The Growth Rate of the Social Cost of Carbon

in an Optimal Growth Model

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Abstract   This paper derives analytically the social cost of carbon (SCC) in a deterministic Ramsey model of optimal economic growth with carbon emissions from burning fossil fuels. In addition, it derives that the growth rate of the SCC equals the growth rate of the marginal product of fossil fuel use in output production. Alternatively, the growth rate of the SCC can be derived to equal the sum of the social discount rate and the negative marginal climate damage. In an unregulated market economy without climate policy the use of fossil fuel use as well as carbon emissions are higher than in a regulated market economy with climate policy. As a consequence, as is shown in the paper, the growth rate of the SCC declines ceteris paribus in a transition from an unregulated market economy towards a regulated market economy, provided the marginal climate damage is not too strongly rising in the stock of carbon in the atmosphere.

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1. Introduction

The social cost of carbon (SCC) is defined as the present value of the marginal damage from carbon emission, where the damage is caused by climate change. It represents an externality that is not considered by market agents in their decision making process. The externality can however be corrected with a Pigovian tax. Complete internalization of the externality requires the Pigovian tax to equal the SCC on the optimal carbon emission path.[1] As a consequence, using Pigovian taxation or alternative climate change policies requires understanding of the determinants of the SSC on the optimal carbon emission path.

The SCC is usually estimated in integrated assessment models (IAMs), i.e. in simulation models that integrate economic and scientific models of global warming. The first step in calculating the SCC is to estimate the stream of future relative marginal damages of carbon. The second step in calculating the SCC is then to employ a discount rate (sometimes labeled consumption discount rate) to convert this stream of future relative marginal damages into a present value.[2] To choose the discount rate, IAMs usually employ a Ramsey rule, i.e. an optimality condition that must be fulfilled on the consumption path that maximizes lifetime utility of a representative household (e.g. in the Ramsey model). For a constant savings rate, the Ramsey rule relates the discount rate to the income growth rate. In turn, since a well-known stylized fact of modern growth of Kaldor is a constant average income growth rate in industrialized countries over periods of at least hundred years (see e.g. Sørensen and Whitta-Jacobsen (2010)), it is standard in IAMs to employ a constant discount rate. It appears that this can also be motivated by the fact that historical data show trendless market rates of return on capital (more precisely, returns on risky stocks or risk-free government bonds). As a consequence, IAMs also usually use in their numerical simulations the historical average market rates of return as the value of the discount rate and use parameter values of the utility function that make the historical average market rates of return consistent with the Ramsey rule, given the historical average income growth rate.

Weitzman (1994) however suggests employing in cost-benefit calculations a social discount rate (see also Groom et al. (2005) for this possibility), which he labels environmental social discount rate. In contrast to market rates of return, such a social discount rate incorporates climate externalities. In turn, if these climate externalities were changing over time, then the social discount rate might have been changing in the past and will do so in the future, despite of constancy of market rates of return. Horowitz (2002), Tol (2003) and Anthoff et al. (2009) however argue that in cost-benefit calculations one should use a discount rate that does not incorporate the climate externality and that therefore equals the historical market rate of return because the climate externality should be fully incorporated in the aforementioned stream of future relative marginal damages of carbon.[3] Therefore, if we were to calculate the actual social cost of carbon, we should employ as the discount rate the value that follows from application of the Ramsey rule to constant historical average income growth rates in industrialized countries.

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[1] See e.g. Nordhaus (2011, p. 2).

[2] See e.g. Kousky et al. (2011), Marten (2011), and Pycroft et al. (2011) in the special issue of this journal.

[3] In contrast to this argumentation, in the Stern Review (cf. Stern et al. (2006)) it is believed that market agents have the "wrong preferences". More specifically, the Stern Review argues that in the Ramsey rule one should incorporate a "moral" concern for future generations and therefore employ a lower pure rate of time preference than selfish market agents would have. Use of this lower pure rate of time preference in the Ramsey rule then gives rise to a lower discount rate than the historical market rate of return.

