The social cost of carbon in the Stokey model

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Abstract This paper derives analytically the social cost of carbon (SSC) in a version of the Stokey model of optimal economic growth, where pollution is assumed to be a stock variable and where there is exogenous technical progress. As is shown in the present paper, the special feature of this version of the Stokey model is that with logarithmic utility it allows, at a moderate income level, for a quasi-steady state with a constant discount rate and a rising stock of carbon in the atmosphere. This is consistent with the stylized facts of modern growth and is a standard implicit assumption in integrated assessment models. The model implies that once income passes a threshold income level, it is optimal to gradually switch to cleaner production technologies and the economy leaves the quasi-steady state and moves towards a steady state with a constant discount rate and a stabilized stock of carbon in the atmosphere. Furthermore, this paper shows that in the transition from the quasi-steady state towards the steady state the growth rate of the SCC rises from a value close to zero to a value that equals the rate of income growth.

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1. Introduction

The social cost of carbon (SCC) is defined as the present value of the marginal damage from carbon emission, where the damage is caused by climate change. It represents an externality that is not considered by market agents in their decision making process. The externality can however be corrected with a Pigovian tax. Complete internalization of the externality requires the Pigovian tax to equal the SCC on the optimal carbon emission path.[1] As a consequence, using Pigovian taxation or alternative climate change policies requires understanding of the determinants of the SSC on the optimal carbon emission path.

The SCC is usually estimated in integrated assessment models (IAMs), i.e. in simulation models that integrate economic and scientific models of global warming. The first step in calculating the SCC is to estimate the stream of future relative marginal damages of carbon. The second step in calculating the SCC is then to employ a discount rate to convert this stream of future relative marginal damages into a present value. To choose the discount rate, IAMs usually employ a Ramsey rule, i.e. an optimality condition that must be fulfilled on the consumption path that maximizes lifetime utility of a representative household (e.g. in the Ramsey model). For a constant savings rate, the Ramsey rule relates the discount rate to the income growth rate. In turn, since a well-known stylized fact of modern growth of Kaldor is a constant average income growth rate in industrialized countries over periods of at least hundred years (see e.g. Sørensen and Whitta-Jacobsen (2010)), it is standard in IAMs to employ a constant discount rate. This can also be motivated by the fact that historical data show trendless market rates of return (i.e. returns to risky stocks or risk-free government bonds).

Weitzman (1994) suggests employing in cost-benefit calculations a social discount rate (see also Groom et al. (2005) for this possibility), which he labels environmental discount rate. In contrast to market rates of return, such a social discount rate should incorporate climate externalities. In turn, if these climate externalities have been increasing over time, then the social discount rate might have been declining in the past or will do so in the future, despite of constancy of market rates of return. Horowitz (2002), Tol (2003) and Anthoff et al. (2009) however argue that in cost-benefit calculations one should use a discount rate that does not incorporate the climate externality and that therefore equals the historical market rate of return because the climate externality should be fully incorporated in the aforementioned stream of future relative marginal damages of carbon. Indeed, if the climate externality is fully internalized with a Pigovian tax levied on carbon emissions and if therefore the Pigovian tax equals the SCC on the optimal carbon emission path, then the market rate of return to capital will be equal to the social marginal product of capital. Therefore, a social planner would set the discount rate equal to the social marginal product of capital.[2]

The assumption of a constant discount rate seems well-found, as it is compatible with the stylized facts of modern economic growth and since the discount rate must be constant in the steady

[1] See e.g. Nordhaus (2011, p. 2).

