Kraken equation development

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Draft version 3

We begin with the classical banking multiplier

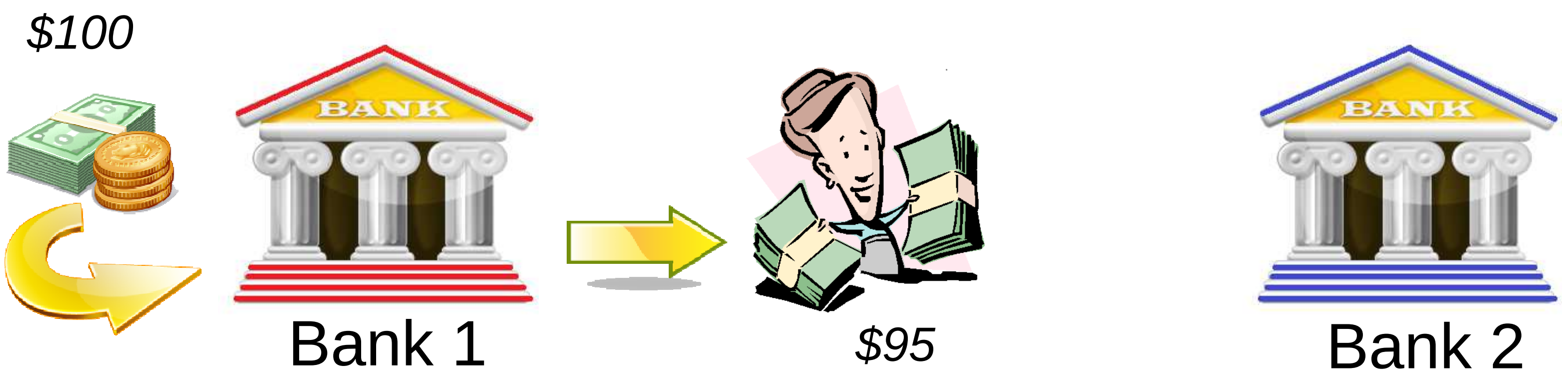
- The classic banking multiplier starts with the concept of reserves.
- Reserves allow new money to be created by banks through the issuance of loans. This happens because the requirement for physical money representation is eliminated.
- The banking multiplier is taught as:

$$m = \frac{1}{R} \tag{1}$$

where $\boldsymbol{R}$ = capital reserve fraction

How does banking create money?

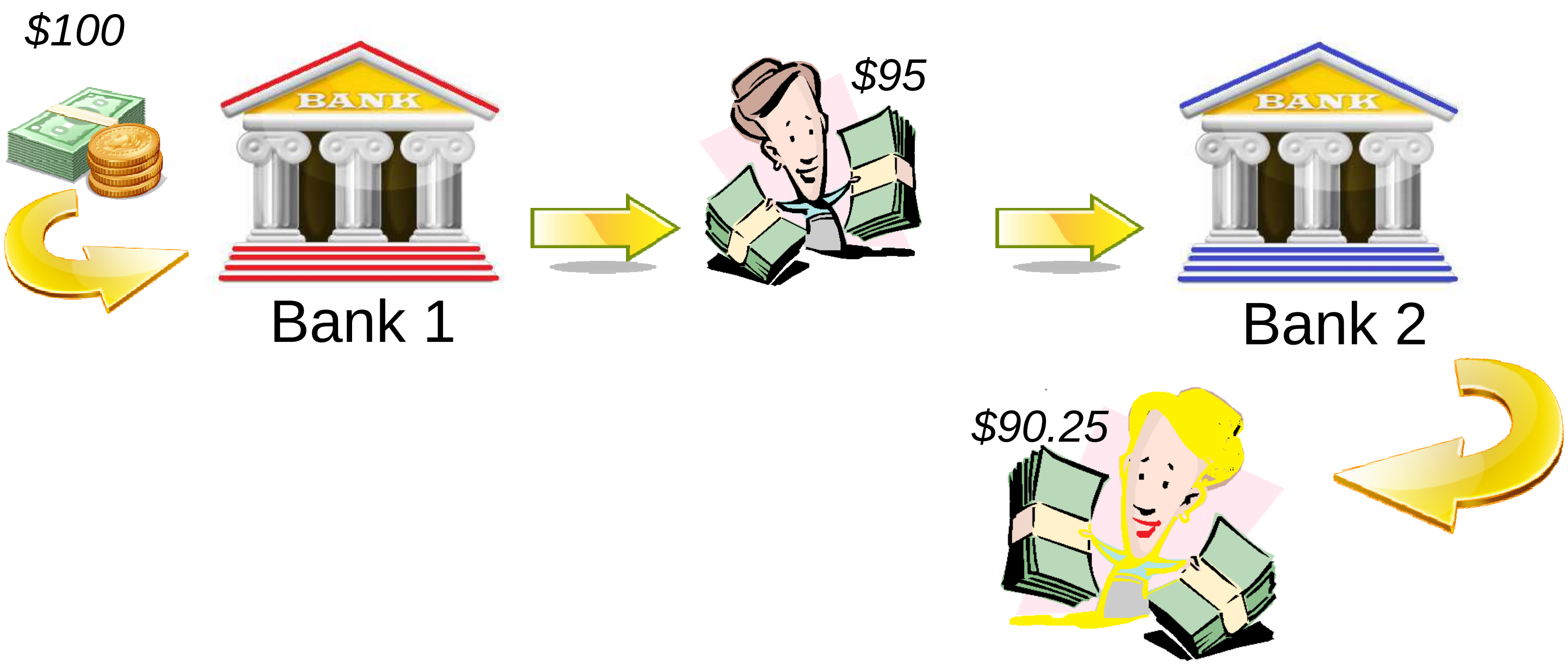
- To understand this we will use a simplified system with Zeke and Jane and two banks, bank 1 and bank 2.
- Zeke borrows from Bank 1 and deposits to Bank 2.
- Jane borrows from Bank 2 and deposits to Bank 1.
- These banks have a 5% reserve requirement.



- We will start with an initial deposit of $100 into Bank 1. With 5% reserve, Bank 1 can loan $95 to Zeke.

What happens with the first loan of $95?

