Referee Report on "Circuit Theory Extended: The Role of Speculation in Crises"

This is an interesting extension of Circuit Theory and I recommend publication.

General comments

While the paper is predicated on the method developed in Keen 2011, it has actually been implemented using the methodology of Godley & Lavoie. These are related, but are also very different. There would be far too much work involved in replicating the paper using my method rather than Godley & Lavoie, but I think it is important to point out the differences here. I would urge the author to consider using this methodology for a subsequent paper on the same topic.

Godley & Lavoie's method involves deriving a set of difference equations from a flow table in which all rows and all columns sum to zero. The equations of the system then have to be derived by imposing dynamic relations on components of the table, and deriving a set of equations out of which one is redundant because it is implied by all the others. It is therefore a non-trivial exercise to derive the equations of the model from the table

My method involves specifying the financial accounts in a banking system, and specifying the flows between them in continuous time--ie, each entry (say, flowi) in a column (say column X) is an entry in a differential equation. The equation for that column--which is a financial account and a system state in the dynamic model of the economy--is the $\frac{d}{dt}X(t) = \sum_{i=1}^{n} flow_i$, where there are n terms in the equation. Clearly the sum of the columns cannot be zero (since then all the equations would be $\frac{d}{dt}X(t) = 0$).

The advantage of my method over Godley and Lavoie's is that the financial equations of the model can be derived automatically, by symbolically summing each column. This makes what is a difficult exercise in Godley's method a trivial one in mine--even though the equations derived themselves may seem daunting to those not used to differential equations.

Since publishing "Solving the Paradox of Monetary Profits", I have modified my method to make it double-entry consistent. The Appendix to this report sets out the programs used to do this in the computer algebra program Mathcad. There is now a "beta" version of simulation program Minsky, which provides a GUI method to build these models:

http://www.debtdeflation.com/blogs/minsky/

The method only derives the financial equations for an economic simulation--equations for the physical processes set in train by the financial flows must be developed separately.

Specific comments

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Page 2, 3rd paragraph. "UIP..." should be "The empirical failure of UIP";

Page 3, paragraph 3. The argument that Graziani's non-commodity approach only applied after the failure of Bretton-Woods is incorrect. It is more accurate to ground Graziani's analysis in the view--well expressed in Graeber's "Debt--the first 5,000 years"--that money has always been credit and non-commodity in nature. So this wasn't a new role after the collapse of Bretton Woods, but a return to normal (if unstable!) practice after the collapse of an attempt to ground money in relation to a commodity (gold).

Appendix: The Keen method for deriving models of financial dynamics

Programs

System definitions

Account (system state) names	$\text{AccountNames}(x) \equiv \text{submatrix}(x, 1, 1, 1, \text{cols}(x) - 1)$
System states	$\text{Functions}(x) \equiv \text{submatrix}(x, 3, 3, 1, \text{cols}(x) - 1)$
Assets, Liabilties, Equity	$\text{Types}(x) \equiv \text{submatrix}(x, 0, 0, 1, \text{cols}(x) - 1)$
Flow equations	$\text{Equations}(x) \equiv \text{submatrix}(x, 4, \text{rows}(x) - 1, 1, \text{cols}(x) - 1)$
Initial conditions	$\text{Initial}_{\text{Conditions}}(x) \equiv \text{submatrix}(x, 2, 2, 1, \text{cols}(x) - 1)^{T}$
Operations between system states	$\text{Operations}(x) \equiv \text{submatrix}(x, 4, \text{rows}(x) - 1, 0, 0)$
Intangible Asset	$A_I := \infty$
Tangible Asset	$A_T := 1$
Liability owned by entity	$\text{Liab}_{\text{own}} := 0$
Liability of entity to others	$\text{Liab} := -1$

Equity of entity $Equity := -0.1$

To number rows & columns $lbl(i,j) := j$

The basic structure of a financial table is as shown in S0.

$$S_0 := \begin{pmatrix} \text{"Priv. Bank"} & A_I & A_T & Liab_{own} & Liab & Liab & Liab & Equity \\ \text{"Account"} & \text{"G'will"} & \text{"Loan"} & \text{"Bk Vault"} & \text{"Firms"} & \text{"Wkrs"} & \text{"Caps"} & \text{"Bk Equity"} \\ \text{"Value"} & Goodwill & 0 & -Goodwill & 0 & 0 & 0 & 0 \\ \text{"Symbol"} & B_G(t) & F_L(t) & B_V(t) & F_D(t) & W_D(t) & C_D(t) & B_E(t) \\ \text{"Lend"} & 0 & 0 & Loan & -Loan & 0 & 0 & 0 \\ \text{"Rec. Loan"} & -Loan & Loan & 0 & 0 & 0 & 0 & 0 \\ \text{"Restore G'will"} & Loan & 0 & -Loan & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$S_0 \rightarrow \begin{pmatrix} \text{"Priv. Bank"} & \infty & 1 & 0 & -1 & -1 & -1 & -0.1 \\ \text{"Account"} & \text{"G'will"} & \text{"Loan"} & \text{"Bk Vault"} & \text{"Firms"} & \text{"Wkrs"} & \text{"Caps"} & \text{"Bk Equity"} \\ \text{"Value"} & Goodwill & 0 & -Goodwill & 0 & 0 & 0 & 0 \\ \text{"Symbol"} & B_G(t) & F_L(t) & B_V(t) & F_D(t) & W_D(t) & C_D(t) & B_E(t) \\ \text{"Lend"} & 0 & 0 & Loan & -Loan & 0 & 0 & 0 \\ \text{"Rec. Loan"} & -Loan & Loan & 0 & 0 & 0 & 0 & 0 \\ \text{"Restore G'will"} & Loan & 0 & -Loan & 0 & 0 & 0 & 0 \end{pmatrix}$$