The present paper does however not aim to derive the actual SCC. Instead, it aims to calculate the SCC that would result if a social planner would choose the socially optimal emission level. As is well-known, the social planner solution can be replicated with optimal climate policy, such as a Pigovian tax set equal to the SCC on the optimal carbon emission path. The social discount rate in the social planner solution does in general however not be equal to the constant historical average market rate of return. This is so because equality of this social discount rate with the constant historical average market rate of return would require that the climate externality has in the past hundred years been fully internalized with optimal climate policy.[4] Such full internalization over the past hundred years (or even the past sixty years) had however not been the case. It might be tempting to argue that the Kaldor facts are only observed for industrialized countries and so far there have been only minor climate damages for industrialized countries. It must however been noted that carbon in the atmosphere is a stock variable. Hence, carbon emissions remain in the atmosphere for centuries. As a consequence, climate externalities include future climate damages and it cannot be taken for granted that there will also be minor climate damages for industrialized countries in the future. Moreover, future climate damages in developing countries will also reduce future income growth in industrialized countries and often IAMs aim to calculate the SCC for the world economy, rather than just the industrialized countries. As a consequence, when deriving the SCC in the social planner economy, we must leave the actual value and the time path of the social discount unspecified because it cannot be ruled out that a social planner solution would have implied a quite different value and time path of the social discount rate than the historical average market rate of return. Analogously, it cannot be ruled out that the value and the time path of the market rate of return on capital would have been quite different from its historical value and time path if the climate externality had in the past been fully internalized with optimal climate policy.

The present paper derives the SCC and its growth rate in a deterministic Ramsey model of optimal economic growth with carbon emissions from burning fossil fuels, similar to Krautkrämer (1985), Sinn (2008) and van der Ploeg and Withagen (2011). Moreover, technical progress is modeled as in Valente (2005), while the future marginal disultility from pollution is derived similarly to Aronsson and Löfgren (1998). Section 2 of the present paper presents the social planner model structure and derives the optimality conditions. Section 3 derives the SCC and its growth rates in the social planner economy. Section 4 derives the growth rate of the SCC in the unregulated market economy without climate policy and contrasts it with the growth rate of the SCC in the regulated market economy with climate policy. Finally, section 5 concludes.

2. The model

To derive the SSC analytically, the paper assumes a social planner with perfect foresight, who maximizes in period 0 lifetime utility, $W(O)$, of an infinitely lived representative household subject to the economy's resource constraints. In a competitive market economy without externalities there exists a market equilibrium equivalent to the social planner solution, while in the presence of a climate externality the social planner solution can be replicated in a regulated market economy with climate policy such as Pigovian taxation. Following Krautkrämer (1985), but adapted to climate damage, life-time utility is assumed to be:[5]

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[4] I owe thanks to a referee for making me aware of this point.
[5] In the paper, the time index t is mostly omitted.

$$W(0) = \int_0^\infty U(C,P)e^{-\rho t}dt, \tag{1}$$

where U(C,P) represents instantaneous utility, C denotes consumption and P denotes the stock of carbon in the atmosphere.[6] It is assumed that $U_C > 0, U_P < 0, U_{CC} < 0,$ and $U_{PP} \leq 0,$ while we make no assumptions on the sign of U_{CP} and U_{PC}.[7] Moreover, ρ denotes the pure rate of time preference. For simplicity it is assumed that there exists only one household in the economy. Note that it is therefore abstracted from population growth, which seems not to be too unrealistic for the very long-run, as population growth in industrialized countries is low and world population growth is predicted to slow down in the distant future. For simplicity it is also abstracted from uncertainty, leaving its consideration to future research.