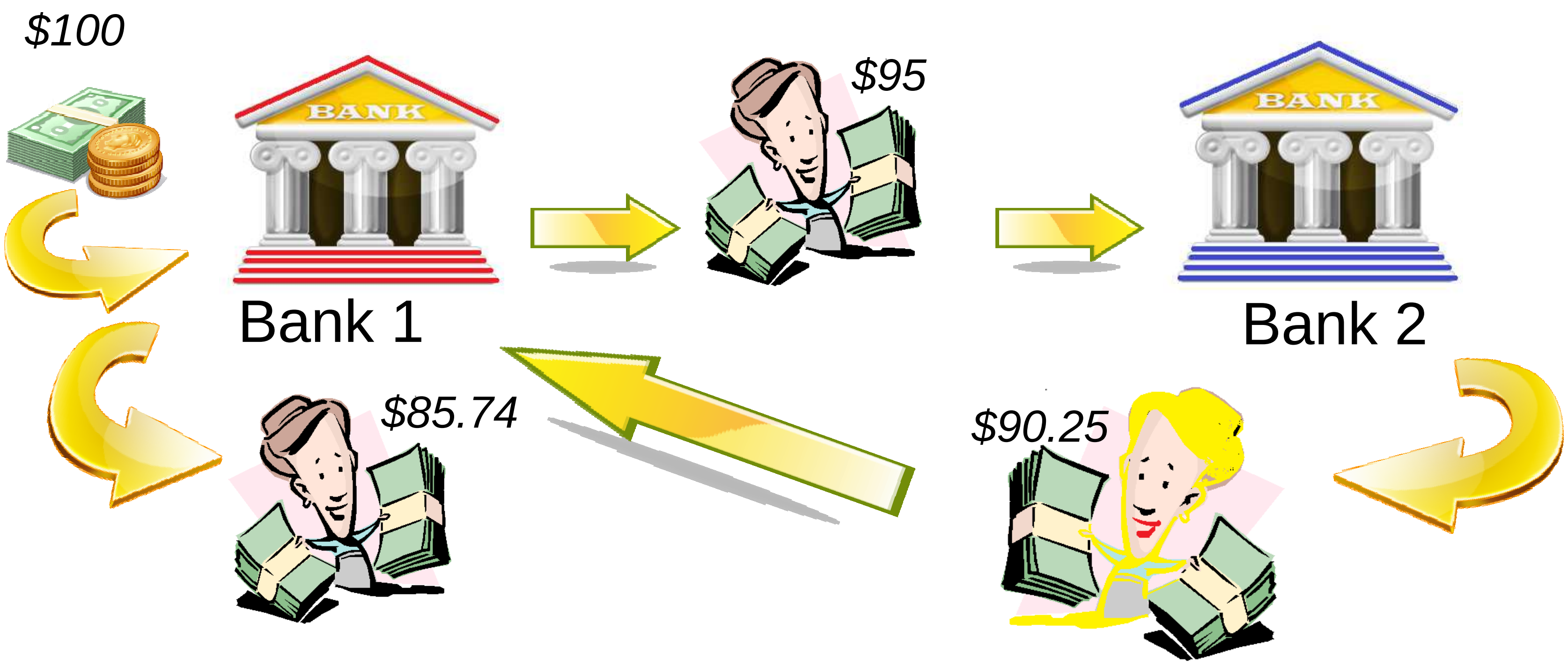
- Zeke deposits his newly created $95 into Bank 2.



- So now Bank 2 can loan Jane 95% of that new deposit originating from Zeke's loan he got from Bank 1. 95% of $95 = $90.25

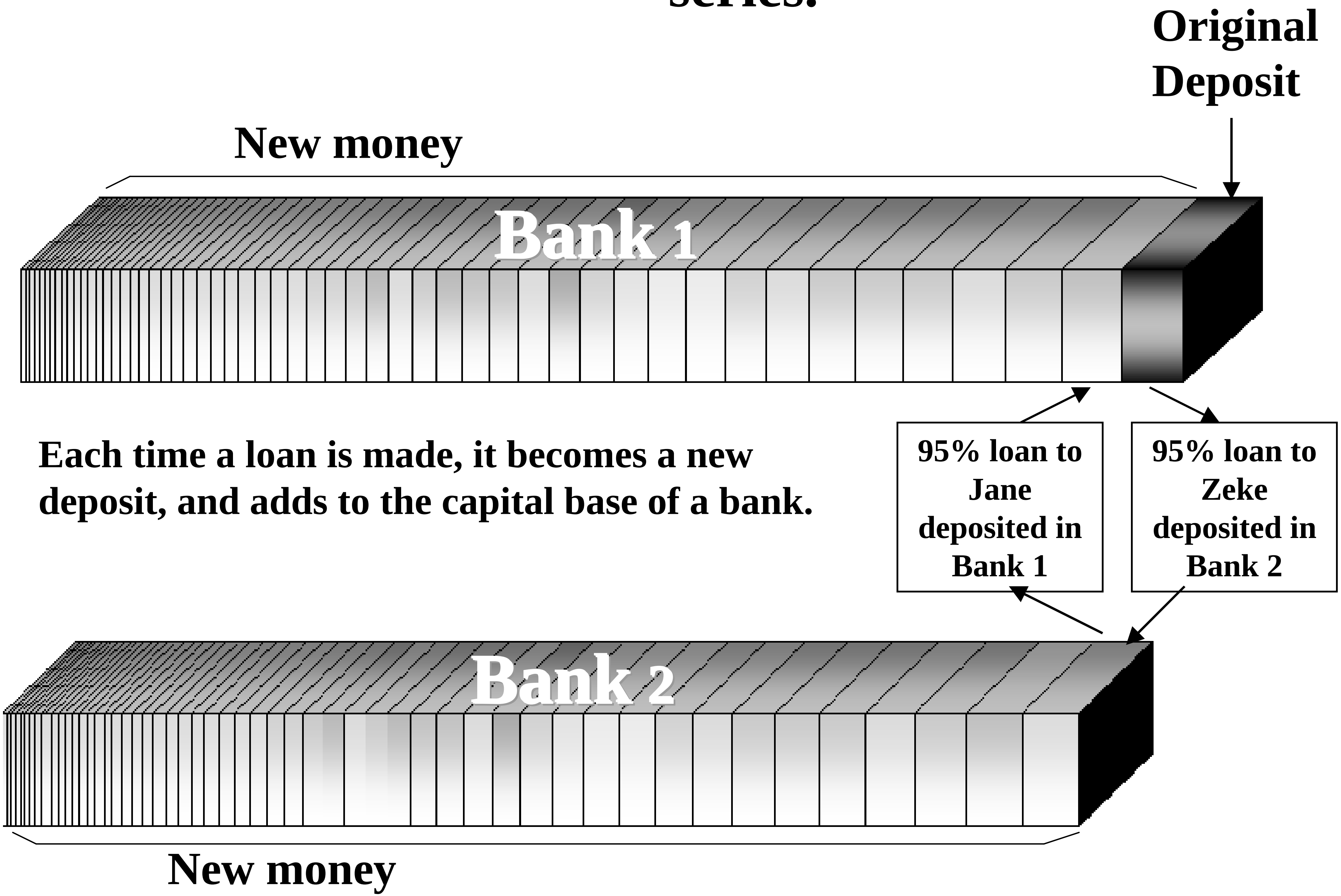
And so the $90.25 Jane deposits into Bank 1 becomes the basis for another loan, and the cycle repeats.