[2] In contrast to this argumentation, in the Stern Review (cf. Stern et al. (2006)) it is believed that market agents have the “wrong preferences”. More specifically, the Stern Review argues that in the Ramsey rule one should incorporate a “moral” concern for future generations and therefore employ a lower utility discount rate than selfish market agents would have. Use of this lower utility discount rate in the Ramsey rule then gives rise to a lower discount rate than the historical market rate of return.

state of optimal growth models without carbon emissions, such as the Ramsey model.[3] The situation becomes trickier, however, if one introduces carbon emissions into models of optimal growth. This is so because carbon emissions are in analytical growth models usually assumed to arise from burning fossil fuels that are used as input into output production and such models imply in general that in the steady state the stock of carbon in the atmosphere must be constant. During the long periods of constant average income growth and trendless market rates of returns, however, the stock of carbon in the atmosphere has instead certainly been rising. For this reason, optimal growth models with emission from burning fossil fuel are in general inconsistent with the observed trendless market rates of return.

In work that is related to the present paper, Golosov et al. (2011), however, succeed in presenting an optimal growth model with emission from burning fossil fuel that does allow for a constant discount rate despite of a rising stock of carbon in the atmosphere. The assumptions that do the trick are to assume climate damage in output production to be proportional to output (an assumption that is similarly made in the DICE model – i.e. a particular IAM as presented e.g. in Nordhaus (2008)) and to assume that climate damages do not directly enter the utility function. As Golosov et al. emphasis, however, their model results survives if one allows for disutility from climate change, provided one makes the knife-edge assumption of linear disutility from climate change. Due to the authors' assumptions and if in their model uncertainty and substitutability between two forms of fossil fuels (oil and coal) is ignored, then the model of Golosov et al. allows for a quasi-steady state with a constant discount rate despite of a rising stock of carbon. Moreover, in this quasi-steady state the SCC grows with the rate of income growth (although this is not explicitly derived in Golosov et al.). These two results arises irrespectively of whether or not it is allowed for disutility from climate change that is linear.

The present paper takes another route than Golosov et al. to find a model that is consistent with a constant discount rate despite of a rising stock of carbon. It does so by applying a model version of Stokey (1998), namely the last model version in Stokey's paper, in which pollution is a stock variable and in which there is exogenous technical progress. In all model versions of Stokey, emissions arise as a by-product of output production rather than from burning a fossil fuel input. Moreover, the emissions associated with a particular output level can be reduced by switching to a cleaner production technology. Therefore, the model structure is similar to the model structure in the DICE model.[4] The model implies that at a moderate income level it is optimal to use the dirtiest technology. Moreover, disutility from pollution and utility from consumption are assumed to be additively separable. This assumption guarantees that in the Ramsey rule a rising stock of carbon in the atmosphere does not reduce the discount rate. One can however show that the Stokey model also requires fulfillment of a modified Hotelling rule, which states that the discount rate must also be equal to the social return to switching to cleaner technologies. The present paper shows that, at a moderate income level, the Hotelling rule allows the economy to be in a quasi-steady state with a constant discount rate despite of a rising stock of carbon, provided instantaneous utility is logarithmic in consumption as well as in pollution. Moreover, the model implies that once income passes a threshold level, it become optimal to gradually switch to cleaner technologies and the

[3] Henceforth, the word steady state is used interchangeable with the word balanced growth path.

[4] See Nordhaus (2008, Appendix: Equations of the DICE-2007 Model). Moreover, as is emphasized in Brock and Taylor (2005), gradually switching to cleaner production technologies is a model-equivalent to gradually intensified abatement, the latter feature being also included in the model structure of the DICE model.

economy then leaves the quasi-steady state and moves to a steady state with again a constant discount rate and constant income growth, but now also a constant stock of carbon in the atmosphere. While the steady state might be a realistic prediction of the distant future, what we currently observe in reality is more consistent with the model's quasi-steady state. An important difference to the model result in Golosov et al. (2011) is that, while the model of Golosov et al. implies the SCC to grow at the rate of income growth in the quasi-steady state as well as in a steady state, in the present model, only in the steady state does the SCC grow at the rate of income growth. In the quasi-steady state of the present model, however, the SCC grows at a lower rate that only slightly exceeds the value zero.

