Unexpected Consequences of Ricardian Expectations

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Abstract

Economists are widely familiar with the Ricardian equivalence thesis. It maintains that, given the time-path of government spending, a change in taxation does not alter the set of feasible life-time consumption plans of the households and affects neither the demand for commodities and services nor the rate of interest, provided the households act rationally.

In this note a surprising finding is established. Assuming that the agents in a standard infinite horizon growth model hold the very expectations the thesis proposes ("Ricardian expectations"), it is shown that these expectations are disappointed.

This divergence from the Ricardian equivalence thesis is traced to the omission of interest payments on public debt as part of the households' disposable income. The non-equivalence is valid in a wide class of models.

Keywords: Barro-Ricardo equivalence, Ricardian equivalence, fiscal policy, debt, taxation, rational expectations, Ricardian expectations, Barro expectations
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1   Introduction

The Ricardian equivalence thesis, originally developed by Robert Barro (1974, 1979) has become a standard topic in every macroeconomic textbook. It establishes the set of feasible life-time consumption plans of the households as determined by the time-path of government spending. Provided the households act rationally, it is asserted that this set remains unaltered by a change in taxation. Therefore, neither the demand for commodities and services nor the rate of interest are affected.

In this note, the Ricardian equivalence thesis is re-examined in detail. At the start, it is assumed that the agents exhibit "Ricardian behavior": behavior that is rational if "Ricardian expectations" prevail. Ricardian expectations are the expectations Barro suggests as being rational. The analysis produces an unexpected result: the Ricardian expectations are disappointed.

First, the original formulation of Barro's argument is provided as a point of reference (Section 2). Then, an example of an economy in steady-state growth is is outlined where all households exhibit Ricardian behavior, but their Ricardian expectations are not fulfilled (Section 3).

To elucidate the reason for this odd finding, Barro's original argument is examined step by step (Section 4). It is shown that Barro's thesis depends on the unwarranted implicit assumption that interest payments on public debt do not contribute to the households' disposable income.

The example shows that there is no *economic* necessity to balance less taxes to-day with offsetting larger taxes in the future, but there may be *political* reasons for introducing limits on public debt. If a zero limit on government debt is enforced by law, this may render Ricardian behavior prudent *ex post,* but this does not salvage the Ricardian equivalence thesis (Section 5). Finally it is shown that the criticism of the Ricardian equivalence thesis developed in the example carries over to the general case of arbitrary growth paths (Section 6).

2 Barro's Argument

Let us start by recapitulating the Ricardian equivalence thesis. Robert Barro (1979, 38f.) explains it as follows.

> The Ricardian ... analysis begins with the observation that, for a given path of government spending, a deficit-financed cut in current taxes leads to higher future taxes that have the same present value as the initial cut. This result follows from the government's budget constraint, which equates total expenditures for each period (including interest payments) to revenues from taxation or other sources and the net issue of interest-bearing public debt. Abstracting from chain-letter cases where the public debt can grow forever at the rate of interest or higher, the present value of taxes (and other revenues) cannot change unless the government changes the present value of its expenditures. ... Hence, holding fixed the path of government expenditures and non-tax revenues, a cut in today's taxes must be matched by a corresponding increase in the present value of future taxes.

> Suppose now that households' demands for goods depend on the expected present value of taxes - that is, each household subtracts its share of this present value from the expected present value of income to determine a net wealth position. Then fiscal policy would affect aggregate consumer demand only if it altered the expected present value of taxes. But the preceding argument was that the present value of taxes would not change as long as the present value of spending did not change. Therefore, the substitution of a budget deficit for current taxes (or any other rearrangement of the timing of taxes) has no impact on the aggregate demand for goods. In this sense, budget deficits and taxation have equivalent effects on the economy - hence the term, "Ricardian equivalence theorem."

> To put the equivalence result another way, a decrease in the government's saving (that is, a current budget deficit) leads to an offsetting increase in desired private saving, and hence to no change in de-

> sired national saving. Since desired national saving does not change,
> the real interest rate does not have to rise in a closed economy to
> maintain balance between desired national saving and investment
> demand. Hence, there is no effect on investment, and no burden of
> the public debt or social security. (Barro, 1989, 38f.)

This explanation can hardly be improved upon. Due to its transparency and lucidity, the argument has logical appeal (Romer, 1995, 72). It is so convincing that the Ricardian equivalence thesis became a staple topic in public finance.