- Jane deposits her newly created $90.25 into Bank 1.



- So now Bank 1 can loan Zeke another 95% of that $90.25 new deposit originating from Jane's loan she got from Bank 2. 95% of $90.25 = $85.74

Schematically, we can visualize this series.



Each time a loan is made, it becomes a new deposit, and adds to the capital base of a bank.

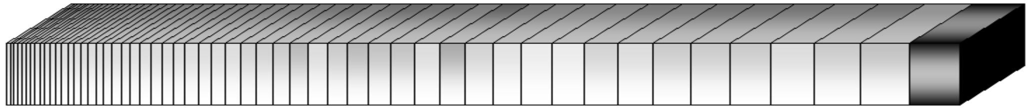
$ 100.00

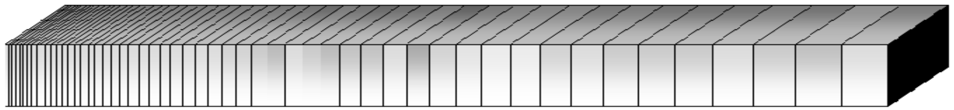
Table of deposits to banks 1 and 2

$ 95.00	$ 56.88	$ 34.06	$ 20.39	$ 12.21	$ 7.31	$ 4.38
$ 90.25	$ 54.04	$ 32.35	$ 19.37	$ 11.60	$ 6.94	$ 4.16
$ 85.74	$ 51.33	$ 30.74	$ 18.40	$ 11.02	$ 6.60	$ 3.95
$ 81.45	$ 48.77	$ 29.20	$ 17.48	$ 10.47	$ 6.27	$ 3.75
$ 77.38	$ 46.33	$ 27.74	$ 16.61	$ 9.94	$ 5.95	$ 3.56
$ 73.51	$ 44.01	$ 26.35	$ 15.78	$ 9.45	$ 5.66	$ 3.39
$ 69.83	$ 41.81	$ 25.03	$ 14.99	$ 8.97	$ 5.37	$ 3.22
$ 66.34	$ 39.72	$ 23.78	$ 14.24	$ 8.53	$ 5.10	$ 3.06
$ 63.02	$ 37.74	$ 22.59	$ 13.53	$ 8.10	$ 4.85	$ 2.90
$ 59.87	$ 35.85	$ 21.46	$ 12.85	$ 7.69	$ 4.61	$ 2.76

In this table, n = 70 and R = 5%

Mathematically, the banking multiplier (*m*) is a summation





$$m = \sum_{i=1}^{n} (1 - R)^{i} \qquad \textbf{(2)}$$

- where ***R*** = capital reserve fraction
- ***i*** = iteration number on loans/deposits
- ***n*** = iteration limit

- This equation has an asymptote at equation 1.

$$m = \frac{1}{R} \qquad \textbf{(1)}$$

How does this equation behave?

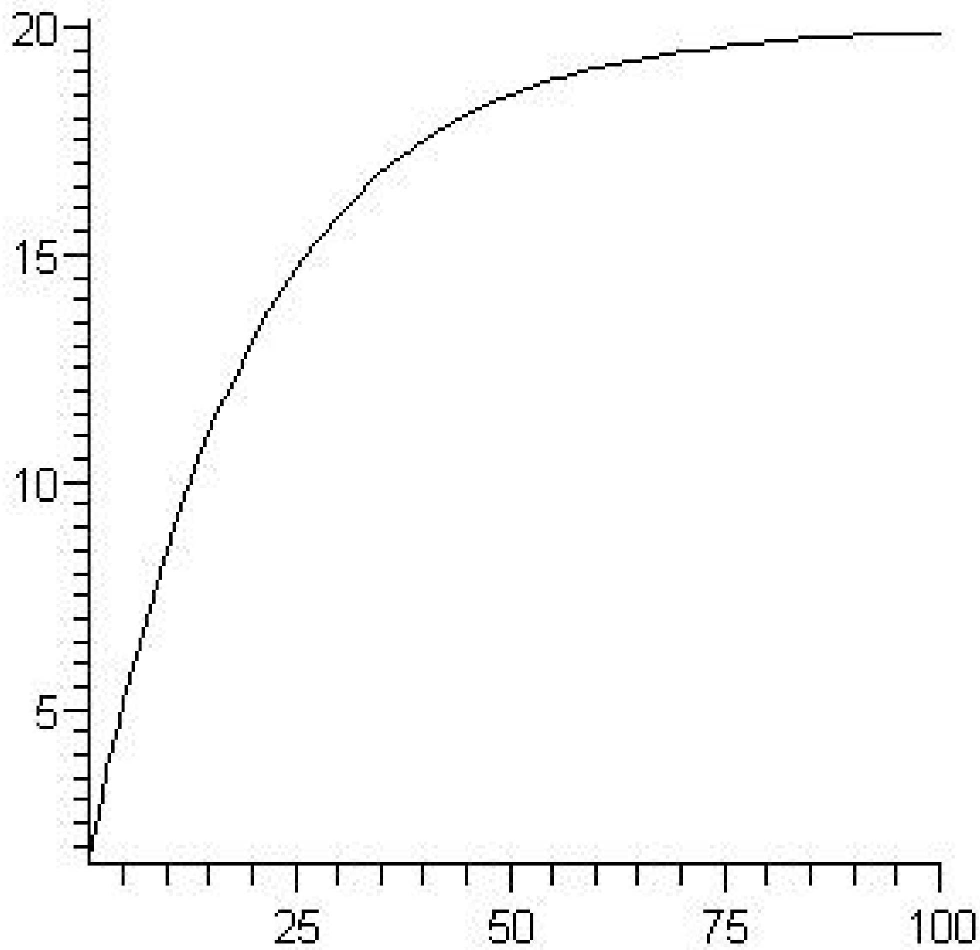
$$\sum_{i=1}^{10} (0.95)^i = 8.623998154$$

$$\sum_{i=1}^{20} (0.95)^i = 13.18876747$$

$$\sum_{i=1}^{40} (0.95)^i = 17.55826903$$

$$\sum_{i=1}^{80} (0.95)^i = 19.68620789$$

Etc.