Following van der Ploeg and Withagen (2011) and similarly to Sinn (2008), the stock of carbon in the atmosphere is assumed to evolve according to the following equation:

$$P = P(0) + S(0) - S, \tag{2}$$

where S denotes the stock of fossil fuel left in the ground, while P(0) and S(0) denotes the initial stock of carbon in the atmosphere and the initial stock of fossil fuel in the ground. Depletion of the stock of fossil fuel in the ground follows according to:

$$\dot{S} \equiv \frac{\partial S}{\partial t} = -R, \tag{3}$$

where R denotes the use of fossil fuel in output production. Related to Sinn (2008), production of output, Y, takes place according to the following aggregate production function:

$$Y = F(K, R, P, A), \tag{4}$$

where K denotes the capital stock and A denotes technology that is assumed to grow exogenously and to increase productivity in output production. For simplicity we abstract from use of labor in output production. It is assumed that $F_K > 0, F_R > 0, F_P < 0, F_A > 0,$ while $F_{KK} < 0, F_{RR} < 0,$ and $F_{PP} \leq 0.$ Equation (1) and (4) show that the stock carbon in the atmosphere causes direct disutility as well as output loss.[8] Finally, capital accumulation is assumed to evolve according to the following differential equation:

$$\dot{K} = Y - C, \tag{5}$$

where it is for simplicity abstracted from fossil fuel extraction costs and capital depreciation .

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[6] For simplicity it is ignored that instantaneous utility is actually affected by world temperature rather than the stock of carbon in the atmosphere, where the former is affected by the latter.

[7] I also owe thanks to a referee for making me aware that usually in climate economics it is assumed that $U_{PP} \leq 0.$

[8] Sinn (2008) suggests interpreting F_P as the direct marginal output loss from climate change as well as the indirect marginal output loss from devoting output to mitigation of climate change that is therefore not available for consumption or capital accumulation.

Combining (1)-(5) the present value Hamiltonian that the social planner maximizes is therefore:

$$H = U(C, P(0) + S(0) - S)e^{-\rho t} + \lambda[F(K, R, P(0) + S(0) - S, A) - C] + \mu[-R].$$

This gives rise to the following first order conditions:

$$\frac{\partial H}{\partial C} = 0 \quad \Rightarrow \quad U_C e^{-\rho t} = \lambda, \tag{6a}$$

$$\frac{\partial H}{\partial R} \geq 0 \quad \Rightarrow \quad \lambda F_R = \mu, \tag{6b}$$

$$\frac{\partial H}{\partial K} = -\dot{\lambda} \quad \Rightarrow \quad \lambda F_K = -\dot{\lambda}, \tag{6c}$$

$$\frac{\partial H}{\partial S} = -\dot{\mu} \quad \Rightarrow \quad -\left(U_P e^{-\rho t} + \lambda F_P\right) = -\dot{\mu}. \tag{6d}$$

As is shown in Appendix A, if we define the social discount rate, r, to be equal to the social marginal product of capital, F_K, then from (6a) and (6c) we find the following modified Ramsey rule:[9]

$$r = \rho + \eta_{CC}\hat{C} + \eta_{CP}\hat{P}, \tag{7}$$

with $\eta_{CC} \equiv -\dfrac{U_{CC}C}{U_C}$ and $\eta_{CP} \equiv -\dfrac{U_{CP}P}{U_C}$,

where η_{CC} and η_{CP} denote the elasticity of marginal utility of consumption with respect to C and P.

Moreover, as is shown in Appendix B, using (6a)-(6d), we can derive the modified Solow-Stiglitz efficiency condition as:[10]

$$r \equiv F_K = \hat{F}_R - \left(\frac{U_P/U_C + F_P}{F_R}\right), \tag{8}$$

where $-U_P/U_C$ represents the household's marginal rate of substitution between consumption and climate damage and F_P represents the marginal output loss from climate change. Rearranging (8) yields:

$$\dot{F}_R = F_K F_R + \left(\frac{U_P}{U_C} + F_P\right). \tag{9}$$

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[9] A hat on a variable represents the growth rate of that variable.
[10] Use of a Solow-Stiglitz efficiency condition to describe the equilibrium of a resource-capital optimal growth model can for example also be found in Sinn (2008).