Section 2 of the present paper presents the model structure and derives the optimality conditions. Section 3 derives the equilibrium conditions in the quasi-steady state with a constant discount rate and a rising stock of carbon in the atmosphere. Section 4 derives the equilibrium conditions in the steady state with a constant discount rate and a constant stock of carbon. Finally, section 5 concludes.

2. The model

To derive the SSC analytically, we follow Stokey (1998) to assume a social planner with perfect foresight, who maximizes in period 0 lifetime utility, $W(O)$, of an infinitely lived representative household subject to the economy's resource constraints. In a competitive economy without externalities there exists a market equilibrium equivalent to the social planner solution, while in the presence of externalities the social planner solution can be replicated in a market economy upon use of Pigovian taxation. As mentioned before, instantaneous utility is assumed to be additively separable and logarithmic in consumption, C, and the pollution stock, X:[5]

$$W(0) = \int_0^\infty [\ln C - \gamma \ln X] e^{-\rho t} dt, \tag{1}$$

where instantaneous utility is represented by the expression within brackets and ρ denotes the constant utility discount rate. The assumption of logarithmic utility is not made in the paper of Stokey because she is not interested in allowing for a quasi-steady state and instead only aims to derive an Environmental Kuznets curve, i.e. to derive that at a threshold income level pollution starts to improve. For simplicity it is assumed that there exists only one household in the economy. Note that it is therefore abstracted from population growth, which seems not to be too unrealistic for the very long-run, as population growth in industrialized countries is low and world population growth is predicted to slow down in the distant future. For simplicity it is also abstracted from uncertainty, leaving its consideration to future research.

Accumulation of the stock of carbon in the atmosphere is assumed to evolve according to the following differential equation:

$$\dot{X} = z^{\beta-1} Y - \delta X, \quad \text{where } \beta > 1, \tag{2}$$

[5] The time index t is omitted.

where $x = z^{\beta-1}Y$ represents the flow of carbon emissions and δ represents a constant dissipation rate of X each period and Y denote production of a consumption.[6] The index $z \in [0,1]$ measures how dirty technology is, where the degree of dirtiness is rising in z. Production of Y takes place according to the following Cobb-Douglas production function:

$$Y = zK^{\alpha}(AL)^{1-\alpha}, \quad \text{where} \quad 0 < \alpha < 1, \tag{3}$$

where K denotes the capital stock, A denotes technology that is assumed to grow exogenously and to increase productivity in output production. Moreover, L denotes labor that is exogenously supplied by the economy's single household. Equation (3) shows that technology is more productive in producing output if technology is dirtier. As in Stokey, pollution is assumed not to affect productivity in output production. Finally, capital accumulation is assumed to evolve according to the following differential equation:

$$\dot{K} = Y - C, \tag{4}$$

where it is for simplicity abstracted from capital depreciation.

Combining (1)-(4) the present value Hamiltonian that the social planner maximizes is therefore:

$$H = [\ln C - \gamma \ln X]e^{-\rho t} + \lambda[zK^{\alpha}(AL)^{1-\alpha} - C] + \mu[\delta X - z^{\beta}K^{\alpha}(AL)^{1-\alpha}].$$

This gives rise to the following first order conditions:

$$\frac{\partial H}{\partial C} = 0 \quad \Rightarrow \quad \frac{1}{C}e^{-\rho t} = \lambda, \tag{5a}$$

$$\frac{\partial H}{\partial z} \geq 0 \quad \Rightarrow \quad \lambda \geq \mu\beta z^{\beta-1}, \text{ with equality for z<1,} \tag{5b}$$

$$\frac{\partial H}{\partial K} = -\dot{\lambda} \quad \Rightarrow \quad (\lambda - \mu z^{\beta-1})\alpha z K^{\alpha-1}(AL)^{1-\alpha} = -\dot{\lambda}, \tag{5c}$$

$$\frac{\partial H}{\partial X} = \dot{\mu} \quad \Rightarrow \quad -\frac{\gamma}{X}e^{-\rho t} + \mu\delta = \dot{\mu} \tag{5d}$$

