than just relegating to the appendix part of this analysis). Thus, after reading Sections 2 and 3 the reader must know which are, a) the main theoretical results, b) the specific hypotheses that are

going to be tested in the empirical application, and c) the contribution of these findings to the relevant literature.

3.- The choice of the particular measures proposed, and also the decomposition approach seems quite ad hoc. I suggest the authors to provide strong arguments for their choices or at least comment on the extent to which their results would have changed (the impact of different sources of income on each measure differs) had these measures be different to the ones proposed.

4.- I believe there are lots of arguments for studying the role of the various sources of income on the measures proposed per se (that is, for the decomposition analysis undertaken), for instance in terms of policy analysis. However, there is a complete absence of motivation to this analysis. This is important given the topic of the paper.

5.- One of the interesting parts of the paper, in my view, is the analysis presented in Section 4. I think it is a rather novel approach. In this sense I think that income might be just an application and thus the author could try to provide a more general analysis of decomposition these measures on other variable distributions. Another application might be the study of education results (score) distribution in the population.

However, I also have some concerns regarding the analysis proposed here. In particular, and to check the robustness of their empirical results, the authors propose to consider a wider set of income sources to the one consider in the first study, namely, seven instead of just three. I suggest the authors to better explain their robustness strategy (why would one expect the results to be different?) before proceeding to describe their results. In addition the author should clarify to what extent the result depends on other particular properties of the income distribution (dispersion, symmetry, etc.)

## Some minor comments

- Page 6: The authors use different notation for the same variable, namely,  $G_B$  and  $G^B$  for the between group Gini index (the same for  $G_W$  and  $G^W$ ).
- Page 8: parameter alpha is not described (this is done lately in page 15, but it should be done when it appears for the first time in the text).
- Page 11: It should be equations (8) and (9) instead of (10) and (11).
- Page 16. In Table 2 the authors should explicitly mention how the new seven sources of income relate to the previous three ones, and try to maintain the same order as in Table 1. This would enormously facilitate the comparison of the results in both cases.