Programs

Check that all rows sum to zero

$$
\mathrm{Audit}(x) := \left|
\begin{array}{l}
\mathrm{Acs} \leftarrow \mathrm{AccountNames}(x) \\
\mathrm{Fns} \leftarrow \mathrm{Functions}(x) \\
\mathrm{Type} \leftarrow \mathrm{Types}(x) \\
\mathrm{Eqns} \leftarrow \mathrm{Equations}(x) \\
\mathrm{IC} \leftarrow \mathrm{Initial}_{\mathrm{Conditions}}(x) \\
\text{for } i \in 0 .. \mathrm{rows}(\mathrm{Eqns}) - 1 \\
\quad \left| \begin{array}{l}
\mathrm{Ans}_{k,0} \leftarrow \mathrm{Operations}(x)_i \\
\mathrm{Ans}_{k,1} \leftarrow \displaystyle\sum_{j=0}^{\mathrm{cols}(\mathrm{Eqns})-1} \mathrm{Eqns}_{i,j} \\
k \leftarrow k + 1
\end{array} \right. \\
\mathrm{Ans}_{k,0} \leftarrow \text{"Initial Conditions"} \\
\mathrm{Ans}_{k,1} \leftarrow \displaystyle\sum_{j=0}^{\mathrm{rows}(\mathrm{IC})-1} \mathrm{IC}_j \\
\text{return } \mathrm{stack}[(\text{"Operation"} \quad \text{"Total"}), \mathrm{Ans}]
\end{array}
\right.
$$

Return a set of coupled ordinary differential equations from the table

$$\text{ODEs}(x) := \left| \begin{array}{l} \text{Fns} \leftarrow \text{Functions}(x) \\ \text{Type} \leftarrow \text{Types}(x) \\ \text{Eqns} \leftarrow \text{Equations}(x) \\ \text{IC} \leftarrow \text{Initial}_{\text{Conditions}}(x) \\ \text{for } i \in 0 .. \text{cols}(\text{Fns}) - 1 \\ \quad \left| \begin{array}{l} E_i \leftarrow \frac{d}{dt}\text{Fns}_i = \sum \text{Eqns}^{\langle i \rangle} \quad \text{if } \text{Type}_i > 0 \\ E_i \leftarrow \frac{d}{dt}\text{Fns}_i = -\sum \text{Eqns}^{\langle i \rangle} \quad \text{otherwise} \end{array} \right. \\ \text{return } E \end{array} \right.$$

Return the initial conditions for the equations (using the symbolic substitution operator to replace t with 0)

$$\text{ICs}(x) := \left| \begin{array}{l} \text{Fns} \leftarrow \text{Functions}(x) \\ \text{Type} \leftarrow \text{Types}(x) \\ \text{Eqns} \leftarrow \text{Equations}(x) \\ \text{IC} \leftarrow \text{Initial}_{\text{Conditions}}(x) \\ \text{for } i \in 0 .. \text{cols}(\text{Fns}) - 1 \\ \quad \left| \begin{array}{l} I_i \leftarrow \text{Fns}_i = \text{IC}_i \quad \text{if } \text{Type}_i > 0 \\ I_i \leftarrow \text{Fns}_i = -\text{IC}_i \quad \text{otherwise} \end{array} \right. \\ \text{return } I \end{array} \right.$$

Document the table

```
Label(x) := | Name ← x[0,0]
            | x[0,0] ← "Type"
            | numrows ← rows(x)
            | numcols ← cols(x)
            | toprow ← matrix(1, numcols + 1, lbl − 1)
            | frontcol ← matrix(1, numrows, lbl − 3)^T
            | labelled ← stack(toprow, augment(frontcol, x))
            | labelled[0,0] ← Name
            | labelled[0,1] ← "Column"
            | labelled[1,0] ← "Rows"
            | labelled[2,0] ← "Variables"
            | labelled[3,0] ← "Init Conds"
            | return labelled
```