3   An Example

Consider the standard infinite horizon model of a closed economy in steady state growth that grows at the nominal rate g. Production at time t is X_t, private expenditure (consumption plus investment) is E_t, taxes are T_t and government expenditure is G_t.

Production and government expenditure grow both with rate g. So we have

$$X_t = (1+g)^t X_0 \tag{1}$$
$$G_t = (1+g)^t G_0 \tag{2}$$

X constitutes the sum-total of pre-tax income, that is, wages plus income from capital ownership, but it does not include income from interest on government bonds, to be introduced later.

The economy is initially in full equilibrium with an interest rate $r > g$. The rate of interest is assumed to exceed the growth rate. This assumption is also made by Barro; otherwise the present values used in his argument would not exist.

Private expenditure E and government expenditure G add up to total production, and the expectation held by all parties is that this will continue in the future:

$$E_t + G_t = X_t. \tag{3}$$

Up to $t = -1$, the government budget was balanced, and the taxes levied in any period $t < 0$ were equal to government spending G_t in that period. Call this the "pay-as-you-go regime." All parties have expected and expect that this policy would continue throughout the future, but government changes its policy and decides to run a permanent deficit of a fraction $\alpha \in (0, 1)$ of its expenditure G_t in each period, beginning at $t = 0$ while leaving government expenditure G_t unchanged. So government expenditure remains as described in equation (13). Call this the "debt regime."

Let D_t denote government debt. Initially there is no government debt:

$$D_0 = 0. \tag{4}$$

In line with Barro's (1974; 1989) analysis, the households and firms expect that the change in policy does not affect the present value of their lifetime income stream. Hence they believe that "rearrangements of the timing of taxes – as implied by budget deficits – have no first-order effect on the economy" (Barro, 1989, 51). They conclude that, sooner or later, the government has to increase taxes, leaving the present value of their incomes unaltered. So they change neither consumption nor investment. In short, everybody in the private sector holds "Ricardian expectations" and behaves accordingly – everyone exhibits "Ricardian behavior."

In each period $t = 0, 1, 2...$ the deficit is αG_t, and outstanding government debt D increases by this amount. Therefore we have

$$D_0 = 0 \tag{5}$$
$$D_{t+1} = D_t + \alpha G_t. \tag{6}$$

This implies together with (2)

$$D_t = \frac{1}{g}\left((1+g)^t - 1\right)\alpha G_0. \tag{7}$$

Hence debt grows asymptotically in proportion with production. The ratio of government debt to government expenditure approaches α/g and the ratio of government debt to production approaches α/g times the share of government

expenditure in total production.

$$\lim_{t\to\infty} \frac{D_t}{G_t} = \frac{\alpha}{g}, \quad \lim_{t\to\infty} \frac{D_t}{X_t} = \frac{\alpha}{g} \frac{G_0}{X_0}. \tag{8}$$

As the government keeps its expenditure on goods, services and manpower G unaltered and runs a deficit, its budget, denoted by B, will exceed expenditure G by interest payments rD on public debt:

$$B_t = G_t + rD_t. \tag{9}$$

The share of interest payments in the government budget is

$$\frac{rD_t}{B_t} = \alpha \frac{r}{g} \frac{(1+g)^t - 1}{(1+g)^t} \tag{10}$$

and approaches $\alpha \frac{r}{g}$:

$$\lim_{t\to\infty} \frac{rD_t}{B_t} = r\frac{\alpha}{g}. \tag{11}$$

So for a growth rate of 2 percent ($g = 0.02$), a rate of interest of 4 percent ($r = 0.04$), and a deficit rate of 10 per cent ($\alpha = 0.1$) this ratio would approach 17 percent.

The present value of government debt D_t is

$$\left(\frac{1}{1+r}\right)^t D_t = \frac{\alpha}{g}\left(\frac{(1+g)^t - 1}{(1+r)^t}\right) G_0. \tag{12}$$

As $r > g$ is assumed, the present value of the debt is a positive number and goes to zero for $t \to \infty$ although debt is never retired.