Equation (9) allows for an interpretation along the line of reasoning in van der Ploeg and Withagen (2011). The left hand side of (9) represents the social return from leaving a marginal unit fossil fuel in the ground. In the social optimum this social return must be equal to the right hand side of (9). The first term on the right hand side of (9) is the social return from extracting a marginal unit fossil fuel, allocating the marginal unit fossil fuel to output production for the return F_R and investing the proceeds, F_R, in capital to be used in output production for the return F_K. Due to climate change, we have to add to this return the second term on the right hand side of (9). The second term is the negative marginal climate damage from burning the unit fossil fuel. This latter negative term represents an instantaneous climate externality, which a social planner considers in his allocation decision, while it is not considered in an unregulated market economy by market agents. Therefore, in an unregulated market economy, the left hand side of (9) is higher than would be socially optimal and therefore in such an unregulated market economy the extraction rate of fossil fuel is higher than the socially optimal extraction rate.[11]

3. The SCC and its growth rate in the social planner economy

As is shown in Appendix C, upon use of (6a), (6c) and (6d) the SCC - i.e. the present value of the future relative marginal damage from carbon emission - can be derived to be:[12]

$$\text{SCC} \equiv \frac{\mu}{\lambda} = -\int_t^{\infty} \left(\frac{U_P(z)}{U_C(z)} + F_P(z) \right) e^{-\int_t^z r(z')dz'} dz. \tag{10}$$

Furthermore, combining (10) with (6b), we find:

$$F_R = \frac{\mu}{\lambda} \equiv \text{SCC} = -\int_t^{\infty} \left(\frac{U_P(z)}{U_C(z)} + F_P(z) \right) e^{-\int_t^z r(z')dz'} dz. \tag{11}$$

Equation (11) is the familiar condition for efficiency that the marginal benefit from fossil fuel extraction must be equal to the present value of the marginal damage from fossil fuel extraction.[13]

It is straightforward to see from (11) that:

$$\overset{\wedge}{\text{SSC}} \equiv \hat{\mu} - \hat{\lambda} = \hat{F}_R. \tag{12}$$

Alternatively, it is derived in Appendix D from (6a)-(6d) that:[14]

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[11] See also Sinn (2007).

[12] See analogously in van der Ploeg and Withagen (2011, Proposition 6) for the case with, firstly, instantaneous utility that is additively separable in consumption and climate damage and, secondly, without output loss from climate change.

[13] See similar in Perman et al. (2003, p. 549-550).

[14] See somewhat similarly van der Ploeg and Withagen (2011, eq. (7)), with differences as mainly explained in footnote 12 and for the case with extraction costs and capital depreciation.

$$\overset{\wedge}{\text{SSC}} \equiv \hat{\mu} - \hat{\lambda} = r + \left(\frac{U_P / U_C + F_P}{F_R} \right). \tag{13}$$

Equation (13) shows that the growth rate of the SCC equals the sum of the social discount rate and the negative marginal climate damage. Since the social discount rate equals F_K, the first term on the right hand side of (13) equals the social return from extracting $(1/F_R)$ units fossil fuel, allocating the $(1/F_R)$ units fossil fuel to output production for the return F_R and investing the proceeds, which is one unit output, in capital to be used in output production for the return F_K. The second term on the right hand side of (13) equals the negative marginal climate damage from burning $(1/F_R)$ units fossil fuel. Comparison with equation (9) reveals that therefore the growth rate of the SCC equals the social return from fossil fuel extraction on the right hand side of (9) times $(1/F_R)$, where $(1/F_R)$ units fossil fuel must be used to produce one unit output. Consistent with this, it is straightforward to see that (12) and (13) are compatible with (8) for $r \equiv F_K$.

