Referee report on "A Parsimonious Model for Intraday European Option Pricing" by E. Scalas and M.Politi

This paper discusses option pricing in an intra-day model. The log-price of the underlying stock follows the dynamics of a compound renewal process.

We start with the following two remarks.

- The distribution of sums of two independent random variables (convolutions) is a topic treated in every introductory course in probability and statistics, see e.g. Section 3.6.1 in Rice (1995) or Section 2.5.3 in Ross (2009). Therefore the computations on p.4 can be removed completely.
- The case when the time point t does not coincide with a renewal epoch time is in our view irrelevant (case 2 on p.7). The reason for this is that during a typical trading day there are several thousands of tick-by-ticks (renewal epoch times) and one can therefore without any further loss of accuracy approximate the time point t with the closest renewal epoch time during that day (as in case 1 on p.7). Then all of the computations on pp.9-10 becomes rather irrelevant.

Thus, removing the above two remarks then makes the paper left with the results on pp.5-8. On these pages the authors observe that if the log-price of the underlying stock follows the dynamics of a compound renewal process given by Equation (11), then the option pricing formula reduces to the problem of computing the distribution $F_{\tilde{S}(T_M)}(u)$ in Equation (24) on p.7. Let us now discuss this proposed method in more detail. First, note that $F_{\tilde{S}_n}(u)$ in Equation (23) on p.7 is the distribution of a product of n random variables. To this end the authors refer to methods proposed in the papers Springer & Thompson (1966) and Lomnicki (1967). A closer study of Springer & Thompson (1966) reveals that their method only works for very special cases, where $n \leq 10$ when Y_i is Cauchy distributed, or $n \leq 6$ where Y_i is normally distributed. Furthermore, Lomnicki (1967) only considers the case when Y_i is normally distributed, and it is not clear to us if the method in Lomnicki (1967) works in practice for $n \ge 8$. We also remark that the authors may need to compute $F_{\tilde{S}_n}(u)$ for very large values of n, in theory $n \to \infty$. Also note that the quantity $P(N(T_M) = n)$ in Equation (24) is suggested to be computed by using the distribution $F_I^n(x)$, see Equation (26) on p.8 in the paper. But $F_I^n(x)$ is the distribution of a sum of n random variables. We remark that computing the distribution $F_J^n(x) = P(J \le x)$ for a sum of random variables $J = \sum_{i=1}^n T_i$ is numerically quite challenging, especially if this has to be done simultaneously for $n = 2, 3, \dots, K$ with potentially very large Ks (in theory $K \to \infty$) as in Equation (24) p.7. Thus, computing $F_J^n(x)$ is at least as challenging as computing $F_{\tilde{S}_n}(u)$, for $n=2,3\ldots,K$ where $K\to\infty$. So unless the authors prove convincing arguments in terms of numerical implementations of the model, we strongly disagree that the pricing formula is "parsimonious" as proposed by the authors. In fact, given the above remarks we find it even misleading to call the model (or the formula) for parsimonious. Therefore, until the authors provide the reader with convincing numerical examples and pointing out the efficiency in the numerical implementations, we do not recommend the paper to be published at this stage.

Besides the above remarks we also recommend that the following issues should be addressed before the paper can be considered for a potential resubmission.

• The authors points out that the durations J_i in Equation (6) should not follow exponential distributions, according to previous studies. To this end we would like the authors to con-

cretize the distribution, providing at least one (hopefully more than one) explicit model for J_i . Furthermore, we would like the authors to demonstrate how to realistically calibrate or estimate the parameters in these explicit models. Finally, some numerical studies should be provided showing the model in practice (as already mentioned above).

- Does options with a one-day maturity really exists on the market? If so, please provide a reference to which exchange such options are traded at etc.
- There is a typo in the first and third equality in Equation (14) on p.5: e_i^Y should be e^{Y_i}

In conclusion, we do not believe the paper to be ready for publication and we recommend that the authors should address all of the above remarks before a possible resubmission.

References

Lomnicki, Z. A. (1967), 'On the distribution of products of random variables', *Journal of the Royal Statistical Society. Series B (Methodological)* **29**(3), 513–524.

Rice, J. (1995), Mathematical statistics and data analysis, Duxbury Press, New York.

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