Calculate liabilities

$$
\mathrm{Liabilities}(x) \equiv \left[\begin{array}{l}
\mathrm{Acs} \leftarrow \mathrm{AccountNames}(x) \\
\mathrm{Fns} \leftarrow \mathrm{Functions}(x) \\
\mathrm{Type} \leftarrow \mathrm{Types}(x) \\
\mathrm{Eqns} \leftarrow \mathrm{Equations}(x) \\
\mathrm{IC} \leftarrow \mathrm{Initial}_{\mathrm{Conditions}}(x) \\
\text{for } i \in 0 .. \mathrm{cols}(\mathrm{Type}) - 1 \\
\quad \text{if } \mathrm{Type}_i \le 0 \\
\qquad \left| \begin{array}{l}
\mathrm{Ans}_{j,0} \leftarrow \mathrm{Acs}_i \\
\mathrm{Ans}_{j,1} \leftarrow \frac{d}{dt}\mathrm{Fns}_i \\
\mathrm{Ans}_{j,2} \leftarrow \sum \mathrm{Eqns}^{\langle i \rangle} \\
j \leftarrow j + 1
\end{array} \right. \\
\mathrm{Ans}_{j,0} \leftarrow \text{"Sum"} \\
\mathrm{Ans}_{j,1} \leftarrow \sum \mathrm{Ans}^{\langle 1 \rangle} \\
\mathrm{Ans}_{j,2} \leftarrow \sum \mathrm{Ans}^{\langle 2 \rangle} \\
\text{return } \mathrm{stack}[(\text{"Account"} \quad \text{"Change"} \quad \text{"Amount"}), \mathrm{Ans}]
\end{array}\right]
$$

Calculate Loans

$$
\mathrm{Loans}(x) := \left|
\begin{array}{l}
\mathrm{Acs} \leftarrow \mathrm{AccountNames}(x) \\
\mathrm{Fns} \leftarrow \mathrm{Functions}(x) \\
\mathrm{Type} \leftarrow \mathrm{Types}(x) \\
\mathrm{Eqns} \leftarrow \mathrm{Equations}(x) \\
\mathrm{IC} \leftarrow \mathrm{Initial}_{\mathrm{Conditions}}(x) \\
\text{for } i \in 0 .. \mathrm{cols}(\mathrm{Type}) - 1 \\
\quad \text{if } \mathrm{Type}_i = 1 \\
\qquad \left|
\begin{array}{l}
\mathrm{Ans}_{j,0} \leftarrow \mathrm{Acs}_i \\
\mathrm{Ans}_{j,1} \leftarrow \frac{d}{dt}\mathrm{Fns}_i \\
\mathrm{Ans}_{j,2} \leftarrow \sum \mathrm{Eqns}^{\langle i \rangle} \\
j \leftarrow j + 1
\end{array}
\right. \\
\mathrm{Ans}_{j,0} \leftarrow \text{"Sum"} \\
\mathrm{Ans}_{j,1} \leftarrow \sum \mathrm{Ans}^{\langle 1 \rangle} \\
\mathrm{Ans}_{j,2} \leftarrow \sum \mathrm{Ans}^{\langle 2 \rangle} \\
\text{return } \mathrm{stack}[(\text{"Account"} \quad \text{"Change"} \quad \text{"Amount"}), \mathrm{Ans}]
\end{array}
\right.
$$

Calculate Assets

$$
\text{Assets}(x) := \left|
\begin{array}{l}
\text{Acs} \leftarrow \text{AccountNames}(x) \\
\text{Fns} \leftarrow \text{Functions}(x) \\
\text{Type} \leftarrow \text{Types}(x) \\
\text{Eqns} \leftarrow \text{Equations}(x) \\
\text{IC} \leftarrow \text{Initial}_{\text{Conditions}}(x) \\
\text{for } i \in 0 .. \text{cols}(\text{Type}) - 1 \\
\quad \text{if } \text{Type}_i > 0 \\
\qquad \left|
\begin{array}{l}
\text{Ans}_{j,0} \leftarrow \text{Acs}_i \\
\text{Ans}_{j,1} \leftarrow \dfrac{d}{dt}\text{Fns}_i \\
\text{Ans}_{j,2} \leftarrow \sum \text{Eqns}^{\langle i \rangle} \\
j \leftarrow j + 1
\end{array}
\right. \\
\text{Ans}_{j,0} \leftarrow \text{"Sum"} \\
\text{Ans}_{j,1} \leftarrow \sum \text{Ans}^{\langle 1 \rangle} \\
\text{Ans}_{j,2} \leftarrow \sum \text{Ans}^{\langle 2 \rangle} \\
\text{return } \text{stack}[(\text{"Account"} \quad \text{"Change"} \quad \text{"Amount"}), \text{Ans}]
\end{array}
\right.
$$