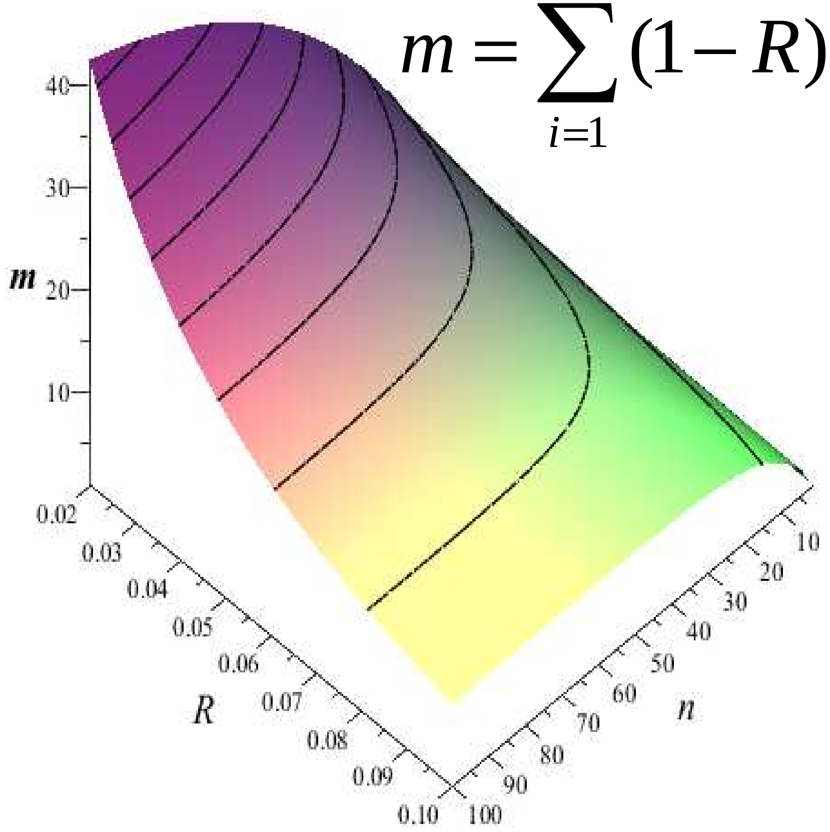


n

We can render this banking multiplier (*m*) as an isosurface

- This isosurface plot shows how the money multiplier varies as iterations (n) go from 1 to 100.
- The reserve (R) parameter starts at 2% and increases t 10%.
- Most reserves in the USA and EU are around 5% to 7%.
- In the GFC, some formal reserves dropped as low as 2.4%.