The deficit in period 0 is αG_0. It is entailed by the tax reduction of the same size. So we have tax receipts of $T_0 = (1-\alpha)G_0$ in period 0. In period 1 government debt is $D_1 = \alpha G_0$. This requires interest payments rD_1. The deficit in period 1 is the sum of government expenditure G_1 plus interest payments rD minus tax receipts T_1. The deficit is to be αG_1. Hence we have

$$G_1 + rD_1 - T_1 = \alpha G_1.$$

A similar consideration applies to all periods:

$$G_t + rD_t - T_t = \alpha G_t. \tag{13}$$

Solving for T_t gives the amount of taxes to be collected in period t:

$$T_t = (1 - \alpha)G_t + rD_t. \tag{14}$$

Furthermore, the ratio of taxes to production approaches

$$\lim_{t \to \infty} \frac{T_t}{X_t} = \left(1 + \frac{(r-g)\alpha}{g}\right) \cdot \frac{G_o}{X_o} > \frac{G_o}{X_o}. \tag{15}$$

For $r > g$ the government collects higher taxes under the debt regime than under the pay-as-you-go regime. Comparing the tax increase entailed by switching from the pay-as-you-go regime to the debt regime (which is $T - G$) with interest payments rD necessary under a debt regime yields

$$\lim_{t \to \infty} \left(\frac{T_t - G_t}{X_t} - \frac{rD_t}{X_t}\right) = \frac{\alpha G_o}{X_o}. \tag{16}$$

In the long term, the tax increases entailed by switching to the debt regime exceed the interest payments necessary to serve the debt.

Now consider the present value of the households disposable income. Define for any time series x the function

$$\Omega(x) = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t x_t \tag{17}$$

which gives the present value of the time series x. It is linear:

$$\begin{aligned}
\Omega(x + y) &= \Omega(x) + \Omega(y) \tag{18}\\
\Omega(ax) &= a\Omega(x) \text{ for any } a \in \mathbb{R}. \tag{19}
\end{aligned}$$

If the government would run a balanced budget all the time, the households discounted disposable income would have been

$$
\begin{aligned}
\Omega\left(X-G\right) &= \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^{t}\left(X_{t}-G_{t}\right) \\
&= \left(X_{0}-G_{0}\right)\sum_{t=0}^{\infty}\left(\frac{1+g}{1+r}\right)^{t} \\
&= \frac{1+r}{r-g}\left(X_{0}-G_{0}\right).
\end{aligned}
\tag{20}
$$

The debt policy, however, results in disposable income

$$
Y_{t} = X_{t} - T_{t} + r D_{t}
\tag{21}
$$

which is

$$
\begin{aligned}
Y_{t} &= \left(1+g\right)^{t}\left(X_{0}-(1-\alpha)\,G_{0}\right). \\
&= X_{t}-(1-\alpha)\,G_{t} > X_{t}-G_{t}.
\end{aligned}
\tag{22}
$$

Under the pay-as-you-go regime, disposable income in each period would have been $X_{t}-G_{t}$. Hence *the switch from the pay-as-you-go regime to the debt regime has increased disposable income for all periods by the fraction α of government expenditure G_{t}.*

The present value of disposable income is

$$
\Omega\left(Y\right) = \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^{t} Y_{t}.
$$

This is calculated as

$$
\Omega\left(Y\right) = \frac{1+r}{r-g}\cdot\left(X_{0}-(1-\alpha)\,G_{0}\right)
\tag{23}
$$

The difference between this present value of disposable income under the debt regime and the corresponding present value under the pay-as-you-go regime (20) is

$$
\Omega\left(Y\right)-\Omega\left(X-G\right) = \frac{1+r}{r-g}\alpha G_{0} > 0.
\tag{24}
$$

Hence the present value of the households' lifetime income has increased by switching from the pay-as-you-go regime to the debt regime. The Barro expectations held by the subjects are not fulfilled. The necessity for tax increases never arises. The households engage in precautionary savings in order to finance tax increases that never occur.

As the value of their lifetime income stream has increased, they could have afforded higher expenditure, with more consumption and more investment, but this would have presumably affected the rate of interest and the value of production and income in turn, and the Barro-Ricardo equivalence thesis would not hold true.

4   Solving the Contradiction

In the above example, Ricardian expectations are not fulfilled in presence of Ricardian behavior. This surprising result is at odds with Barro's argument as given earlier (Section 2). So let us consider Barro's argument in the context of the above example.

First note that the example does not involve what Barro calls a "chain letter case." Government debt grows eventually with the growth rate g that is below the rate of interest. The present value of government debt is well defined and goes to zero for $t \to \infty$; see equation (12).

Barro gives the governments budget constraint as equating total expenditure plus interest payments on government debt with revenue from taxation and the net issue of interest-bearing public debt. In the context of the example this can be written as

$$G_t + r D_t = T + \alpha G_t \tag{25}$$

and is equivalent to equations (13), (14) in the example.

