

RE: Manuscript 580 by Tomas Kögel „The social cost of carbon on an optimal balanced growth path”, now titled “The social cost of carbon in the Stokey model”

## RESPONSE TO THE REFEREE REPORT 1

I first would like to thank the referee for his (or her) very useful report.

### **On “Is it the right question?”**

*The referee objects that my discussion paper only asks the question “What is the growth rate of the social cost of carbon (SCC) in the steady state?”, but that it might take several centuries the climate system to reach a steady state with stabilized climate and that the truly pressing question would be “What is the growth rate of the SCC now?”. As a consequence, my discussion paper would ask the wrong question. The referee therefore suggests to me to derive formally the growth rate of the SCC in the transition phase from current conditions to the steady state (in my discussion paper labeled balanced growth path). The referee, however, appears to be aware of that this task might be difficult and therefore generously allowed me alternatively to give the reader at least some quantitative indication of how the growth rate of the SCC in the near term relates to its growth rate in the steady state.*

I am grateful to the referee for his generosity and I followed, by and large, his latter suggestion because I would not really know how to calculate a formula for the growth rate of an integral outside of the steady state. I was however more ambitious with respect to another aspect of my paper. This is so because - apart from the reason of analytical tractability - I had another reason why I derived the growth rate of the SCC in the steady state. The reason was that according to my understanding a standard assumption in integrated assessment models is the assumption of a constant discount rate. Moreover, I believe it is not a bad idea to make this assumption because it is consistent with the stylized fact of modern economic growth by Kaldor that there are constant market rates to return in industrialized countries since at least the last hundred years. However, the referee is right with his argument that currently the stock of carbon in the atmosphere is rising rather than constant and therefore the steady state in my discussion paper, which required a constant stock of carbon, is inconsistent with what we currently observe in reality. For this reason, I decided to completely change the model in the revised version of my paper and to apply in the revised version another model that allows for a quasi-steady state. I defined a quasi-steady state as an equilibrium in which the discount rate and income growth are constant, but the stock of carbon in the atmosphere is rising. Such a quasi-steady state is what we currently seem to observe in reality. Optimal growth models usually do not allow for such a quasi-steady state. I did however realize that a version of the model by Stokey (1998) allows for such a quasi-steady state if one imposes in her model the restriction utility to be logarithmic in consumption as well as in pollution (a restriction that she did not make because she was not interested in allowing for a quasi-steady state). Hence, in the revised version, I now solve a version of the Stokey model with logarithmic utility and derive the SCC in the (current) quasi-steady state as well as in the (future) steady state. In the Stokey model, emissions arise as a by-product of output production and they can be reduced by switching to a cleaner technology (this seems to me to be similar to the DICE model). In the quasi-steady state of my model adaption, however, it is optimal to use the dirtiest technology. In this quasi-steady state, the growth rate of the SCC is close to zero because in the quasi-steady state consumption and the stock of carbon grow with the same rate and this implies constancy of the household's marginal rate of substitution between consumption and pollution. Once income passes a threshold level, it becomes

optimal to gradually switch to cleaner technologies. As a consequence, the economy will leave the quasi-steady state and move towards a steady state with again a constant discount rate, a constant consumption and income growth rate, but now also a constant stock of carbon in the atmosphere. In the steady state, the growth rate of the SCC is equal to rate of income growth. This is so because in the steady state the savings rate is constant and therefore growing consumption and a constant stock of carbon implies that the household's marginal rate of substitution between consumption and pollution grows at the rate of income growth. In the revised version of my paper, I did not calculate a formula for growth of the SCC in the transition from the quasi-steady state towards the steady state. I did however indicate to the reader that, obviously, the fact that the growth rate of the SCC in the quasi-steady state is close to zero and is equal to the rate of income growth in the steady state implies that the growth rate of the SCC is rising during the transition.

### **On "The SCC on a balanced growth path"**

*The referee cast doubts on my discussion paper's result that in the steady state the growth rate of the SCC is negative. The referee motivates these doubts by solving a restricted model in which the steady state-growth rate of the SCC equals the rate of income growth. He argues that this result should also be true in my more general model. If he is right, then I should correct my error and rewrite my paper accordingly.*

The referee is absolutely correct. In my discussion paper I made an error in calculating the growth rate of the integral in the SCC formula. For whatever reason, I thought the growth rate of this integral should equal the difference between the growth rate of the impact from climate change (i.e. of the relative marginal damage) and the discount rate. Since for plausible parameter restrictions the growth rate of the impact from climate change is lower than the discount rate, I thought the growth rate of the SCC would be negative. However, the referee's intuition is absolutely correct and it is true that after properly solving the integral in the SCC formula, the steady state-growth rate of the SCC must be equal to the rate of income growth. I was eventually able to derive this growth rate in the revised version of my paper after realizing a formal similarity of my model with the dividend discount model in financial economics. Probably the solution in my revised version is obvious to the referee because the referee used identical calculations when he solved an integral in his referee report in the equation on page 3. If not, then the referee is referred to the Appendix of this response, in which I exploit the similarity of my model with the dividend discount model in financial economics.

### **On "Minor comments"**

Since I completely revised my paper, almost none of the "minor comments" were relevant anymore. However, where relevant, I followed the suggestions.

### **Where is the revised paper?**

The revised version of my paper, now titled "The social cost of carbon in the Stokey model" is attached to this response.

## Appendix: The similarity of my model with the dividend discount model in financial economics

Ignoring uncertainty, the dividend discount model implies:<sup>1</sup>

$$P(t) = \int_0^{\infty} D(t+s)e^{-ks}ds, \quad (1)$$

where  $P(t)$  denotes the stock price,  $D(t+s)$  denotes dividends and  $k$  is the discount rate. If  $k$  is constant and dividends grow at the constant rate  $\mu$ , then financial economics literature has shown that solving the integral in (1) implies:

$$P(t) = \frac{D(t)}{k - \mu}, \quad (2)$$

where  $k > \mu$ . Since  $D(t)$  grows at the rate  $\mu$ ,  $P(t)$  grows at the rate  $\mu$  as well.

In a nutshell, the SCC in my paper's model equals:

$$SCC(t) = \int_0^{\infty} MRS(t+s)e^{-ks}ds, \quad (3)$$

where  $MRS(t+s)$  denotes the marginal rate of substitution between consumption and pollution and  $k$  again denotes the discount rate. If we are in period  $t$  in the steady state and if  $k$  is therefore constant and if  $MRS(t+s)$  grows in the steady state at the rate  $\mu$ , then using the formal similarity to the dividend discount model, solving the integral in (3) implies:

$$SSC(t) = \frac{MRS(t)}{k - \mu}, \quad (4)$$

where again  $k > \mu$ . Since  $MRS(t)$  grows in the steady state at the rate  $\mu$ ,  $SCC(t)$  grows in the steady state at the rate  $\mu$  as well. I applied in my paper identical steps as in this Appendix and due to the similarity with the established dividend discount model my revised paper's calculations should be correct with almost 100% certainty.

## References

- Ackert and Hunter (1999). [Intrinsic Bubbles: The Case of Stock Prices: Comment](#). American Economic Review 89: 1372-1376.
- Stokey, N.L. (1998). [Are there limits to growth?](#). International Economic Review 39: 1–31.

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<sup>1</sup> See Ackert and Hunter (1999).