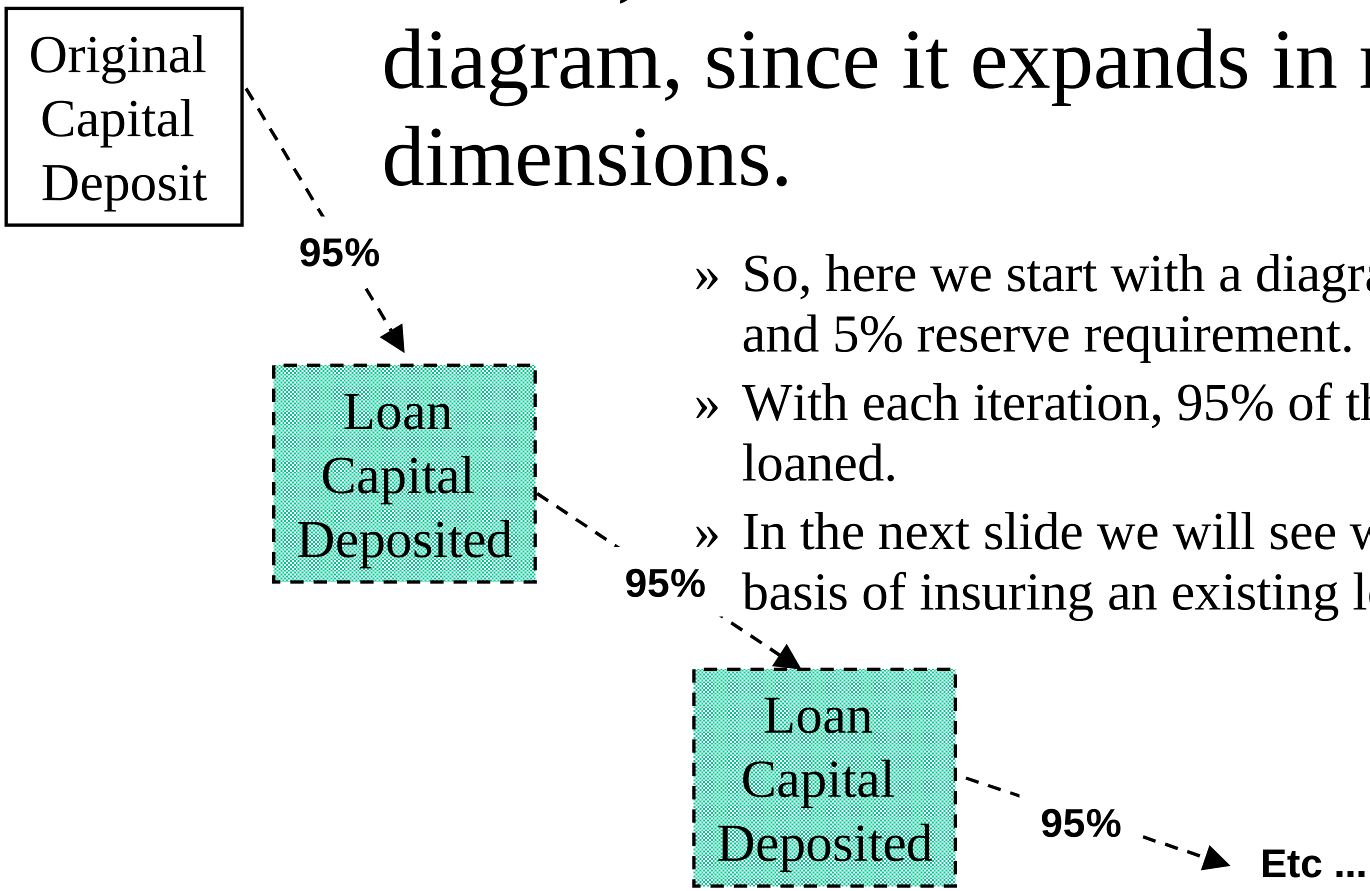
$$m = \sum_{i=1}^{n} (1-R)^i$$



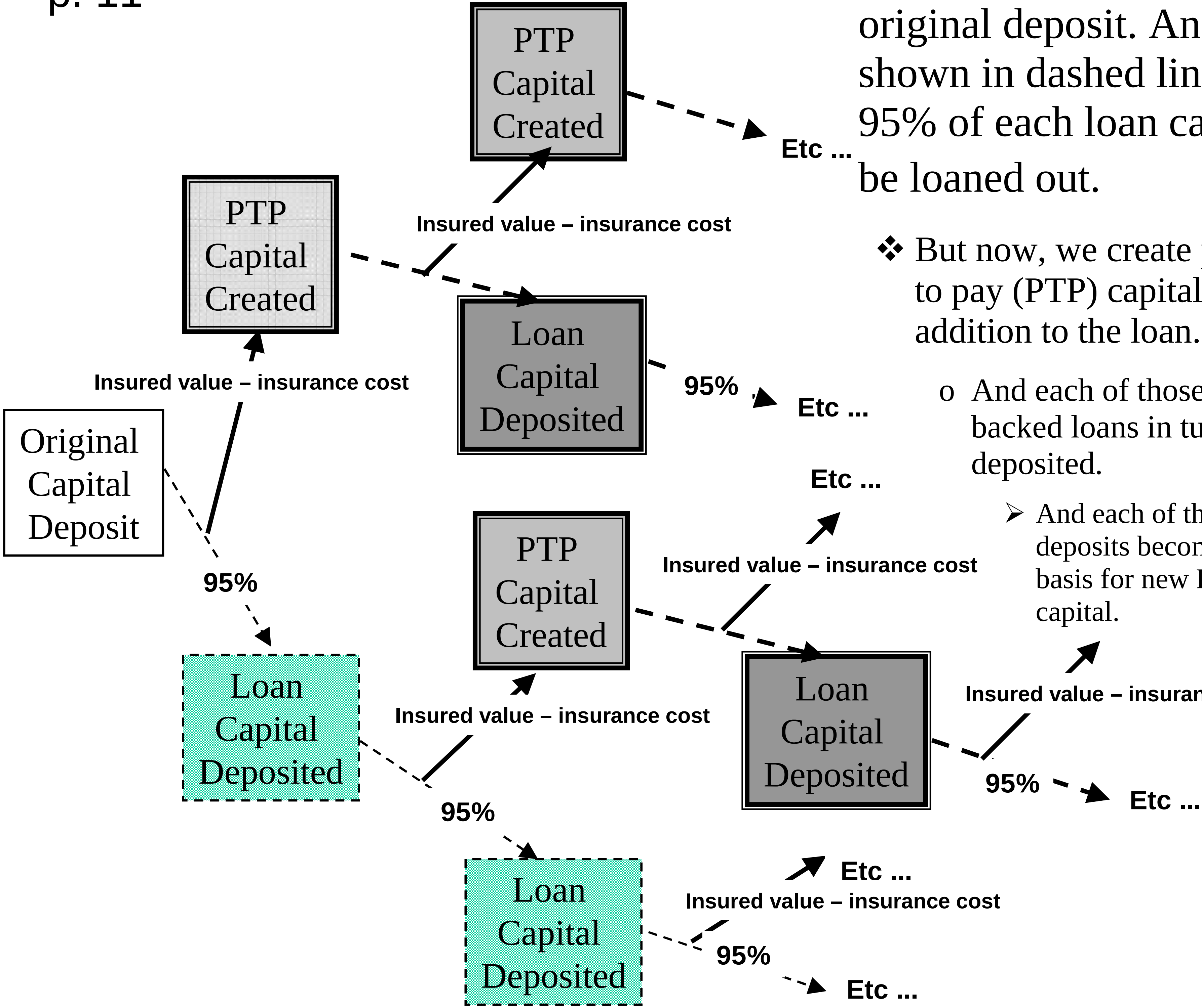
Value of m as reserve and iterations vary
R = reserve fraction
n = iteration limit

So how did using promises to pay to insure loans change the hoary banking multiplier?

- To answer this we need to compare normal banking with this insured loan method. To do that, I will use more than one type of diagram, since it expands in more than three dimensions.
  - » So, here we start with a diagram of ordinary banking and 5% reserve requirement.
  - » With each iteration, 95% of the capital deposited can be loaned.
  - » In the next slide we will see what loans made on the basis of insuring an existing loan accomplishes.



- As before, we start with an original deposit. And, as shown in dashed lines, 95% of each loan can then be loaned out.
  - ❖ But now, we create promise to pay (PTP) capital in addition to the loan.
    - o And each of those PTP backed loans in turn gets deposited.
      - ➢ And each of those deposits becomes the basis for new PTP capital.



Let’s start on a toy PTP example.