4. The growth rate of the SCC in the unregulated market economy versus in the regulated market economy

Since in a competitive market economy without externalities there exists a market equilibrium equivalent to the social planner solution, the unregulated market economy without climate policy can be analyzed by assuming in the first order conditions (6a)-(6d) that $\partial H / \partial S = 0$. In turn, taking time derivatives of (6b) we find:

$$\dot{\lambda} F_R + \lambda \dot{F}_R = \dot{\mu}. \tag{14}$$

Using $\partial H / \partial S = 0$ in (6d) gives $\dot{\mu} = 0$. Upon use of the latter in (14), rearranging and use of (6c), we obtain the Solow-Stiglitz efficiency condition:[15]

$$\hat{F}_R = F_K. \tag{15}$$

As is discussed in Sinn (2007), comparison of the Solow-Stiglitz efficiency condition (15) with the modified Solow-Stiglitz efficiency condition (8) reveals that in an unregulated economy market agents fail to consider the instantaneous climate externality (i.e. the second term on the right hand side of (8)). As mentioned in the last section, in an unregulated market economy, $\hat{F}_R$ is therefore higher than would be socially optimal and therefore in the unregulated market economy the extraction rate of fossil fuel is higher than the socially optimal extraction rate.

Furthermore, as is derived in Appendix E, in the unregulated economy as well as in the social planner economy, the growth rate of the SCC equals:

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[15] See Sinn (2008, p. 367-368). In addition, using (6b), defining $\omega \equiv \mu / \lambda$ as the price of a unit fossil fuel and defining $i \equiv F_K$ as the market rate of return on capital, (15) gives the familiar Hotelling rule $\hat{\omega} = i$ (cf. Sinn (2008, p. 367) and van der Ploeg and Withagen (2011, p. 7).

$$\overset{\wedge}{SSC} = F_K + \left[\frac{\left(U_P + U_C F_P\right)e^{-\rho t}}{-\int_t^\infty \left(U_P(z) + U_C(z)F_P(z)\right)e^{-\rho(z-t)}dz} \right], \tag{16}$$

where in the social planner economy the social planner allocates resources such that the right hand side of eq. (16) equals the right hand side of eq. (13), while in the unregulated market economy only the right hand side of eq. (16) equals the growth rate of the SCC. In turn, due to more fossil fuel use in the unregulated market economy and due to the fact that it is plausible to assume that $F_{KR} > 0$, the first term on the right hand side of (16) is larger in the unregulated market economy than in the social planner economy. In contrast, if $U_{PP} = 0, U_{CC} = 0, U_{CP} = 0$ and $F_{PP} = 0$, then the second term on the right hand side of (16), the instantaneous climate externality, has the same value in the unregulated market economy and in the social planner economy. As a consequence, if $U_{PP} = 0, U_{CC} = 0, U_{CP} = 0$ and $F_{PP} = 0$, then the growth rate of the SCC is unambiguously larger in the unregulated market economy than in the social planner economy.

Matters are more complicated if $U_{PP} < 0$ and $F_{PP} < 0$. In this case there are two opposite effects from the unregulated market economy's forward shifting of fossil fuel use on the second term on the right hand side of (16). On the one hand, forward shifting of fossil fuel use implies that the absolute value of the numerator is larger in the unregulated market economy. On the other hand, because future climate damages are heavier discounted in $-\int_t^\infty \left(U_P(z) + U_C(z)F_P(z)\right)e^{-\rho(z-t)}dz$, forward shifting of fossil fuel use also implies that the denominator is larger in the unregulated market economy. Most likely the increase in the absolute value of the numerator exceeds the increase in the denominator. Nevertheless, the increase in the absolute value of the instantaneous climate externality in not very large, provided the absolute values of U_P and F_P, the marginal climate damages, are not too strongly rising in the stock of carbon in the atmosphere. Since the instantaneous climate externality is negative, eq. (16) therefore implies that the growth rate of the SCC is larger in the unregulated market economy than in the social planner economy (i.e. the effect from the larger F_K to dominate), provided the marginal climate damage is not too strongly rising in the stock of carbon in the atmosphere.

