Reply to the Second Referee

Thank you very much for your constructive and thorough evaluation of my note, and for your time and attention. I appreciate that you checked the algebra and, apart from the error in my equation (9p), found no mistake. Thank you also for the detailed analysis you have prepared in the report. The argument developed there is, I think, at the very core of the topic and provides a perfect basis for analyzing the problem further. I am happy to say that I am in full agreement with the algebra given in your report. Thus we are of one mind on the mechanical aspects of the issue and can proceed trying to determine the basic issues which the mathematics hides rather than exposes.

In the following, I will denote equation numbers referring to the the original paper by adding a "p" to the respective number, and to equations in the referee report by adding an "r". Equations in this reply are referred to by their number without any addition. So (1r) refers to equation (1) in the report, (1p) refers to equation (1) in the original paper, and (1) refers to equation (1) given below.

Let me state right at the beginning that I did not, in my note, intend to judge the step-by-step logic of Barro's analysis. Rather, I wanted to take an agnostic stand on that question and just analyze by means of a definite example what would happen if people actually behave as Barro suggests, and simply look at the consequences. The background for this agnostic approach is that I harbor some doubt regarding the soundness of Barro's analysis in the sense that I consider his budget constraints as something akin to tautologies, valid in many other models that actually look at first glance quite incompatible with Barro's analysis (Schlicht, 2006, Section 9).

The follow-up analysis I performed in order to better understand your comments confirmed, rather than weakened, these doubts. All equations you give are valid in my example. In particular, the budget constraint you suggest and which I am happy to accept – equation (3r) – is satisfied in the example. Likewise, equation (5r) is satisfied in the example, also under the debt regime. So the problem relates to interpreting equation (5r) as a budget constraint describing the choice set of the household sector. Strictly speaking it is not a budget constraint, but rather a result obtained from combining the households' budget constraint and the government's budget constraint. While I am skeptical regarding this issue, I expect the results I present to be nevertheless informative for the reader who does not follow me in my skepticism. In this case my analysis may be read as establishing a (perhaps interesting) paradox.

Some more detailed comments follow. Yet one single example suffices to disprove the general validity of the Barro-Ricardo equivalence. If I am right, my analysis provides just such an example, quite independent of these comments. The time-paths given in my example may involve substantial government deficits, but no period-by-period budget constraint is ever violated. As all intertemporal budget constraints are derived from the

period-by-period budget constraints, all intertemporal budget constraints are observed as well.

The budget constraint in the example

I assume the economy being in steady state. Under a pay-as-you-go regime the households budget constraint is met. Hence

$$E_t = Y_t - G_t. \tag{1}$$

is the period-by-period budget constraint. We may split this up as you suggested. Denote consumption at time t by C_t, the capital stock by K_t investment by $I_t = K_{t+1} - K_t$ and wage income by L_t. (I replace W with L, as the former symbol has a different definition in my example.) Take the case of no depreciation, as considered in my example – otherwise r_t in budget constraint (3r) would have to be replaced by i_t and this would complicate the discussion. With this understanding, equation (1) can be re-written as

$$C_t + I_t = L_t + iK_t - T_t \tag{2}$$

The pay-as-you-go regime characterized by $T_t = G_t$. I assume here that all requirements for any relevant budget constraint are met.

The intertemporal budget constraint implied by (2) is

$$\sum_{t=0}^{\infty} (1+i)^{-t} (C_t + I_t) \, dt \quad = \quad \sum_{t=0}^{\infty} (1+i)^{-t} (L_t + iK_t - T_t) \, dt \tag{3}$$

$$= \quad \sum_{t=0}^{\infty} (1+i)^{-t} (L_t + iK_t - G_t) \, dt \tag{4}$$

In the case of the debt regime, and with Barro expectations, the primary deficit is always αG_t. Taxes cover the fraction $(1 - \alpha)$ of government expenditure G_t plus interest payments on government debt iD_t. These interest payments constitute income for the private households. So the net effect of all this is that disposable income increases in each period by the primary deficit.