Calculate the change in the money level in circulation

$$
\mathrm{Circulation}(x) \equiv \left|
\begin{array}{l}
\mathrm{Acs} \leftarrow \mathrm{AccountNames}(x) \\
\mathrm{Fns} \leftarrow \mathrm{Functions}(x) \\
\mathrm{Type} \leftarrow \mathrm{Types}(x) \\
\mathrm{Eqns} \leftarrow \mathrm{Equations}(x) \\
\mathrm{IC} \leftarrow \mathrm{Initial}_{\mathrm{Conditions}}(x) \\
\text{for } i \in 0..\mathrm{cols}(\mathrm{Type}) - 1 \\
\quad \text{if } \mathrm{Type}_i < 0 \\
\qquad \left| \begin{array}{l}
\mathrm{Ans}_{j,0} \leftarrow \mathrm{Acs}_i \\
\mathrm{Ans}_{j,1} \leftarrow \frac{d}{dt}\mathrm{Fns}_i \\
\mathrm{Ans}_{j,2} \leftarrow \sum \mathrm{Eqns}^{\langle i \rangle} \\
j \leftarrow j + 1
\end{array} \right. \\
\mathrm{Ans}_{j,0} \leftarrow \text{"Sum"} \\
\mathrm{Ans}_{j,1} \leftarrow \sum \mathrm{Ans}^{\langle 1 \rangle} \\
\mathrm{Ans}_{j,2} \leftarrow \sum \mathrm{Ans}^{\langle 2 \rangle} \\
\text{return } \mathrm{stack}[(\text{"Account"} \quad \text{"Change"} \quad \text{"Amount"}), \mathrm{Ans}]
\end{array}
\right.
$$

Documentation of sample model

$$L_1 := \mathrm{Label}(S_0)$$

$$L_1 \rightarrow \begin{pmatrix} \text{"Priv. Bank"} & \text{"Column"} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \text{"Rows"} & \text{"Type"} & \infty & 1 & 0 & -1 & -1 & -1 & -0.1 \\ \text{"Variables"} & \text{"Account"} & \text{"G'will"} & \text{"Loan"} & \text{"Bk Vault"} & \text{"Firms"} & \text{"Wkrs"} & \text{"Caps"} & \text{"Bk Equity"} \\ \text{"Init Conds"} & \text{"Value"} & \text{Goodwill} & 0 & -\text{Goodwill} & 0 & 0 & 0 & 0 \\ 0 & \text{"Symbol"} & B_G(t) & F_L(t) & B_V(t) & F_D(t) & W_D(t) & C_D(t) & B_E(t) \\ 1 & \text{"Lend"} & 0 & 0 & \text{Loan} & -\text{Loan} & 0 & 0 & 0 \\ 2 & \text{"Rec. Loan"} & -\text{Loan} & \text{Loan} & 0 & 0 & 0 & 0 & 0 \\ 3 & \text{"Restore G'will"} & \text{Loan} & 0 & -\text{Loan} & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{Assets}(S_0) \rightarrow \begin{pmatrix} \text{"Account"} & \text{"Change"} & \text{"Amount"} \\ \text{"G'will"} & \frac{d}{dt}B_G(t) & 0 \\ \text{"Loan"} & \frac{d}{dt}F_L(t) & \text{Loan} \\ \text{"Sum"} & \frac{d}{dt}B_G(t) + \frac{d}{dt}F_L(t) & \text{Loan} \end{pmatrix}$$

$$\mathrm{Liabilities}(S_0) \rightarrow \begin{pmatrix} \text{"Account"} & \text{"Change"} & \text{"Amount"} \\ \text{"Bk Vault"} & \frac{d}{dt}B_V(t) & 0 \\ \text{"Firms"} & \frac{d}{dt}F_D(t) & -\mathrm{Loan} \\ \text{"Wkrs"} & \frac{d}{dt}W_D(t) & 0 \\ \text{"Caps"} & \frac{d}{dt}C_D(t) & 0 \\ \text{"Bk Equity"} & \frac{d}{dt}B_E(t) & 0 \\ \text{"Sum"} & \frac{d}{dt}B_E(t) + \frac{d}{dt}B_V(t) + \frac{d}{dt}C_D(t) + \frac{d}{dt}F_D(t) + \frac{d}{dt}W_D(t) & -\mathrm{Loan} \end{pmatrix}$$

$$\mathrm{Loans}(S_0) \rightarrow \begin{pmatrix} \text{"Account"} & \text{"Change"} & \text{"Amount"} \\ \text{"Loan"} & \frac{d}{dt}F_L(t) & \mathrm{Loan} \\ \text{"Sum"} & \frac{d}{dt}F_L(t) & \mathrm{Loan} \end{pmatrix}$$

$$\text{Circulation}(S_0) \rightarrow \begin{pmatrix} \text{"Account"} & \text{"Change"} & \text{"Amount"} \\ \text{"Firms"} & \frac{d}{dt}F_D(t) & -\text{Loan} \\ \text{"Wkrs"} & \frac{d}{dt}W_D(t) & 0 \\ \text{"Caps"} & \frac{d}{dt}C_D(t) & 0 \\ \text{"Bk Equity"} & \frac{d}{dt}B_E(t) & 0 \\ \text{"Sum"} & \frac{d}{dt}B_E(t) + \frac{d}{dt}C_D(t) + \frac{d}{dt}F_D(t) + \frac{d}{dt}W_D(t) & -\text{Loan} \end{pmatrix}$$

$$\text{Audit}(S_0) \rightarrow \begin{pmatrix} \text{"Operation"} & \text{"Total"} \\ \text{"Lend"} & 0 \\ \text{"Rec. Loan"} & 0 \\ \text{"Restore G'will"} & 0 \\ \text{"Initial Conditions"} & 0 \end{pmatrix}$$

$$\mathrm{ODEs}\left(S_0\right) \rightarrow \begin{pmatrix} \frac{d}{dt}B_G(t) = 0 \\ \frac{d}{dt}F_L(t) = \mathrm{Loan} \\ \frac{d}{dt}B_V(t) = 0 \\ \frac{d}{dt}F_D(t) = \mathrm{Loan} \\ \frac{d}{dt}W_D(t) = 0 \\ \frac{d}{dt}C_D(t) = 0 \\ \frac{d}{dt}B_E(t) = 0 \end{pmatrix}$$