Next consider the present value of taxes. From equations (7) and (14) the present value of taxes is calculated as

$$\Omega(T) = \frac{(1+r)}{r-g} G_0. \tag{26}$$

The expression is independent of α. Hence Barro's assertion that "the present value of taxes would not change as long as the present value of spending did not change" is satisfied in the example. Further, the present value of government expenditure, as obtained from equation (2) has the same value

$$\Omega(G) = \frac{(1+r)}{r-g}G_0. \tag{27}$$

Barro goes on to explain that each household subtracts its share of the present value of taxes from the expected present value of income to determine a net wealth position. This formulation does not consider the interest payments on government debt that are paid out of taxes under the debt regime. The correct statement would be: each household subtracts its share of present value of *net* taxes (taxes minus interest payments on government debt) from the expected present value of income to determine a net wealth position.

While the present value of the tax burden remains unchanged by a switch in the tax regime, the present value of *net* taxes changes with such a switch. This destroys the Barro-Ricardo equivalence.

Using equation (7), the present value of interest payments on government bonds rD is calculated as

$$\Omega(rD) \quad = \quad \frac{(1+r)}{r-g}\alpha G_0. \tag{28}$$

This expression is identical to the difference between the present values of income under the debt regime and under the pay-as-you-go regime as given in equation (24). The argument given in Section 2 does not take into account that, under the debt regime, part of the tax finances interest payments that increase the disposable income of the households. Although the present value of taxes remains unaffected by a switch in the tax regime, *net* taxes and the present value of disposable income are affected by such a switch.

5   Political Restrictions on Government Debt

Ricardian expectations rest on the thesis that tax reductions to-day entail, by *economic* necessity, offsetting tax increases later. This thesis – the Ricardian

equivalence thesis – turns out to be not true. Still there may arise political reasons for such tax increases, and people may harbor Ricardian expectations and consequently exhibit Ricardian behavior for political reasons. After all, the absence of tax increases in the past does not rule out tax increases at some point of time in the indefinite future. There is not, and can never be, direct evidence shaking Ricardian beliefs as long as the debt regime is kept in place.

So assume that new politicians attain power. They write the pay-as-you-go policy into the constitution. With such an austerity shock, the Ricardian expectations seem to be vindicated and the Ricardian behavior fully justified. Assume that the shock occurs at time T. At that time, government debt is $D_T = \frac{1}{g}\left(\left(1+g\right)^T - 1\right)\alpha G_0$, see equation (7). This capital is collected through an additional austerity tax, and is used to repay the government's debt to the debtors. So the households pay D_T in additional taxes and obtain D_T as the repayment for the government debt they hold. Things have developed as expected. The households have acted optimally.

This observation does not vindicate the Ricardian equivalence thesis, however. The thesis says that a change in the fiscal regime does not matter. If the pay-as-you-go policy is cemented in the constitution, it remains still true that a switch to the debt regime would entail real consequences.

It is important to note, however, that the austerity shock is not occasioned by any *economic* necessity as the Ricardian equivalence thesis seems to suggest. It is, in this case, brought about by an "arbbitrary" policy decision. If this decision was inspired by the Ricardian equivalence thesis, it was flawed from the beginning.

6 Arbitrary Growth Paths

The argument made in the example given in Section 3 about Ricardian expectations can be easily generalized to arbitrary growth paths. We keep assumption (3)

$$E_t + G_t = X_t \tag{29}$$

for all t, along with $E_t > 0$ and $G_t > 0$, but drop assumptions (1) and (2). We permit the rate of interest to vary over time, too, and write r_t for the rate at time t.

Equations (5), (6), (14), and (21) remain valid, and we end up with disposable income

$$Y_t = X_t - (1 - \alpha) G_t > X_t - G_t \tag{30}$$

which is identical to the result (22) obtained in the steady-state case: disposable income increases by the primary deficit αG. Note also that inequality (30) is valid independently of the level of interest, and regardless of whether the present values of production and government spending exists or not. This, because net taxes (taxes minus interest income on government bonds received) are $(1 - \alpha) G$, and they, rather than total taxes $T = (1 - \alpha) G + rD$, are to be deduced from pre-tax income X in order to arrive at disposable income.