Because the households hold Barro expectations, they don't change their consumption and investment plans and put the the additional income αG_t in their precautionary savings. They behave as if their intertemporal budget constraint remains (4), while it actually is

$$\sum_{t=0}^{\infty} (1+i)^{-t} (C_t + I_t) \, dt \quad = \quad \sum_{t=0}^{\infty} (1+i)^{-t} (L_t + iK_t - (1-\alpha) G_t) \, dt \tag{5}$$

$$> \quad \sum_{t=0}^{\infty} (1+i)^{-t} (L_t + iK_t - G_t) \, dt. \tag{6}$$

The households act in this way because they expect that they have to dissolve their precautionary savings but never have to do that. I consider their behavior as not being rational and maintain that (5) rather than (4) is the relevant budget constraint.

The budget constraint in the referee report

The equations from (1r) to (2r) including the unnumbered intermediate step are all satisfied by the solutions given in my example (R_{s+1} being replaced by R_s in equation (2r)). You then suggest equation (3r) as the relevant period-to-period budget constraint of the households. In the following I show that (3r) is actually implied by the equations given in my note.

Start with equation (4p) of my paper and write

$$
\begin{aligned}
E_t &= Y_t - G_t. & (7)\\
&= Y_t - G_t + iD_t - iD_t + \alpha G_t - \alpha G_t & (8)\\
&= Y_t + iD_t - \alpha G_t - ((1-\alpha)G_t + iD_t). & (9)
\end{aligned}
$$

According to equation (11p) taxes are

$$
T_i = (1-\alpha)G_t + iD_t. \tag{10}
$$

and we can write equation (9) as

$$
E_t = Y_t + iD_t - \alpha G_t - T_t. \tag{11}
$$

I have assumed a tax policy

$$
D_{t+1} - D_t = \alpha G_t \tag{12}
$$

in equation (6p). Hence we obtain

$$
E_t = Y_t + iD_t - D_{t+1} + D_t - T_t \tag{13}
$$

Private expenditure E is defined as consumption C_t plus investment I_t. So we obtain

$$
C_t + I_t = Y_t + iD_t - D_{t+1} + D_t - T_t. \tag{14}
$$

Let me assume for simplicity a depreciation rate δ of zero. Otherwise equation (3r) may need modification. Without depreciation, investment gives the change in the capital stock:

$$
I_t = K_{t+1} - K_t \tag{15}
$$

and we can write

$$C_t + K_{t+1} - K_t = Y_t + iD_t - D_{t+1} + D_t - T_t \tag{16}$$

and, after re-arrangement

$$(K_{t+1} + D_{t+1}) - (K_t - D_t) = Y_t + iD_t - C_t - T_t. \tag{17}$$

You correctly observe that what I denote as "income" is to be understood as the sum of labor income L_t and interest income on capital iK_t with K_t as the stock of capital at time t. (I use the symbol L_t for labor income rather than following your usage of W_t in order to avoid conflict with the different use of the symbol W in my paper.) With this understanding we can write

$$(K_{t+1} + D_{t+1}) - (K_t - D_t) = L_t + i(K_t + D_t) - C_t - T_t. \tag{18}$$

The assets are the sum of capital and debt holdings: $A_t = K_t + D_t$. The return on assets r is equal to the rate of interest i , and we arrive at equation (3r):

$$A_{t+1} - A_t = L_t + rA_t - T_t - C_t. \tag{19}$$

You denote this as the "period-by-period budget constraint of the households". It holds true in my example.

OLG or Ramsey?

You mention that I have not been clear about whether I am concerned with an OLG model or a Ramsey model. I accept Barro's (1974) argument that these are, for the purposes at hand, equivalent. I consider, for simplicity, only the basic infinite horizon model. I shall make that clear in a revised version.