Programs

Simple example--19th century Free Banking

$$S_{FB} := \begin{pmatrix} \text{"Priv. Bank"} & A_I & A_T & Liab_{own} & Liab & Liab & Equity \\ \text{"Account"} & \text{"G'will"} & \text{"Loan"} & \text{"Bk Vault"} & \text{"Firms"} & \text{"Wkrs"} & \text{"Bk Equity"} \\ \text{"Value"} & Goodwill & 0 & -Goodwill & 0 & 0 & 0 \\ \text{"Symbol"} & B_G(t) & F_L(t) & B_V(t) & F_D(t) & W_D(t) & B_E(t) \\ \text{"Lend"} & 0 & 0 & Loan & -Loan & 0 & 0 \\ \text{"Rec. Loan"} & -Loan & Loan & 0 & 0 & 0 & 0 \\ \text{"Restore G'will"} & Loan & 0 & -Loan & 0 & 0 & 0 \\ \text{"Compound Debt"} & -Int & Int & 0 & 0 & 0 & 0 \\ \text{"Pay Interest"} & 0 & 0 & 0 & Int & 0 & -Int \\ \text{"Record Payment"} & Int & -Int & 0 & 0 & 0 & 0 \\ \text{"Wages"} & 0 & 0 & 0 & Wage & -Wage & 0 \\ \text{"Consume"} & 0 & 0 & 0 & -Cons_W & Cons_W & 0 \\ \text{"Consume"} & 0 & 0 & 0 & -Cons_B & 0 & Cons_B \end{pmatrix}$$

$$Audit(S_{FB}) \rightarrow \begin{pmatrix} \text{"Operation"} & \text{"Total"} \\ \text{"Lend"} & 0 \\ \text{"Rec. Loan"} & 0 \\ \text{"Restore G'will"} & 0 \\ \text{"Compound Debt"} & 0 \\ \text{"Pay Interest"} & 0 \\ \text{"Record Payment"} & 0 \\ \text{"Wages"} & 0 \\ \text{"Consume"} & 0 \\ \text{"Consume"} & 0 \\ \text{"Initial Conditions"} & 0 \end{pmatrix}$$

$$\mathrm{ODEs}\left(S_{FB}\right) \rightarrow \begin{pmatrix} \frac{d}{dt}B_G(t) = 0 \\ \frac{d}{dt}F_L(t) = \mathrm{Loan} \\ \frac{d}{dt}B_V(t) = 0 \\ \frac{d}{dt}F_D(t) = \mathrm{Cons}_B + \mathrm{Cons}_W - \mathrm{Int} + \mathrm{Loan} - \mathrm{Wage} \\ \frac{d}{dt}W_D(t) = \mathrm{Wage} - \mathrm{Cons}_W \\ \frac{d}{dt}B_E(t) = \mathrm{Int} - \mathrm{Cons}_B \end{pmatrix}$$

Symbolic Analysis

$$\mathrm{Assets}\left(S_{FB}\right) \rightarrow \begin{pmatrix} \text{"Account"} & \text{"Change"} & \text{"Amount"} \\ \text{"G'will"} & \frac{d}{dt}B_G(t) & 0 \\ \text{"Loan"} & \frac{d}{dt}F_L(t) & \mathrm{Loan} \\ \text{"Sum"} & \frac{d}{dt}B_G(t) + \frac{d}{dt}F_L(t) & \mathrm{Loan} \end{pmatrix}$$

$$\text{Liabilities}\left(S_{FB}\right) \rightarrow \begin{pmatrix} \text{"Account"} & \text{"Change"} & \text{"Amount"} \\ \text{"Bk Vault"} & \frac{d}{dt}B_V(t) & 0 \\ \text{"Firms"} & \frac{d}{dt}F_D(t) & Int - Cons_W - Cons_B - Loan + Wage \\ \text{"Wkrs"} & \frac{d}{dt}W_D(t) & Cons_W - Wage \\ \text{"Bk Equity"} & \frac{d}{dt}B_E(t) & Cons_B - Int \\ \text{"Sum"} & \frac{d}{dt}B_E(t) + \frac{d}{dt}B_V(t) + \frac{d}{dt}F_D(t) + \frac{d}{dt}W_D(t) & -Loan \end{pmatrix}$$

$$\text{Circulation}\left(S_{FB}\right) \rightarrow \begin{pmatrix} \text{"Account"} & \text{"Change"} & \text{"Amount"} \\ \text{"Firms"} & \frac{d}{dt}F_D(t) & Int - Cons_W - Cons_B - Loan + Wage \\ \text{"Wkrs"} & \frac{d}{dt}W_D(t) & Cons_W - Wage \\ \text{"Bk Equity"} & \frac{d}{dt}B_E(t) & Cons_B - Int \\ \text{"Sum"} & \frac{d}{dt}B_E(t) + \frac{d}{dt}F_D(t) + \frac{d}{dt}W_D(t) & -Loan \end{pmatrix}$$