As is shown in Appendix A, defining the discount rate, r, to be equal to the social marginal product of capital, then we have:

$$r = \alpha\left(\frac{\beta-1}{\beta}\right)\frac{Y}{K}. \tag{6}$$

[6] Such a linear accumulation stock of carbon in the atmosphere, albeit with emission represented by x rather than $z^{\beta-1}Y$, is the standard textbook pollution stock accumulation equation and can also be found in Perman et al. (2003, p. 182) or Pindyck and Rubinfeld (2009, Ch. 18). A dot on a variable represents the derivative of that variable with respect to time.

As is derived in Appendix B, using (5a)-(5c) we can derive the Ramsey rule to be:[7]

$$\mathrm{r} = \rho + \hat{\mathrm{C}}, \tag{7}$$

Finally, as is shown in Appendix C, using (5a)-(5d), we can derive the modified Hotelling rule as:[8]

$$\mathrm{r} = -(\beta - 1)\hat{\mathrm{z}} - \delta + \beta\gamma\left(\frac{\mathrm{C}}{\mathrm{X}}\right)\mathrm{z}^{\beta-1}, \tag{8}$$

where $\gamma(\mathrm{C}/\mathrm{X})$ represents the household's marginal rate of substitution between consumption and pollution. The left hand side of (8) represents the social return to investment in K, while the right hand side of (8) represents the social return to switching to cleaner technologies. Moreover, upon use of (8), the SSC – i.e. the present value of the marginal damage from carbon emission - can be derived to be:[9]

$$\mathrm{SCC} = \int_{\mathrm{t}}^{\infty} \beta\gamma\left(\frac{\mathrm{C}}{\mathrm{X}}\right)\frac{\mathrm{dX}}{\mathrm{dx}}\mathrm{e}^{-\int_{\mathrm{t}}^{\mathrm{s}}\mathrm{r(s')ds'}}\mathrm{ds}, \tag{9}$$

Finally, (5b) implies the following:

$$\mathrm{z} = \begin{cases} 1, & \text{if} \quad \lambda \geq \mu\beta, \\ \left(\dfrac{\lambda}{\mu\beta}\right)^{\frac{1}{\beta-1}}, & \text{if} \quad \lambda < \mu\beta. \end{cases} \tag{10}$$

3. The quasi-steady state with a constant discount rate and a rising stock of carbon

As follows from (5a), λ is the higher, the lower the level of consumption, C, and therefore the lower the income level. As a consequence, for a moderate income level we have $\lambda \geq \mu\beta$. In turn, eq. (10) implies that for such a moderate income level we have z=1, i.e. that it is optimal to use the dirtiest technology. Since z is not declining at this income level, while output grows, eq. (2) allows for a rising stock of carbon in the atmosphere even if the economy might be in a quasi-steady state with a constant discount rate, which is what we currently seem to observe in reality.[10]

Use of z=1 in (8), the modified Hotelling rule becomes:

$$\mathrm{r} = -\delta + \beta\gamma\frac{\mathrm{C}}{\mathrm{X}}, \tag{11}$$

[7] A hat on a variable represents the growth rate of that variable.

[8] Use of a modified Hotelling rule to describe the equilibrium in the Stokey model follows the lecture notes of Susanne Soretz's course "Endogenes Wachstum und Nachhaltigkeit" (in English "Endogenous growth and Sustainability") at the University of Greifswald, Germany.

[9] See analogously in van der Ploeg and Withagen (2011, Proposition 6).

[10] Under certain parameter constellations, the model implies $\lambda < \mu\beta$ before the economy reached the quasi-steady state. However, if the model is supposed to explain reality, then parameter constellation must be such that the economy reaches the quasi-steady state before λ declines to such a level that $\lambda < \mu\beta$.