- Let us assume that we go one level deep. This means that for each conventional loan, we insure that conventional loan, and make one PTP loan from it.
- The value of that new PTP loan for each conventional loan will be assumed to be equal to the value of the insured loan. (e.g. a zero cost of insurance)

$$m = \sum_{i_1=1}^{n} ((1-R)^{i_1} + (1-R)^{i_1})$$

Decomposing the first toy equation

$$m = \sum_{i_1=1}^{n} \left(\underline{(1-R)}^{i_1} + \underline{(1-R)}^{i_1} \right)$$

Classical banking multiplier term

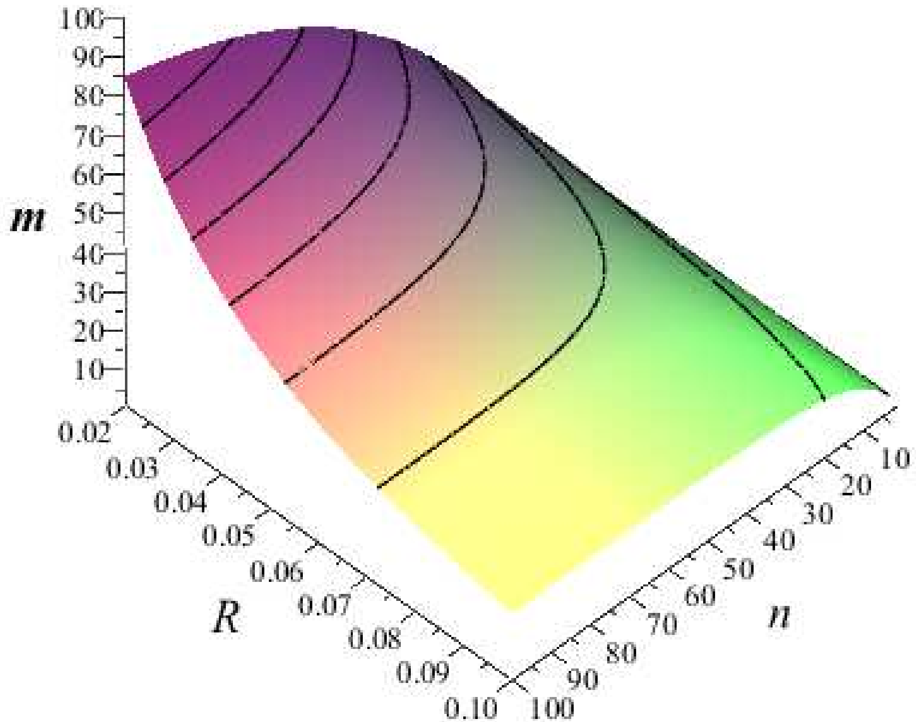
The new single PTP originated loan if it were equal to the Insured loan amount.

This equation will not be reduced to simplest form because in the next steps the second term is modified and becomes quite important.

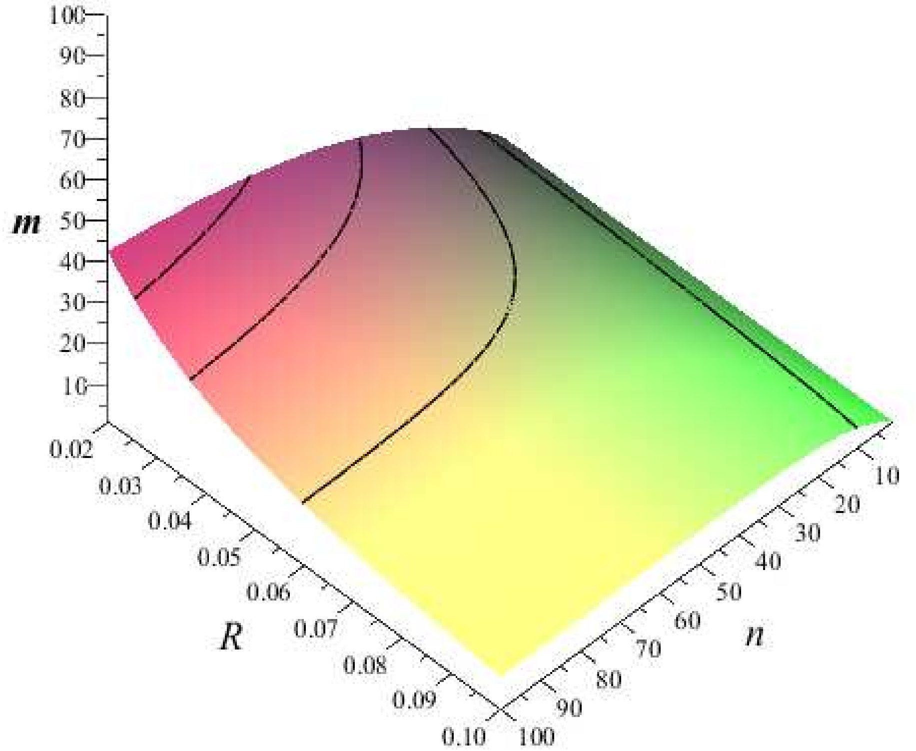
Working out the first toy equation where cost of insurance is set to zero

$$m=\sum_{i_1=1}^{n}((1-R)^{i_1}+(1-R)^{i_1})$$

$$m=\sum_{i=1}^{n}(1-R)^{i}$$



Value of m as reserve and iterations vary
R = reserve fraction
n = iteration limit



Value of m as reserve and iterations vary
R = reserve fraction
n = iteration limit

Now let us go more than one level deep. We will call this "Toy 2" example.

- To go two levels deep means that for each conventional loan, we insure it, and then make a PTP loan. Then, we make one conventional loan and a new PTP loan from the PTP loan deposit.
- Again, the cost of insurance on each new PTP loan will be set to zero.
- Notice that to do this, one must nest the equations because this new layer is only created from the second term.

$$m = \sum_{i_1=1}^{n} ((1-R)^{i_1} + ((1-R)^{i_1} \cdot \sum_{i_2=1}^{n} ((1-R)^{i_2} + ((1-R)^{i_2})))$$

Decomposing the “Toy 2” equation

$$m = \sum_{i_1=1}^{n} ((1-R)^{i_1} + ((1-R)^{i_1} \cdot \sum_{i_2=1}^{n} ((1-R)^{i_2} + ((1-R)^{i_2}))$$

Classical banking
multiplier terms

The new
PTP originated loan
if it were equal to the
Insured loan amount.