Functional substitutions

$$\text{Loan} := \frac{F_L(t)}{\tau_L} \qquad \text{Repay} := \frac{F_L(t)}{\tau_R} \qquad \text{Wage} := \frac{(1-s)}{\tau_S}\cdot F_D(t) \qquad \text{Cons}_W := \frac{W_D(t)}{\tau_W} \qquad \text{Cons}_B := \frac{B_E(t)}{\tau_B} \qquad \text{Repay} := \frac{F_L(t)}{\tau_R} \qquad \text{Int} := r_L \cdot F_L(t)$$

$$S_{FB} := \begin{pmatrix} \text{"Priv. Bank"} & A_I & A_T & \text{Liab}_{own} & \text{Liab} & \text{Liab} & \text{Equity} \\ \text{"Account"} & \text{"G'will"} & \text{"Loan"} & \text{"Bk Vault"} & \text{"Firms"} & \text{"Wkrs"} & \text{"Bk Equity"} \\ \text{"Value"} & \text{Goodwill} & \text{Init}_{Loan} & -\text{Goodwill} & -\text{Init}_{Loan} & 0 & 0 \\ \text{"Symbol"} & B_G(t) & F_L(t) & B_V(t) & F_D(t) & W_D(t) & B_E(t) \\ \text{"Lend"} & 0 & 0 & \text{Loan} & -\text{Loan} & 0 & 0 \\ \text{"Rec. Loan"} & -\text{Loan} & \text{Loan} & 0 & 0 & 0 & 0 \\ \text{"Restore G'will"} & \text{Loan} & 0 & -\text{Loan} & 0 & 0 & 0 \\ \text{"Compound Debt"} & -\text{Int} & \text{Int} & 0 & 0 & 0 & 0 \\ \text{"Pay Interest"} & 0 & 0 & 0 & \text{Int} & 0 & -\text{Int} \\ \text{"Record Payment"} & \text{Int} & -\text{Int} & 0 & 0 & 0 & 0 \\ \text{"Wages"} & 0 & 0 & 0 & \text{Wage} & -\text{Wage} & 0 \\ \text{"Consume"} & 0 & 0 & 0 & -\text{Cons}_W & \text{Cons}_W & 0 \\ \text{"Consume"} & 0 & 0 & 0 & -\text{Cons}_B & 0 & \text{Cons}_B \\ \text{"Repay"} & 0 & 0 & -\text{Repay} & \text{Repay} & 0 & 0 \\ \text{"Record Repay"} & \text{Repay} & -\text{Repay} & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{Functions}(S_{FB})^T \rightarrow \begin{pmatrix} B_G(t) \\ F_L(t) \\ B_V(t) \\ F_D(t) \\ W_D(t) \\ B_E(t) \end{pmatrix} \qquad \text{ICs}(S_{FB}) \text{ substitute}, t = 0 \ \rightarrow \begin{pmatrix} B_G(0) = \text{Goodwill} \\ F_L(0) = \text{Init}_{\text{Loan}} \\ B_V(0) = \text{Goodwill} \\ F_D(0) = \text{Init}_{\text{Loan}} \\ W_D(0) = 0 \\ B_E(0) = 0 \end{pmatrix}$$

$$\text{ODEs}(S_{FB}) \rightarrow \begin{bmatrix} \frac{d}{dt}B_G(t) = \frac{F_L(t)}{\tau_R} \\ \frac{d}{dt}F_L(t) = \frac{F_L(t)}{\tau_L} - \frac{F_L(t)}{\tau_R} \\ \frac{d}{dt}B_V(t) = \frac{F_L(t)}{\tau_R} \\ \frac{d}{dt}F_D(t) = \frac{B_E(t)}{\tau_B} - r_L \cdot F_L(t) - \frac{F_L(t)}{\tau_R} + \frac{F_L(t)}{\tau_L} + \frac{W_D(t)}{\tau_W} + \frac{F_D(t)\cdot(s-1)}{\tau_S} \\ \frac{d}{dt}W_D(t) = -\frac{W_D(t)}{\tau_W} - \frac{F_D(t)\cdot(s-1)}{\tau_S} \\ \frac{d}{dt}B_E(t) = r_L \cdot F_L(t) - \frac{B_E(t)}{\tau_B} \end{bmatrix}$$

Parameters

$\text{Goodwill} := 100 \quad \text{Years} := 100 \qquad \text{Init}_{\text{Loan}} := 10$

$\tau_L := 4$ $\tau_R := 6$ $\tau_S := \frac{1}{4}$ $s := 30\%$ $\tau_W := \frac{1}{26}$ $\tau_B := 1$ $r_L := 4\%$