Consider the case that the relevant present values exist. The present value function (17) is now defined as

$$\Omega(x) = \sum_{t=0}^{\infty} \left(\prod_{\tau=0}^{t} \frac{1}{(1 + r_\tau)} \right) x_t. \tag{31}$$

Assume that the present value of production is finite:

$$\Omega(X) < \infty \tag{32}$$

This assumption corresponds to the case that the rate of interest exceeds the growth rate in the in the steady state case. As $0 < G_t < X_t$ for all t holds true by assumption (29), the present values of disposable income under the pay-as-you-go regime and under the debt regime are finite

$$\Omega(X - G) \quad = \quad \Omega(X) - \Omega(G) < \infty \tag{33}$$

$$\Omega(Y) \quad = \quad \Omega(X) - (1 - \alpha) \Omega(G) < \infty. \tag{34}$$

The present value of disposable income under the debt regime exceeds that under the pay-as-you-go regime by the present value of the deficit:

$$\Omega(Y) - (\Omega(X - G)) = \alpha \Omega(G) > 0. \tag{35}$$

As Ricardian equivalence requires Ricardian behavior and Ricardian expectations, the violation of Ricardian expectations induced by Ricardian behavior in such a very general setting proves that the Ricardian equivalence thesis cannot rationally be upheld.

7   Conclusion

Martin Feldstein (1976, 323f.) has shown that the Ricardian equivalence thesis does not apply when the rate of interest does not exceed the growth rate. This note has shown that that Ricardian equivalence is violated also for the case that the rate of interest exceeds the growth rate. The argument developed here leads to the conclusion that regardless of the level of the interest rate, Ricardian equivalence does not hold true in a growing economy. Simply assuming that the agents exhibit Ricardian behavior leads to a violation of their Ricardian expectations. The contradiction with Barro's argument is traceable to Barro's omission of interest payments on government debt as part of the households' disposable income.

It has been urged elsewhere that Ricardian equivalence is quite irrelevant regarding fiscal policy, both from a theoretical and an empirical perspective (Romer 2011, 579-598, Schlicht 2006, Sect. 9). It is shown here that the thesis is internally contradictory. From this it would follow that the thesis cannot even provide a "useful theoretical baseline" (Romer, 2011, 598). Despite its logical appeal and wide-spread acceptance as a theoretical proposition, it is wrong.

comments on an earlier version of the manuscript that helped me to improve the presentation of the argument.

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Note : all calculations are elementary and can be done by hand. I have added the calculations done in *Mathematica* for making it easier to check the reults.

An Example (Section 3)

■ **Production X and government expenditure G (Eqs. 2 and 3)**

```
X[t_] := (1 + g)^t X_0;
G[t_] := (1 + g)^t G_0;
```

■ **Government debt (Eqs. 5 to 7)**

```
Unprotect[D];
```

Note: The symbol D is natively used by *Mathematica* for the differential operator. The command Unprotect[D] permits the use of the symbol D for government debt, in conformity with the symbol in the paper.

$$D[t\_] := \frac{\left(-1 + (1+g)^t\right) \alpha G_0}{g}$$

Note: This defines the function D(t). The following commands check whether D(t) is actually the solution to equation (6).

```
D[0] == 0
```

True

Note: The command expression1 == expression2 returns True if exporession1 and expression2 are mathematically the same.

```
Simplify[D[t + 1] == D[t] + α G[t]]
```

True

■ **Asymptotic ratio of debt to government expenditure (Eq.8)**

$$\text{Limit}\left[\frac{D[t]}{G[t]}, t \to \infty, \text{Assumptions} \to (i > 0) \&\& (g > 0)\right]$$

$\frac{\alpha}{g}$

■ **Asymptotic ratio of debt to production (Eq. 8)**

$$\text{Limit}\left[\frac{D[t]}{Y[t]}, t \to \infty, \text{Assumptions} \to (i > 0) \&\& (g > 0)\right]$$

$$\text{Limit}\left[\frac{\left(-1 + (1+g)^t\right) \alpha G_0}{g Y[t]}, t \to \infty, \text{Assumptions} \to i > 0 \&\& g > 0\right]$$

■ **Share of interest on government debt in the government budget (Eqs. 9 and 10)**

$$\text{Simplify}\left[\frac{r D[t]}{G[t] + r D[t]} == \frac{\left((1+g)^t - 1\right) r \alpha}{(1+g)^t g + \left((1+g)^t - 1\right) r \alpha}\right]$$