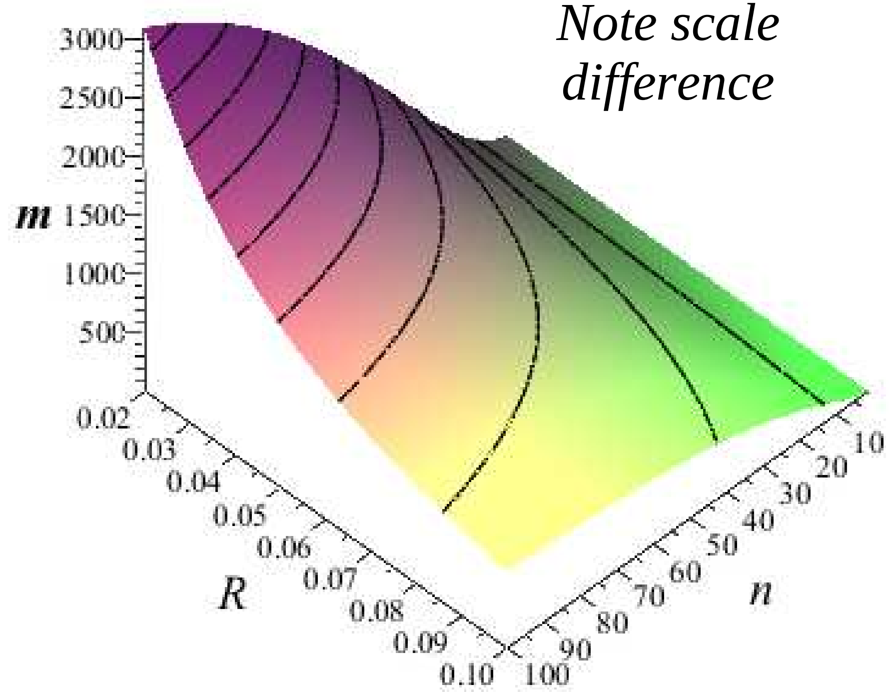
Note that to implement it is necessary to create a new chain of conventional loans plus PTP loans for each of the PTP originated loans in the previous layer.
Refer to the previous diagrams for clarification.

Working out the "Toy 2" equation where cost of insurance is set to zero

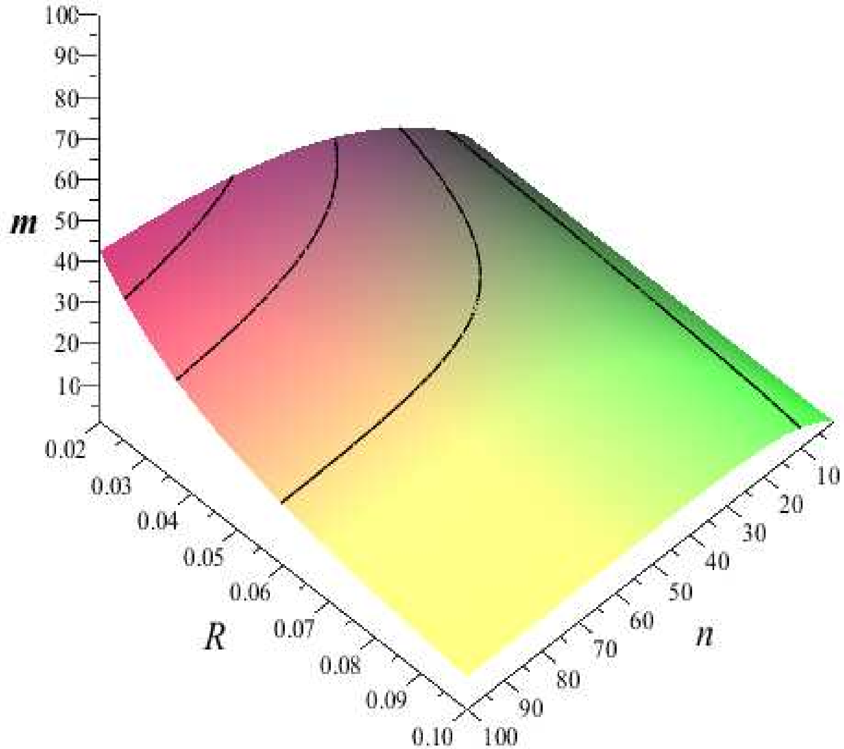
$$m=\sum_{i_1=1}^{n}((1-R)^{i_1}+((1-R)^{i_1}\cdot\sum_{i_2=1}^{n}((1-R)^{i_2}+((1-R)^{i_2}))$$

$$m=\sum_{i=1}^{n}(1-R)^{i}$$

Note scale difference



Value of m as reserve and iterations vary
R = reserve fraction
n = iteration limit



Value of m as reserve and iterations vary
R = reserve fraction
n = iteration limit

Now let us continue with "Toy 3" example.

- Again, the cost of insurance on each new PTP loan will be set to zero.
- Again, we will nest the equations, because for each loan in the second layer, we will now allocate a new standard loan in the third layer, and a new PTP based loan also in the third layer.

$$m = \sum_{i_1=1}^{n}((1-R)^{i_1} + ((1-R)^{i_1} \cdot \sum_{i_2=1}^{n}((1-R)^{i_2} + ((1-R)^{i_2} \cdot \sum_{i_3=1}^{n}((1-R)^{i_3} + ((1-R)^{i_3}))))$$

Decomposing the "Toy 3" equation

$$m=\sum_{i_1=1}^{n}((1-R)^{i_1}+((1-R)^{i_1}\cdot\sum_{i_2=1}^{n}((1-R)^{i_2}+((1-R)^{i_2}\cdot\sum_{i_3=1}^{n}((1-R)^{i_3}+((1-R)^{i_3}))))$$

Classical banking multiplier terms

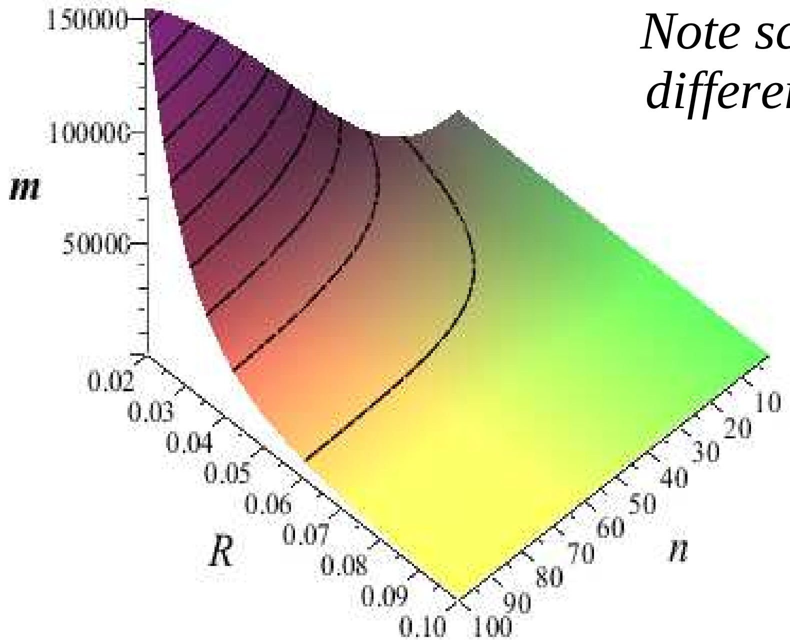
The new PTP originated loan if it were equal to the Insured loan amount.