Simulation

Given

$$\frac{d}{dt}B_G(t) = \frac{F_L(t)}{\tau_R} \qquad B_G(0) = \text{Goodwill}$$

$$\frac{d}{dt}F_L(t) = \frac{F_L(t)}{\tau_L} - \frac{F_L(t)}{\tau_R} \qquad F_L(0) = \text{Init}_{\text{Loan}}$$

$$\frac{d}{dt}B_V(t) = \frac{F_L(t)}{\tau_R} \qquad B_V(0) = \text{Goodwill}$$

$$\frac{d}{dt}F_D(t) = \frac{B_E(t)}{\tau_B} - r_L \cdot F_L(t) - \frac{F_L(t)}{\tau_R} + \frac{F_L(t)}{\tau_L} + \frac{W_D(t)}{\tau_W} + \frac{F_D(t)\cdot(s-1)}{\tau_S} \qquad F_D(0) = \text{Init}_{\text{Loan}}$$

$$\frac{d}{dt}W_D(t) = -\frac{W_D(t)}{\tau_W} - \frac{F_D(t)\cdot(s-1)}{\tau_S} \qquad W_D(0) = 0$$

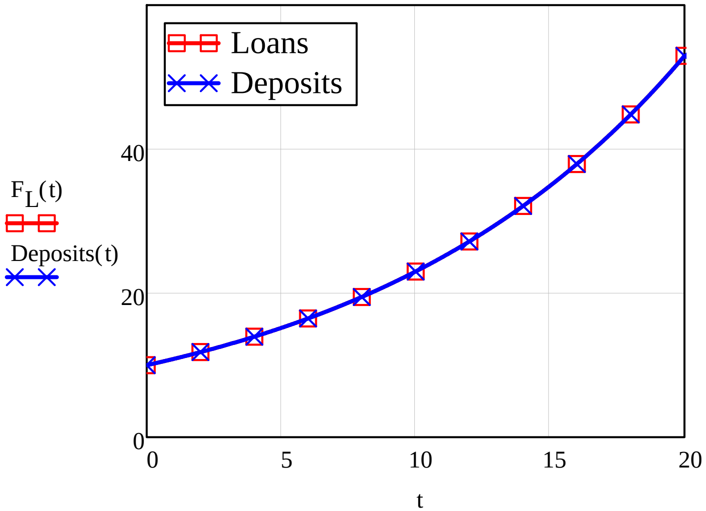
$$\frac{d}{dt}B_E(t) = r_L \cdot F_L(t) - \frac{B_E(t)}{\tau_B} \qquad B_E(0) = 0$$

$$Years := 20 \qquad Sim := Odesolve\left[\begin{pmatrix} B_G \\ F_L \\ B_V \\ F_D \\ W_D \\ B_E \end{pmatrix}, t, Years\right] \qquad \begin{pmatrix} B_G \\ F_L \\ B_V \\ F_D \\ W_D \\ B_E \end{pmatrix} := Sim$$

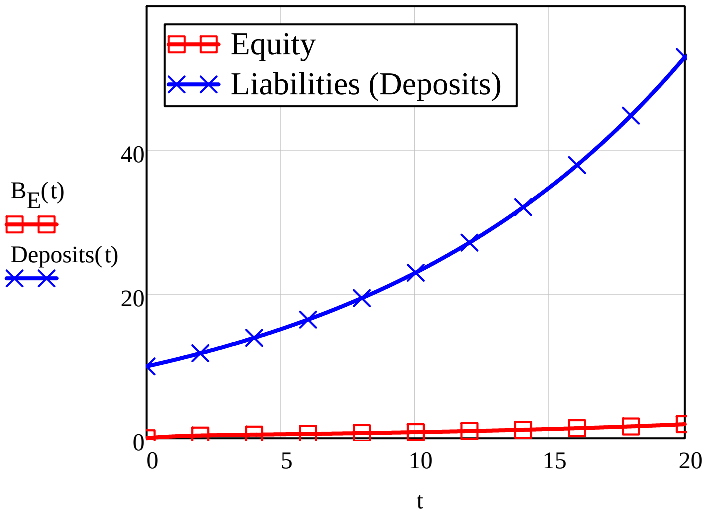
$$Deposits(t) := F_D(t) + W_D(t) + B_E(t)$$