In turn, according to (11), consistency with the suggested constancy of the discount rate in the quasi-steady state requires that:[11,12]

$$\hat{X}^* = \hat{C}^*, \tag{12}$$

because this implies in (11) constancy of the marginal rate of substitution between consumption and pollution, $\gamma(C/X)$. Moreover, taking natural logarithms of (3) with z=1, taking time derivatives of the resulting expression and using $\hat{A} = g$ and $\hat{L} = 0$ yields:

$$\hat{Y} = \alpha\hat{K} + (1-\alpha)g. \tag{13}$$

In turn, a quasi-steady state requires constancy of the capital-output ratio, K/Y, which implies $\hat{K} = \hat{Y}$. Upon substitution of this latter requirement in (13) and rearranging, we get:

$$\hat{Y}^* = \hat{K}^* = g. \tag{14}$$

Furthermore, (4) implies $\dot{K} = sY$, where s denotes the savings rate, S/Y, with S being aggregate savings. Division of $\dot{K} = sY$ by K and using (14) in turn yields s=g(K/Y), which implies, due to constancy of K/Y, a constant savings rate in the quasi-steady state. A constant savings rate in turn implies:

$$\hat{C}^* = \hat{Y}^* = g. \tag{15}$$

In addition, according to (2), $\hat{X}^* = \left(x^* / X^*\right) - \delta = g$ requires:

$$X^* = \left(\frac{1}{g+\delta}\right)x^* \quad \Rightarrow \quad \frac{dX^*}{dx^*} = \frac{1}{g+\delta}. \tag{16}$$

Substituting the latter equation of (16) in (9) implies for the SCC:

$$SCC^* = \left(\frac{\beta\gamma}{g+\delta}\right)\int_t^\infty \frac{C}{X} e^{-\int_t^s r(s')ds'} ds. \tag{17}$$

In the next section, it will be shown that once the income level passes a threshold level, the economy will leave the quasi-steady state and move to a steady state in which there is constancy of X, Y is growing at a constant rate, s is constant and therefore C is growing at the same constant rate as Y and r is constant and is somewhat lower than it is in the quasi-steady state. On the other hand, we know from (12) and (15) that in the quasi-steady state $\hat{C} = \hat{X} = g$. As a consequence, (17) implies that in the transition from the quasi-steady state towards the steady state the growth rate of the SCC will rise from a value close to zero towards the growth rate of Y. That the growth rate of the SCC is close to zero in the quasi-state, in turn, is due to the fact that the assumption of logarithmic utility implies

[11] Henceforth the index (*) denotes the quasi-steady state-level or the quasi-steady state-path of the variable in the quasi-steady state.

[12] In addition, as is shown in Appendix D, X adjusts such that the Ramsey rule (7) and the modified Hotelling rule (11) imply the same value of r (see similarly in Smulders (2007)).

that for $\hat{C} = \hat{X} = g$ the marginal rate of substitution between consumption and pollution, $\gamma(C/X)$, is constant.

4. The steady state with a constant discount rate and a constant stock of carbon

Eq. (5a) implies that λ declines as consumption and therefore income rises. Hence, once income passes a threshold level, we have $\lambda < \mu\beta$. As a consequence, eq. (10) implies that z declines once income passes this threshold level, i.e. it becomes optimal to gradually switch to cleaner technologies. In this case taking natural logarithms of (3), taking time derivatives of the resulting expression and using $\hat{A} = g$ and $\hat{L} = 0$ yields now:

$$\hat{Y} = \hat{z} + \alpha\hat{K} + (1-\alpha)g. \tag{18}$$

Moreover, upon use in (2) of the presumption that $\dot{X}^{**} = 0$ it follows that $(z^{**})^{\beta-1}Y^{**} = \delta X^{**}$ or: [13]

$$z^{**} = \delta^{\frac{1}{\beta-1}}(X^{**})^{\frac{1}{\beta-1}}(Y^{**})^{\frac{-1}{\beta-1}}. \tag{19}$$