As was mentioned in the introduction, the world economy over the past hundred years can be best described as an unregulated market economy without climate policy (or at least insufficient climate policy). In addition, as was mentioned in section 2, in the presence of a climate externality the social planner solution can be replicated in a regulated market economy with climate policy such as Pigovian taxation. Further, rising ecological awareness suggests that future climate policy will be stricter and will therefore be closer to the social planner solution than climate policy has been in the past. As a consequence, in a very stylized fashion, future climate policy can be viewed as a transition from an unregulated market economy towards a regulated market economy, where the latter replicates the social planner solution. As a consequence, our current situation can be viewed as a transition from an unregulated economy towards a regulated economy with a ceteris paribus declining growth rate of the SCC, provided the marginal climate damage is not too strongly rising in the stock of carbon in the atmosphere.

5. Conclusion

The present paper derived the SCC and its growth rate in a deterministic Ramsey model of optimal economic growth with carbon emissions from burning fossil fuels. It was shown that the growth rate of the SCC is larger in an unregulated market economy than in a social planner economy, provided the marginal climate damage is not too strongly rising in the stock of carbon in the atmosphere. It was argued that, in a very stylized fashion, our current situation can be viewed as a transition from an unregulated economy towards a regulated economy, where the latter replicates the social planner solution. Therefore, ceteris paribus, the current growth rate of the SCC declines, provided the marginal climate damage is not too strongly rising in the stock of carbon in the atmosphere. Of course, the ceteris paribus clause is important because if the time path of the SCC is steep, then the effect of the steep time path of the SCC might dominate the effect from switching from an unregulated market economy towards a regulated market economy.

Appendix A: Derivation of the modified Ramsey rule (eq. (7))

Taking time derivatives of (6a) we obtain:

$$U_{CC}\dot{C}e^{-\rho t} + U_{CP}\dot{P}e^{-\rho t} - \rho U_C e^{-\rho t} = \dot{\lambda}. \tag{A1}$$

Upon substituting (6a) in (6c) we get:

$$\dot{\lambda} = -U_C e^{-\rho t} F_K. \tag{A2}$$

Substituting (A2) in (A1) yields:

$$U_{CC}\dot{C}e^{-\rho t} + U_{CP}\dot{P}e^{-\rho t} - \rho U_C e^{-\rho t} = -U_C e^{-\rho t} F_K. \tag{A3}$$

Rearranging (A3) and using the definition $r \equiv F_K$ gives rise to equation (7) in the text.

Appendix B: Derivation of the modified Solow-Stiglitz efficiency condition (eq. (8))

Division of both sides of (6d) by μ gives:

$$-\left(\frac{U_P e^{-\rho t} + \lambda F_P}{\mu} \right) = -\frac{\dot{\mu}}{\mu}. \tag{B1}$$

Taking time derivatives of (6b) we obtain:

$$\dot{\lambda} F_R + \lambda \dot{F}_R = \dot{\mu}. \tag{B2}$$

Substituting (B2) and (6b) in (B1) yields:

$$-\left(\frac{U_P e^{-\rho t}}{\lambda F_R} + \frac{F_P}{F_R} \right) = -\hat{\lambda} - \hat{F}_R. \tag{B3}$$

Upon combining (6a) and (6c) with (B3) und using the definition $r \equiv F_K$, we find after rearranging equation (8) in the text.