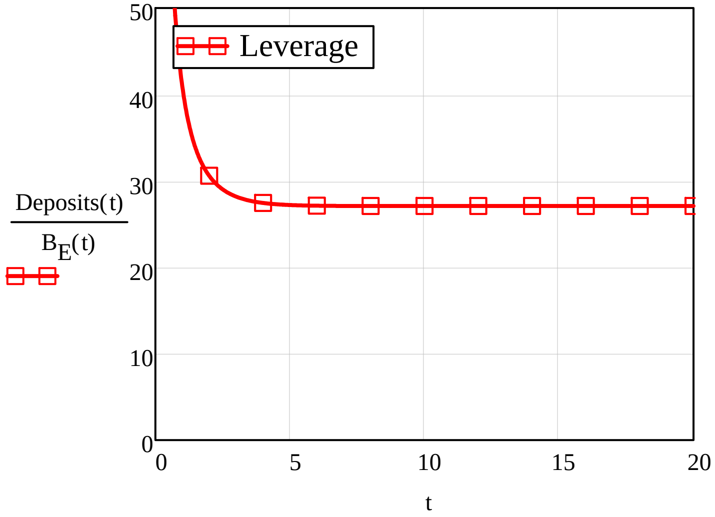
Results

$G_S := 0 \quad G_E := Years \quad G_B := 0$

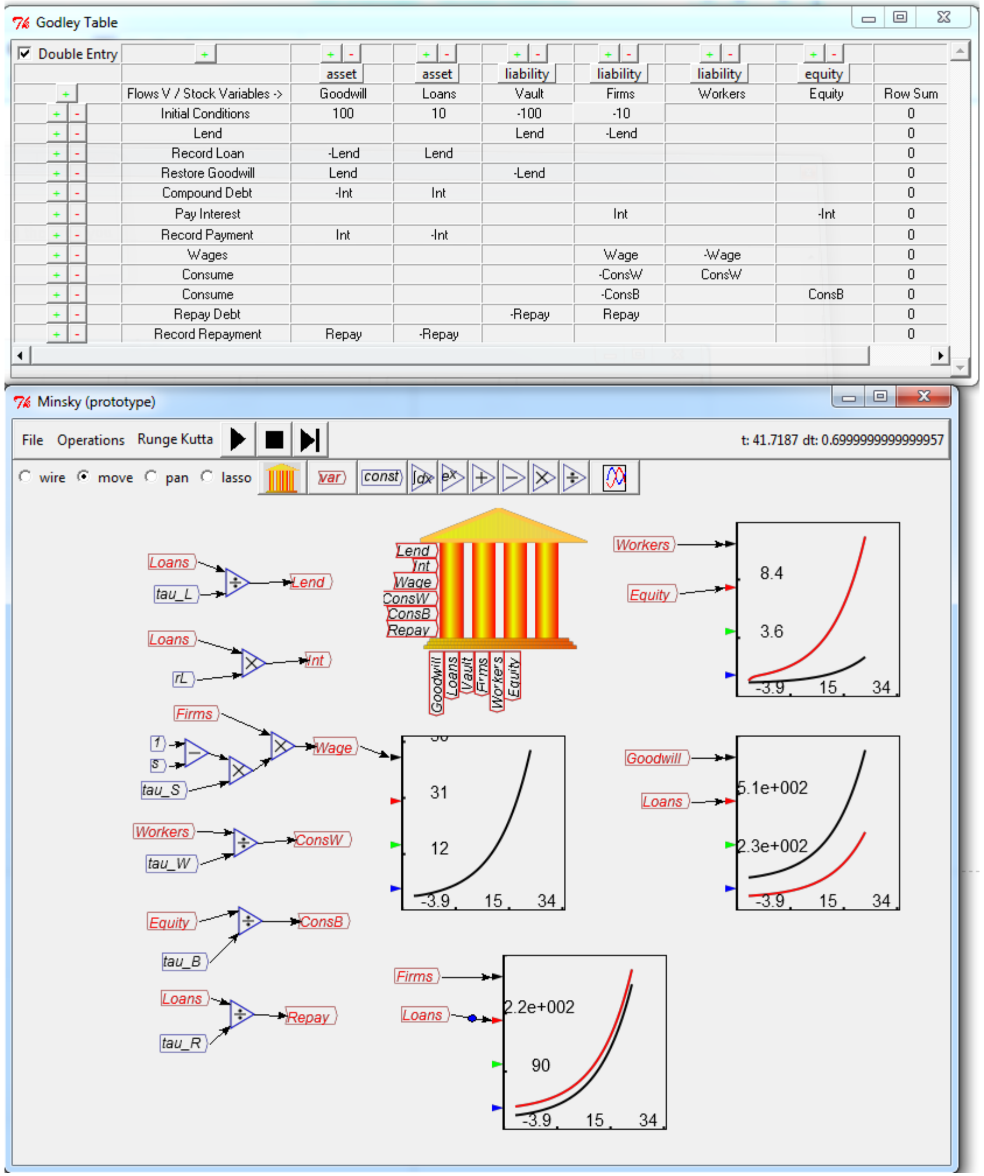


$G_S := 0 \quad G_E := Years \quad G_B := 0$


Equity
Liabilities (Deposits)
$B_E(t)$
Deposits(t)
40
20
0
0
5
10
15
20
t


Leverage
$\frac{\text{Deposits}(t)}{B_E(t)}$
50
40
30
20
10
0
0
5
10
15
20
t

Implementation in Minsky



Flows V / Stock Variables ->	Goodwill	Loans	Vault	Firms	Workers	Equity	Row Sum
	asset	asset	liability	liability	liability	equity	
Initial Conditions	100	10	-100	-10			0
Lend			Lend	-Lend			0
Record Loan	-Lend	Lend					0
Restore Goodwill	Lend		-Lend				0
Compound Debt	-Int	Int					0
Pay Interest				Int		-Int	0
Record Payment	Int	-Int					0
Wages				Wage	-Wage		0
Consume				-ConsW	ConsW		0
Consume				-ConsB		ConsB	0
Repay Debt			-Repay	Repay			0
Record Repayment	Repay	-Repay					0