In addition, taking natural logarithms of (19), taking time derivatives of the resulting expression and using $\hat{X}^{**} = 0$ yields for the steady state:

$$\hat{z}^{**} = -\left(\frac{1}{\beta-1}\right)\hat{Y}^{**}. \tag{20}$$

Use of (20) in (18), using the fact that in a steady state (K/Y)=const. and therefore $\hat{K} = \hat{Y}$ and collecting terms, we find:

$$\hat{Y}^{**} = \hat{K}^{**} = \omega g, \quad \text{where } \omega \equiv \frac{1-\alpha}{1-\alpha+\dfrac{1}{\beta-1}} < 1, \tag{21}$$

which implies that in the steady state income growth is somewhat lower than it was in the quasi-steady state. Moreover, in the steady state the savings rate is constant for the same reasons as those explained for the quasi-steady state. As a consequence, we have:

$$\hat{C}^{**} = \hat{Y}^{**} = \omega g. \tag{22}$$

In addition, substituting (19) in (8) and using C=(1-s)Y, the modified Hotelling rule becomes:

$$r^{**} = -(\beta-1)\hat{z}^{**} - \delta + \beta\gamma(1-s)\delta. \tag{23}$$

Furthermore, upon substituting (22) in (7), we find the Ramsey rule to become: [14]

[13] Henceforth the index (**) denotes the steady state-level or the steady state-path of the variable in the steady state.

$$r^{**} = \rho + \omega g. \tag{24}$$

Moreover, according to (2), $\dot{X} = 0$ implies $X^{**} = (1/\delta)x^{**}$ or $dX^{**}/dx^{**} = (1/\delta)$. Using the latter in (9) together with C=(1-s)Y and constancy of $r = r^{**}$ yields after some rewriting with the time index:

$$SCC^{**}(t) = \left(\frac{\beta\gamma(1-s)}{\delta}\right)\int_0^\infty \frac{Y^{**}(t+s)}{X^{**}(t+s)} e^{-r^{**}s} ds. \tag{25}$$

In addition, (22) and $\dot{X}^{**} = 0$ imply:

$$Y(t+s)^{**} = Y(t)^{**} e^{\omega g t}, \tag{26}$$

$$X(t+s)^{**} = X(t)^{**} = \text{const.} \tag{27}$$

Substituting (26) and (27) in (25) gives rise to:

$$SCC^{**}(t) = \left(\frac{\beta\gamma(1-s)}{\delta}\right)\int_0^\infty \frac{Y(t)^{**}}{X(t)^{**}} e^{(\omega g - r^{**})s} ds. \tag{28}$$

In turn, upon integration of (28) and using the fact that (24) implies $\omega g - r^{**} = -\rho,$ we find:

$$\begin{aligned} SCC^{**}(t) &= -\left[\frac{\beta\gamma(1-s)}{\delta}\right]\left[\frac{Y(t)^{**}/X(t)^{**}}{\rho}\right]\left[e^{-\rho\infty} - e^{-\rho 0}\right] \\ &= -\left[\frac{\beta\gamma(1-s)}{\delta}\right]\left[\frac{Y(t)^{**}/X(t)^{**}}{\rho}\right][0-1], \\ &= \left[\frac{\beta\gamma(1-s)}{\delta}\right]\left[\frac{Y(t)^{**}/X(t)^{**}}{\rho}\right]. \end{aligned} \tag{29}$$

Combining (29) with (26) and (27) and use of the fact that s=constant, we get:

$$\widehat{SCC}^{**}(t) = \hat{Y} = \omega g, \tag{30}$$

i.e. in the steady state the SCC grows with the rate of income growth. This is due to the fact that in the steady state the marginal rate of substitution between consumption and pollution, $\gamma(C/X)$, is, due to constancy of the stock of carbon and the savings rate, rising at the rate of income growth as well.