Again, it is necessary to create a new chain of conventional loans plus PTP loans for each of the PTP originated loans in the previous layer. Refer to the previous diagrams for clarification.

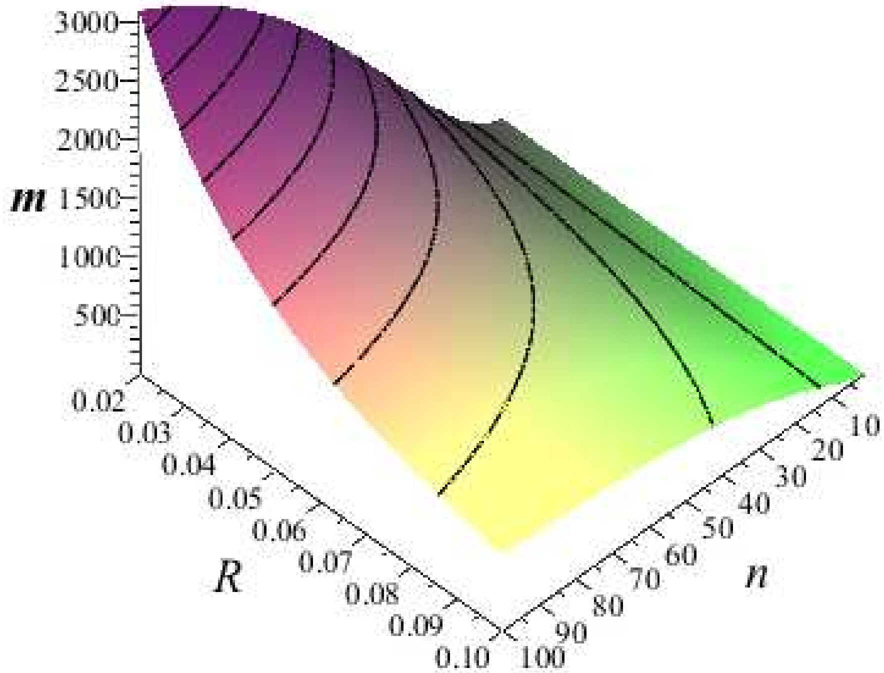
Working out the "Toy 3" equation where cost of insurance is set to zero

$$m=\sum_{i_1=1}^{n}((1-R)^{i_1}+((1-R)^{i_1}\cdot\sum_{i_2=1}^{n}((1-R)^{i_2}+((1-R)^{i_2}\cdot\sum_{i_3=1}^{n}((1-R)^{i_3}+((1-R)^{i_3}))))$$

$$m=\sum_{i_1=1}^{n}((1-R)^{i_1}+((1-R)^{i_1}\cdot\sum_{i_2=1}^{n}((1-R)^{i_2}+((1-R)^{i_2}))$$



Note scale difference



Value of m as reserve and iterations vary
R = reserve fraction
n = iteration limit

Value of m as reserve and iterations vary
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And so on… then some more terms

- The depth of nesting can be arbitrarily large. For this paper maximum nesting was set at 10 and all values shown are for selected values in that range.
- But of course, these equations need some adjustments to make them fit the real world better.
- The new terms to be introduced are:
  - ❖ $\boldsymbol{I} \equiv$ Cost of the insurance.
  - ❖ $\boldsymbol{O} \equiv$ Value of the new loan plus origination fees
  - ❖ $\boldsymbol{T} \equiv$ Tranche fraction

I ≡ Cost of the insurance.

- Cost of insuring the loan by acquiring a promise to pay (PTP) is first shown in equation 3.
- When CDS contracts were bought, they were charged based on the value of the contract.
- Consequently, the ***I*** term represents the fractional cost relative to the loan being insured.
- So, assuming that the PTP contract is equal to the face value of the loan that was issued, the ***I*** term is subtracted from 1.
- In this scheme the 1 is a placeholder for the value of the loan that was issued.
- Thus a new term is introduced to our first toy equation:

$$m = \sum_{i_1=1}^{n} ((1-R)^{i_1} + ((1-R)^{i_1} \cdot (1-I))) \qquad \text{"Toy 1"} + I$$

O ≡ Value of the new loan plus origination fees

- In the real world loans have origination fees. Since these are part of the transaction, they can potentially compensate for some of the cost of the insurance. We will refer to points as *P*.
- So, the ***O*** term represents the value of the loan plus the origination fee points. Calculating this, ***O*** = 1+ ***P***.
- The ***O*** term appears in equation 4 with a discussion.
- Thus a the new ***I*** term introduced into our first toy equation is modified to include the ***O***:

$$m = \sum_{i_1=1}^{n} ((1-R)^{i_1} + ((1-R)^{i_1} \cdot (O-I))) \qquad \text{“Toy 1” + O \& } I$$

T ≡ Tranche fraction

- There is another significant modifier to this equation that comes from how bundled loans were packaged.
- They were packaged in payoff fractions, or “tranches”. Typically, a bundle of loans would be divided into three sections. The first tranche would be paid first. Until everyone in the first tranche was paid, nobody was paid in the second. The second took precedence over the third.
- Typically, only the first tranche was insured. So, the fraction of loans representing that tranche is the limit what can be acquired as insurance.
- The ***T*** term appears in equation 5 with a discussion.
- Thus with the addition of a new ***T*** term into the first toy equation results in:

$$m = \sum_{i_1=1}^{n} ((1-R)^{i_1} + ((1-R)^{i_1} \cdot (O-I) \cdot T)) \qquad \text{“Toy 1” + O, } I \text{ \& } T$$