Response to comments on "The BIP Trilogy (Bipolarization, Inequality and Polarization): One Saga but Three Different Stories"

Joseph Deutsch, Alessio Fusco and Jacques Silber (November 12, 2012)

First of all, we would like to thank the anonymous reader for his useful comments. Below are our answers to each of them.

First comment:

"Negative contributions appear in 5 out of the 10 decomposition exercises. Some readers may be interested in understanding why/how negative contributions may appear. Conceptually, it would seem, that the presence of an income source with a negative contribution reduces inequality/bipolarization/polarization. The authors mention that in their discussion of the results and that's absolutely fine. But I think it would be interesting to provide a relatively simple hypothetical example to show how some sources may come up with positive contributions while some with negative ones in each of the three distributional concepts."

Rather than giving hypothetical examples which would lengthen significantly the paper, in particular in the case of the DER index, we have added a sentence in Section 3 explaining that a contribution can be negative and added some comments in Section 4 where the empirical results are analyzed.

For the Gini index we have clearly explained what the implications of a negative contribution was. Since such a negative contribution occurred only in the case of the two zero income decompositions, we explained on page 15 that a negative contribution simply implies that when total income includes only income from capital and transfers the Gini index of such a total income will be greater than when income from work is also included in total income. This indicates that income of work is more equally distributed than the sum of the two other income sources.

In the case of the bi-polarization index we had already written that income from transfers has a negative impact on bi-polarization and that such a result meant if there had been no transfers, bi-polarization would have been quite higher (around 135% of its actual value). To better explain the meaning of a negative contribution in the case of bi-polarization we

have now added that "Since such a higher bi-polarization means, as was mentioned previously, that there would have been either a greater positive difference between the "between" and the "within" groups Gini index (the numerator of the P_G index) or/and a smaller overall Gini index (the denominator of the P_G index), we may conclude that when transfers are included in total income, either the Gini index of total income is higher, or, what is more likely, the between groups Gini index is smaller (there are less differences between the rich and the poor) and the within groups index is higher (there is more dispersion in the Gini index of the poor and of the rich).

As far as the case of the polarization index DER is concerned, we believe that we had already clearly explained what a negative contribution implies. Such a negative contribution is observed for self-employment and we had written that "if we now take the case where the polarization sensitivity parameter α is equal to 1 (see Table 4), we first observe that only one source has a negative contribution, income from self-employment, but the magnitude of its contribution is small. We can nevertheless say that when the Shapley contribution is negative, this implies that had there been no income from self-employment polarization would have been higher. Income from self-employment seems therefore to smooth the income distribution in the sense that it decreases the extent to which there are local poles. This might indicate that self-employed individuals may come from all strata of the population."

Second comment

"In relation to that, since these contributions are computed using Shapley decompositions, it may be interesting to unpack the computation of one of the negative contributions in order to see how many components turn up negative and how many turn up positive. This might shed some light on what's going on behind a procedure that is certainly sensible, but may have a bit of a "black box" behaviour (that is, before we understand well what's going "inside"). "

Following the clarification requested by an anonymous reader we have added Appendix C which, we hope, explains well why and how a negative contribution may occur. For convenience, we add this appendix at the end of this document.

Third comment

"It may also be interesting to clarify whether the fact that the indices have different ranges/bounds (e.g. Gini between 0 and 1-1/n; Pg and Pfw can take negative values, etc.) may have an impact on the sign of the contributions. Maybe there is no relationship at all, but it might be worth exploring, checking."

After explaining what the Shapley decomposition procedure is all about, we have now added a footnote (footnote 6) where we state that "it should be clear that the interpretation of the contribution of a given component (determinant) to the value of a given index (Gini index, bi-polarization or polarization index) does not depend on the range/bound of this index". We then thanked the anonymous reviewer for asking us to clarify this point.

Fourth comment

"It may be aesthetically better (for many readers) to have the equations outside of their boxes."

We have indeed followed the advice of this reviewer.

Appendix C to be added to the paper: The Shapley decomposition procedure and negative contributions. An illustration.

We give here a simple illustration of the case where negative contributions may be observed when "unpacking" the overall contribution of a factor. This illustration uses the data from which Table 1 was derived. More specifically we "unpack" the negative contribution of income from work (-0.011) in the zero income decomposition of the Gini index in Table 1.

Let a represent income from work, b income from transfers and c income from capital.

Therefore $I(a \neq 0, b \neq 0, c \neq 0)$ will be the Gini index of total income, assuming each of the three income sources (income from work, transfers and capital) are different from zero while $I(a=0, b \neq 0, c=0)$, for example, refers to the Gini index of total income, assuming income from work and capital are equal to zero.

Working with the original database the following results were obtained:

$I(a \neq 0, b \neq 0, c \neq 0)$ = 0.3676

$I(a=0, b \neq 0, c \neq 0)$ = 0.6088

$I(a \neq 0, b=0, c \neq 0)$ = 0.5155

$I(a=0, b=0, c \neq 0)$ = 0.8983

$I(a \neq 0, b \neq 0, c=0)$ = 0.3639

$I(a=0, b \neq 0, c=0)$ = 0.6212

$I(a \neq 0, b=0, c=0)$ = 0.5275

$I(a=0, b=0, c=0)$ = 0 (as expected).

Since the contribution $C(a)$ may be written as

$C(a) = \{ (2/6) [I(a \neq 0, b \neq 0, c \neq 0) - I(a=0, b \neq 0, c \neq 0)]$

$\qquad + (1/6) [I(a \neq 0, b=0, c \neq 0) - I(a=0, b=0, c \neq 0)]$

$\qquad + (1/6) [I(a \neq 0, b \neq 0, c=0) - I(a=0, b \neq 0, c=0)]$

$\qquad + (2/6) [I(a \neq 0, b=0, c=0) - I(a=0, b=0, c=0)] \}$

we derive that

$C(a) = \{ (2/6) [0.3676 - 0.6088] + (1/6) [0.5155 - 0.8983] + (1/6) [0.3639 - 0.6212] + (2/6) [0.5275-0] \}$

C(a) = { (2/6) * -0.2412 + (1/6) * -0.3828 + (1/6) * -0.2573 + (2/6) * 0.5275}

C(a) = -0.01125

Which is exactly the value of the (marginal) contribution of income from work that appears in Table 1 in the case of a zero income decomposition.

In this example, drawn from our data, income from work is more equally distributed (Gini = 0.5275) than income from transfers (0.6212) and income from capital (0.8983). It appears then clearly that the contribution of income from work is negative in all cases except when income from work is eliminated last.