[14] Moreover, Appendix D shows that s adjust such that the Ramsey rule (24) and the modified Hotelling rule (23) imply the same value of r. Appendix D shows that for this to be possible a knife-edge condition must be imposed on the parameter values.

7. Conclusion

The present paper solved a version of the Stokey model to derive the SCC and its growth rate. The paper assumed instantaneous utility to be additively separable as well as logarithmic in consumption and pollution. This assumption allowed for a quasi-steady state with a constant discount rate despite of a rising stock of carbon in the atmosphere. The assumption of logarithmic utility allowed for this because then in the quasi-steady state the household's marginal rate of substitution between consumption and pollution was constant if consumption and the stock of carbon grew at the same rate and therefore the modified Hotelling rule implied a constant discount rate despite of a rising stock of carbon. The existence of such a quasi-steady state was important, as constant market rates of return despite of a rising stock of carbon in the atmosphere is what we currently seem to observe in reality. The paper has also shown that in this quasi-steady state the growth rate of the SCC is close to the value zero. This result is in contrast to the model result of Golosov et al. (2011), which also allows, under certain simplifying assumptions, for a quasi-steady state and in which, in the quasi-steady state, the SCC grows at the rate of income growth. This comparison shows that the value of the current growth rate of the SCC depends very much on the underlying assumptions that guarantee existence of a situation with a constant discount rate despite of a rising stock of carbon in the atmosphere.

Appendix A: Derivation of the social marginal product of capital (eq. 6)

Emissions are in the text assumed to arise according to:

$$\mathrm{x} = \mathrm{z}^{\beta}\mathrm{K}^{\alpha}(\mathrm{AL})^{1-\alpha}. \tag{A1}$$

Rearranging (A1) yields:

$$\mathrm{z} = \mathrm{x}^{\frac{1}{\beta}}\mathrm{K}^{\frac{-\alpha}{\beta}}(\mathrm{AL})^{\frac{\alpha-1}{\beta}}. \tag{A2}$$

Substituting (A2) in (3), we find:

$$\mathrm{Y} = \mathrm{x}^{\frac{1}{\beta}}\mathrm{K}^{\alpha\left(\frac{\beta-1}{\beta}\right)}(\mathrm{AL})^{(1-\alpha)\left(\frac{\beta-1}{\beta}\right)}. \tag{A3}$$

Differentiating (A3) with respect to K and using (A3) in the resulting expression yields eq. (6) in the text.

Appendix B: Derivation of the modified Ramsey rule (eq. (7))

Taking the natural logarithm of (5a) and taking time derivatives of the resulting expression yields:

$$-\hat{\mathrm{C}} - \rho = \hat{\lambda}. \tag{B1}$$

Assuming equality in (5b) and rearranging gives:[15]

[15] For inequality in (5b), the calculations of this Appendix are valid with z=1.

$$\mu z^{\beta-1} = \frac{\lambda}{\beta}. \tag{B2}$$

Upon substituting (A2) in (5c) and division by λ, we find after use of (3) in the resulting expression:

$$\alpha\left(\frac{\beta-1}{\beta}\right)\frac{\mathrm{Y}}{\mathrm{K}} = -\hat{\lambda}. \tag{B3}$$

Use of (6) in (B3) yields:

$$\hat{\lambda} = -\mathrm{r}. \tag{B4}$$

Finally, substituting (B4) in (B1) and rearranging gives eq. (7) in the text.