Appendix C: Derivations of the SCC (eq. (10)

Substituting (6a) in (6d) and rearranging yields:

$$- \dot{\mu} = -\left(U_P + U_C F_P \right) e^{-\rho t}. \tag{C1}$$

Integrating (C1) we find:[16]

$$\mu(t) = -\int_t^\infty \left(U_P(z) + U_C(z) F_P(z) \right) e^{-\rho z} dz. \tag{C2}$$

Collecting terms in (6c) and using the definition $r \equiv F_K$ yields:

$$\hat{\lambda} = -r. \tag{C3}$$

Integrating (C3) between times 0 and t, we find:[17]

$$\lambda(t) = \overline{C} e^{-\int_0^t r(z') dz'}. \tag{C4}$$

Setting in (C4) t=z, (C4) becomes:[18]

$$\lambda(z) = \overline{C} e^{-\int_0^z r(z') dz'}. \tag{C5}$$

Rearranging (C5) we get:

$$\overline{C} = \lambda(z) e^{\int_0^z r(z') dz'}. \tag{C6}$$

Substituting (C6) in (C4) gives rise to:[19]

$$\lambda(t) = \lambda(z) e^{\int_0^z r(z') dz' - \int_0^t r(z') dz'} = \lambda(z) e^{\int_t^z r(z') dz'}. \tag{C7}$$

Upon setting in (6a) t=z and rearranging we obtain:

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[16] More specifically, Aronsson and Löfgren (1998, p. 276-277, eq. (8) and eq. (13)) show that integration of a, by and large, similar differential equation to (C1) yields: $\tilde{\mu} = -\int_t^\infty \left(U_P(z) + U_C(z) F_P(z) \right) e^{-\rho(z-t)} dz$ (I), where $\tilde{\mu} \equiv \mu e^{\rho t}$ (II). In turn, substituting (II) in (I) and rearranging gives rise to (C2).

[17] Cf. Acemoglu (2009, p. 922-923).

[18] Cf. related Jones (2002, p. 205-206).

[19] Cf. related Chiang (2005, p. 459, Property III).

$$\lambda(z) = U_C(z)e^{-\rho z}. \tag{C8}$$

Substituting (C8) in (C7) yields:

$$\lambda(t) = U_C(z)e^{\int_t^z r(z')dz' - \rho z}. \tag{C9}$$

Finally, division of (C2) by (C9) gives rise to equation (10) in the text.

Appendix D: Derivations of the growth rate of the SCC in the social planner solution (eq. (13)

Dividing (6d) by $-\mu$ we find:

$$\hat{\mu} = \frac{U_P + \lambda F_P}{\mu}. \tag{D1}$$

Upon substituting (6b) in (D1) we obtain:

$$\hat{\mu} = \frac{U_P}{\lambda F_R} + \frac{F_P}{F_R}. \tag{D2}$$

Substituting (6a) in (D2) gives rise to:

$$\hat{\mu} = \frac{U_P / U_C + F_P}{F_R}. \tag{D3}$$

Upon rearranging (6c) and using the definition $r \equiv F_K$ we get:

$$\hat{\lambda} = -r. \tag{D4}$$

Combining (D3) and (D4) yields equation (13) in the text.

Appendix E: Derivations of the general formula of the growth rate of the SCC (eq. (16)

Division of (C1) by (C2) and multiplying with -1 yields for the social planner economy:

$$\hat{\mu} = \frac{\left(U_P + U_C F_P\right)e^{-\rho t}}{-\int_t^\infty \left(U_P(z) + U_C(z)F_P(z)\right)e^{-\rho(z-t)}dz}, \tag{E1}$$

which is the second term on the right hand side of eq. (16) and which represents the instantaneous climate externality. While the instantaneous climate externality is unpriced in the unregulated market economy and therefore then $\dot{\mu} = 0$, the instantaneous climate externality is then still a component of the right hand side of eq. (16). Combing (6c) with (E1) or the right hand side of eq. (16) according to eq. (13) in the text, we find eq. (16) in the text.

References

Acemoglu, D. (2009). Introduction to Modern Economic Growth. Princeton University Press, Princeton, NJ and Oxford, UK.