True

- **Asymptotic share of interest on debt in the government budget (Eq. 11)**

```
Limit[ (r D[t]) / (G[t] + r D[t]), t → ∞, Assumptions → (i > 0) && (g > 0)]
```

$$\frac{r\,\alpha}{g + r\,\alpha}$$

- **Numerical example after Eq. 11**

$$\frac{r\,\alpha}{g + r\,\alpha} \; /. \; g \to 0.02 \; /. \; r \to 0.04 \; /. \; \alpha \to 0.10$$

```
0.166667
```

- **Present value of goverment debt (Eq. 12)**

```
Simplify[ (1/(1+i))^t D[t] == α/g ((1+g)^t - 1)/((1+i)^t) G₀, Assumptions → (i > g) && (g > 0)]
```

```
True
```

```
Limit[ (1/(1+i))^t D[t], t → ∞, Assumptions → (i > g) && (g > 0)]
```

```
0
```

- **Taxes (Eq. 14)**

```
T[t_] := (1 - α) G[t] + r D[t]
```

- **Asymptotic ratio of taxes to production (Eq. 15)**

```
Limit[ T[t]/X[t], t → ∞, Assumptions → (i > 0) && (g > 0)]
```

$$-\frac{(g\,(-1 + \alpha) - r\,\alpha)\,G_0}{g\,X_0}$$

```
Simplify[ - ((g (-1 + α) - r α) G₀)/(g X₀) == (1 + ((r - g) α)/g) G₀/X₀ ]
```

```
True
```

- **Tax increase from pay-as-you-go to debt regime minus interest payments necessary under the debt regime (Eq. 16)**

```
Limit[ (T[t] - G[t])/X[t] - (r D[t])/X[t], t → ∞, Assumptions → (i > 0) && (g > 0)]
```

$$-\frac{\alpha\,G_0}{X_0}$$

```
Simplify[ - ((g - i) α G₀)/(g Y₀) == (i - g)/g  (α G₀)/Y₀ ]
```

```
True
```

- **Present value function (Eq. 17)**

$$\Omega[x\_] := \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t x[t];$$

- **Household wealth with balanced budget (Eq. 20)**

```
Simplify[Ω[X] - Ω[G]]
```

$$\frac{(1 + r)\,(G_0 - X_0)}{g - r}$$

■ **Disposable income (Eqs. 21 and 22)**

```
Y[t_] := X[t] - T[t] + r D[t];
```

```
Y[t]
```

$-(1+g)^t (1-\alpha) G_0 + (1+g)^t X_0$

```
Simplify[Y[t] == X[t] - (1 - α) G[t]]
```

True

■ **Present value of disposable income (Eq. 23)**

```
Simplify[Ω[Y]]
```

$-\dfrac{(1+r) \left((-1+\alpha) G_0 + X_0\right)}{g-r}$

■ **Difference of present values (Eq. 24)**

```
Simplify[Ω[Y] - (Ω[X] - Ω[G])]
```

$-\dfrac{(1+r) \alpha G_0}{g-r}$

Solving the Contradiction (Section 4)

■ **Present Value of Taxes (Eq. 26)**

```
Ω[T]
```

$\dfrac{G_0 + r G_0}{-g+r}$

■ **Present Value of Government Expenditure (Eq.27)**

```
Ω[G]
```

$\dfrac{(1+r) G_0}{-g+r}$

Arbitrary Growth Paths (Section 6)

Destroy all previous symbols and results and unprotect the symbol D again so it can again be used for denoting government debt:

```
Quit[];
```

```
Unprotect[D];
```

■ **Replacement for Eq. 7 for arbitrary growth paths**

$$D[t\_] := \sum_{\tau=0}^{-1+t} \alpha G[\tau]$$

```
D[0] == 0
```

True

```
Simplify[D[t + 1] == (D[t] + α G[t])]
```

$$\alpha G[t] + \sum_{\tau=0}^{-1+t} \alpha G[\tau] == \sum_{\tau=0}^{t} \alpha G[\tau]$$

Note: *Mathematica* did not recognize that expression1 equals expression2 here, so it returned an equivalent but simplified statement. It is obviously true.

- **Replacement for Eq. 14 for arbitrary growth paths**

```
T[t_] := (1 - α) G[t] + r[t] D[t]
```

```
T[t]
```

$$(1 - \alpha) \, \text{G}[t] + r[t] \sum_{\tau=0}^{-1+t} \alpha \, \text{G}[\tau]$$

- **Replacement for Eq. 18 for arbitrary growth paths (Eq. 30)**

```
Y[t_] := X[t] - T[t] + r[t] D[t]
```

```
Y[t]
```

$$- (1 - \alpha) \, \text{G}[t] + X[t]$$