The logical problem

Substituting equation (2r) into equation (4r) under my assumption that $D_0 = 0$ and $A_0 = K_0$ gives equation (5r). It is valid in my example, too. As equation (3r) is implied by my analysis, so are equations (4r) and (5r). Thus, the argument up to that point does not invalidate any conclusions I have drawn. Equation (5r) results from combining the households' and the government's budget constraint. As a consequence, it should not be interpreted as a budget constraint in the sense of describing the choice set of the

households. (By the same logic you could substitute from equation (4r) into equation (2r) and obtain (in your notation)

$$0 = K_0 + \sum_{s=0}^{\infty} \frac{G_s + C_s - W_s}{R_s}.$$

I would be hesitant to interpret this as a constraint on the present value of the sum of government spending and private consumption, as determined by the present value of labor income, although the equation is quite true. It is, again, implied by the government's *and* the households' budget constraints – *jointly*.)

You state that equation (5r) establishes the Ricardian position. I do not think so. The fundamental budget constraints are the period-by-period budget constraints. We agree on these. I show in my example that they are never violated. At the same time, equation (5r) is satisfied. So, according to that reasoning, the Ricardian proposition should hold true in my example, but it does not, in the following sense. While households do not change their behavior, they accumulate government bonds as precautionary savings for a case that will never occur. They could plan to spend more, but they don't. Their intertemporal budget constraint, as derived from their period-by-period budget constraints, ceases to be binding. In this sense, the Barro-Ricardo equivalence is violated in my example, notwithstanding that (5r) is satisfied. So (5r) cannot establish the Ricardian proposition. Further, no period-by-period budget constraint is ever violated, but this would have to occur if the argument via (5r) (which is derived from period-by-period budget constraints) were valid.

In your view, my specification of the intertemporal budget constraint poses the main logical problem of my note. I cannot follow you here, as I have shown above that our specifications of the budget constraints are equivalent. Actually the argument I put forth can also be framed by starting from your specification of the budget constraint and reversing the derivation given above. However, the formulation in terms of private expenditure appears more transparent and more straightforward for the problem at hand.

The last issue is one that I do not fully understand. You suggest that the intertemporal consumption choice of the households "does not depend on the time path of disposable income as defined in the paper" and give disposable income in the unnumbered equation on the last page of the report as

$$Y_t + i_t D_t - T_t = W_t + i_t K_t + i_t D_t - T_t \tag{20}$$

with

$$W_t + i_t K_t \equiv Y_t - \delta K_t \tag{21}$$

(in your notation). I fully agree with these equations and this definition of disposable income. It is actually fairly standard:

> Households receive income from their labor and their ownership of capital, pay taxes to the government, and then decide how much of their after-tax income to consume and how much to save. … We define income after payment of all taxes, $Y - T$, as **disposable income.** Households divide their disposable income between consumption and saving. (Mankiw, 1997, 56)

With the understanding that "ownership of capital" is to be replaced in the present context by "ownership of assets", this is exactly the definition I use.

If households divide their disposable income between consumption and saving, the present value of disposable income must be equal to the sum of the present values of consumption and savings, notwithstanding that disposable income changes with the tax regime. The equality of present values gives the households' intertemporal budget constraint. While disposable income is endogenous and changes with the tax regime, it is part of the household's intertemporal budget constraint as commonly understood – see the above quotation. I therefore cannot find the reason for your objection.

I hope that our discussion may lead to fruitful debate and a better understanding of the effects of public debt which is, I think, very much needed. Thank you again for your apt and thoughtful comments.

May 14, 2012

Ekkehart Schlicht

References

Barro, R. J. (1974). Are Government Bonds Real Net Wealth? *Journal of Political Economy*, 82(6): 1095–1117. URL http://www.jstor.org/pss/1830663.

Mankiw, N. G. (1997). *Macroeconomics*. New York: Worth Publishers, 3 edition. ISBN 1572591412.

Schlicht, E. (2006). Public Debt as Private Wealth. *Metroeconomica*, (4): 594–520. URL http://epub.ub.uni-muenchen.de/2143/1/schlicht-public-debt-13-RP.pdf.