Anthoff, D.,Tol, R.S.J., and G.W.Yohe (2009). Discounting for Climate Change. Economics: The Open-Access, Open-Assessment E-Journal 3, 2009-24.

Aronsson, T. and K.-G. Löfgren (1998). Green Accounting in Imperfect Market Economies - A Summary of Recent Research. Environmental and Resource Economics 11, 273-287.

Chiang, A. (2005). Fundamental Methods of Mathematical Economics. 4th Edition, McGraw Hill.

Groom, B., Hepburn, C., Koundouri, P., and D. Pearce (2005). Declining Social discount rates: The Long and the Short of it. Environmental and Resource Economics 32: 445–493.

Horowitz, J.K. (2002). Preferences in the Future. Environmental and Resource Economics 21: 241-259.

Jones, Ch. (2002). Introduction to Economic Growth. 2nd Edition. W.W. Norton & Company, New York and London.

Kousky,C., Kopp, R.E., and R. M. Cooke (2011). Risk Premia and the Social Cost of Carbon: A Review. Economics: The Open-Access, Open-Assessment E-Journal 5, 2011-21.

Krautkrämer, J. A. (1985). Optimal Growth, Resource Amenities and the Reservation of Natural Environments, Review of Economic Studies: 153–170.

Marten, A .L. (2011). Transient Temperature Response Modeling in IAMs: The Effects of Over Simplification on the SCC. Economics: The Open-Access, Open-Assessment E-Journal 5, 2011-18.

Nordhaus, W.D. (2011). Estimates of the Social Cost of Carbon: Background and Results from the RICE-2011 Model. Cowles Foundation Discussion Paper No. 1826.

Perman, R., Ma, Y., McGilvray, J., and M. Common (2003). Natural Resource and Environmental Economics. 3rd Edition. Harlow, England: Pearson/ Addison Wesley.

Pindyck, R., and D. Rubinfeld (2009). Microeconomics. 8th Edition. Harlow, England: Pearson/ Addison Wesley.

Ploeg van der, F., and C. Withagen (2011). Growth and the Optimal Carbon Tax: When to Switch from Exhausible Resources to Renewables? OxCarre Research Paper 55 (revised version).

Pycroft, J., Vergano, L. ,Hope, C. , Paci, D., and J. C. Ciscar (2011). A Tale of Tails: Uncertainty and the Social Cost of Carbon Dioxide. Economics: The Open-Access, Open-Assessment E-Journal 5, 2011-22.

Sinn, H.-W. (2007). Pareto optimality in the extraction of fossil fuels and the greenhouse effect. CESifo working paper no. 2083.

Sinn, H.-W. (2008). Public policies against global warming. International Tax and Public Finance, 15, 360-394.

Sørensen, P.B., and H. J. Whitta-Jacobsen (2010). Introducing Advanced Macroeconomics: Growth and Business Cycles. 2nd edition, Harlow, England: Pearson/ Addison Wesley.

Stern, N., Peters, S., Bakhshi, V., Bowen, A., Cameron, C., Catovsky, S., Crane, D., Cruickshank S., Dietz, S., Edmonson, N., Garbett, S.-L., Hamid, L., Hoffman, G., Ingram, D.,Jones, B., Patmore, N., Radcliffe, H., Sathiyarajah, R., Stock, M., Taylor, C., Vernon, T., Wanjie, H., and D. Zenghelis (2006). Stern Review on the Economics of Climate Change, London: HM Treasury.

Tol, R.S.J. (2003). On Dual-Rate Discounting. Economic Modelling 21: 95-98.

Valente, S. (2005). Sustainable Development, Renewable Resources and Technological Progress. Environmental and Resource Economics. Vol. 30 (1): 115-125.

Weitzman, M. L. (1994). On the Environmental Social discount rate. Journal of Environmental Economics and Management 26: 200-209.