Appendix C: Derivation of the Hotelling rule (eq. (8))

Division of both sides of (5d) by μ gives:

$$-\frac{\gamma e^{-\rho t}}{\mu \mathrm{X}} + \delta = \hat{\mu}. \tag{C1}$$

Assuming equality in (5b) and rearranging yields: [16]

$$\mu = \left(\frac{\lambda}{\beta}\right) z^{1-\beta}. \tag{C2}$$

Substituting (C2) in (C1), we find:

$$-\frac{\beta\gamma e^{-\rho t}}{\lambda \mathrm{X}} z^{\beta-1} + \delta = \hat{\mu}. \tag{C3}$$

Taking the natural logarithm of (C2) and taking time derivatives of the resulting expression yields:

$$\hat{\mu} = \hat{\lambda} + (1-\beta)\hat{z}. \tag{C4}$$

Upon substituting (C4) in (C2) we get:

$$-\frac{\beta\gamma e^{-\rho t}}{\lambda \mathrm{X}} z^{\beta-1} + \delta = \hat{\lambda} + (1-\beta)\hat{z}. \tag{C5}$$

Substituting in (C5) eq. (5a) for λ and eq. (B4) for r and rearranging gives eq. (8) in the text.

[16] Again for inequality in (5b), the calculations of this Appendix are valid with z=1.

Appendix D: Derivations of the value of X – respectively s - that implies the same r for the Ramsey rule and the modified Hotelling rule

<u>Case 1: The quasi-steady state with constant r and rising X</u>

Use of (12) and (15) in (7), we find:

$$\mathrm{r} = \rho + \mathrm{g}. \tag{D1}$$

Eliminating the discount rate between (D1) (the Ramsey rule in case 1) and (11) (the Hotelling rule in case 1), we get:

$$\rho + \mathrm{g} = -\delta + \beta\gamma\frac{\mathrm{C}}{\mathrm{X}} \quad \text{or} \quad \mathrm{X} = \left(\frac{\beta\gamma}{\rho + \mathrm{g} + \delta}\right)\mathrm{C}, \tag{D2}$$

which confirms (12) since the parameters within parenthesis are all constant.

<u>Case 2: The steady state with constant r and constant X</u>

Eliminating the discount rate between (24) (the Ramsey rule in case 2) and (23) (the Hotelling rule in case 2), gives rise to:

$$\rho + \omega\mathrm{g} = -(\beta - 1)\hat{\mathrm{z}}^{**} - \delta + \beta\gamma(1 - \mathrm{s})\delta. \tag{D3}$$

Combing (20) and (22) yields:

$$\hat{\mathrm{z}}^{**} = -\left(\frac{1}{\beta - 1}\right)\omega\mathrm{g}. \tag{D4}$$

Upon substituting (D4) in (D3) we get:

$$\rho + \omega\mathrm{g} = \omega\mathrm{g} - \delta + \beta\gamma(1 - \mathrm{s})\delta. \tag{D5}$$

Solving (D5) for s yields:

$$\mathrm{s} = 1 - \frac{\rho + \delta}{\beta\gamma\delta}. \tag{D6}$$

In addition, (4) implies $\dot{\mathrm{K}} = \mathrm{sY}$ or $\hat{\mathrm{K}} = \mathrm{s}(\mathrm{Y}/\mathrm{K}) = \omega\mathrm{g}$, where in the latter expression use has been made of (21). In turn, upon combining the latter expression with (6), we find:

$$\mathrm{s}\left(\frac{\mathrm{r}^{**}}{\alpha}\right)\left(\frac{\beta}{\beta - 1}\right) = \omega\mathrm{g}. \tag{D7}$$

Substituting (24) in (D7) for r^{**} and collecting terms gives:

$$s = \left(\frac{\beta - 1}{\beta}\right)\left(\frac{\alpha\omega g}{\rho + \omega g}\right). \tag{D8}$$

Eliminating s between (D6) and (D7) yields that we must impose on the parameter values of the model the following knife-edge conditions:

$$\left(\frac{\beta - 1}{\beta}\right)\left(\frac{\alpha\omega g}{\rho + \omega g}\right) = 1 - \frac{\rho + \delta}{\beta\gamma\delta}. \tag{D9}$$

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