Robert Kopp and his colleagues (Kopp, et al., 2011) have authored a provocative paper in which they investigate the implications of imbedding a collection of alternative damage functions into a recent version of the DICE model (Nordhaus, 2007) on estimates of the social cost of carbon (SCC). They focus their readers' attention on the role of time preference and risk aversion by selecting three combinations that hold the overall discount rate at 3% through a variant of the Ramsey discounting equation assuming an annual growth rate (GDP) of 2.03% between 2015 and 2115. With relative risk aversion (denoted by η) set to 0, 1.0 and 1.4, they calculate associated time preferences (denoted by ρ) as 3.0%, 0.95% and 0.14%, respectively.

To the authors, the primary take home message of their contribution, as reported in their abstract, is: "In the absence of risk aversion, SCC values calculated with this function are in agreement with the standard quadratic DICE damage function; with a coefficient of relative risk aversion of 1.4, this damage function yields SCC values more than triple those of the standard function." While this result is not particularly surprising, their paper is worthy of attention because of their inclusion of a comparison of reference case scenarios with scenarios that achieve a 50% likelihood of holding temperature increases to 2.5^{o} C – exactly the sort of comparison that will provide new knowledge to the current stabilization debates.

My comments, which conclude with a discussion of this comparison, are divided into three parts. The first considers context by casting the raw results into the distribution of estimates of the social cost of carbon reported by Tol (2008) in his most recent meta-analysis. The second identifies two places where the modeling structure is not quite standard, and offers some suggestion of how much of a difference they make. The third focuses attention on the comparison of the reference case scenarios to the stabilization alternatives.

1. Placing these estimates into the broader context.

Kopp, et al. (2011) spend a little time comparing their estimates of the social cost of carbon (calibrated for 2015 in terms of $ per ton of CO_2) to the original DICE estimates; their satisfaction in noting that quadratic damages produce comparable estimates confirms that they have anchored their experiment properly.

It is instructive to place their results (for the first two cases, at least) into ranges reported by Tol (2008) on the basis of his estimating a Fisher-Tippett distribution over published SCC estimates that generally work with 2005 estimates with discount rates about 5%. For Case 1 (ρ ~ 3% and η = 0), increasing the Tol (2008) results by 2-4% per year per Hope (2006) to put them into the 2015 timeframe produces estimates of $9-10,

\$7-9 and \$20-25 per ton of CO_2 for the mean, 50th percentile and 95th percentile SCC estimates, respectively For Case 2 ($\rho \sim 1\%$ and $\eta \sim 1$), the corresponding similarly elevated Tol results are \$27-33, \$24-30 and \$68-83 per ton of CO_2. Since the analyses surveyed by Tol generally discount the future by 5%, it is reassuring that the Tol results generally fall well below the range reported by Kopp, et al (2011).

2. Two non-standard modeling decisions.

These comparisons have to be interpreted carefully for two additional reasons. First of all, Kopp, et al. (2011) use a non-standard representation of the Ramsey discounting rule; i.e., they assume that (their equation (5)):

$$1 + r = (1+\rho)(1+g)^{\eta} \qquad (1)$$

rather than the more traditional representation:

$$1 + r = 1 + \rho + \eta g \qquad (2)$$

from Ramsey (1928). In either formulation, g should represent the rate of growth of per capita consumption and not GDP. Using their 2.03% growth rate assumption, equation (2) would suggest that $\rho = 3\%$, 0.98%, and 0.16% would be more appropriate for $\eta = 0$, 1, respectively - not much of a difference, to be sure. But using an estimate of, say, 1.3% for the growth rate of per capita consumption, the corresponding values for ρ should be 3%, 1.7%, and 1.18%, respectively - perhaps a significant difference.

The second source of concern is perhaps more important. Kopp, et al. (2011) do not, for some reason, use the conventional certainty equivalent calculation to estimate the social cost of carbon across the uncertain runs that they employ. Instead, as described in Section 2.3, they use something like:

$$SCC \equiv \{\Delta EU/\Delta CO_2\}/\{\Delta EU/\Delta \text{consumption}\}. \qquad (3)$$

The second term in (3) seems to assume that expected utility can be viewed as a function of average consumption; otherwise how is it consistently defined across multiple states of nature? Of course, expect utility is not a function of expected consumption. It therefore strikes me that something like

$$SCC \equiv \{\Delta EU/\Delta CO_2\}/E\{\Delta U/\Delta \text{consumption}\} \qquad (4)$$

would have been more appropriate - something, in other words, that would have allowed the diversity of marginal utilities across the range of scenarios to be recognized so that η would reflect aversion to inequality as well as aversion to risk.

Anthoff, et al. (2009) produced certainty equivalent SCC estimates from Tol's FUND model assuming a joint distribution for ρ and η reported by Evans and Sezer (2005). Ignoring both risk aversion and inequality aversion (closest to the $\rho = 3\%$ case here), they estimate SCC ~ \$40 per ton of carbon (~ \$13-16 per ton of CO_2 in 2015 with adjustment per Hope (2006)). When uncertainty is viewed through a risk averse lens, only, SCC climbs to ~ \$120 per tone of carbon (~ \$40-49 per ton of CO_2 adjusted for 2015); and adding aversion to inequality that Kopp, et al. (2011) miss, SCC nearly doubles to approximately \$230 per ton of carbon (~ \$77-93 per ton of CO_2 in 2015). Only the last case approaches the SCC values reported for Kopp, et al (2011).

3. The value of stabilization.

Turning now to the value of stabilization, Table 1 reports differences between the Kopp, et al. (2011) Tables 3 and 4 (illustrated in Figure 8) as percentages relative to the reference case estimates. It is important, in interpreting the several notable trends that emerge here, to understand that stabilization produces benefits for three reasons. The first is obvious - lower long run temperatures certainly diminish long term impacts and therefore the discounted economically based damage estimates that are reflected in the SCC. The second two are less obvious - stabilization interventions not only reduce the range of uncertainty at any point in time, but also push the full range of potential damages for a point in time along the reference scenarios farther into the future. Both of these forces should work to lower SCC estimates for cases with no-zero risk aversion (at least for estimates generated by calculating certainty equivalents); and all three should contribute to reducing SCC for all but the first case wherein risk aversion is taken to be zero.

Table 1 shows the expected direction of change for all of the cases except for the SP*, X_c, and W_a damage variants. Explaining exactly why those cases deviate from expectation (especially for case 1 where $\eta = 0$) would be an important addition to the discussion. Nonetheless, note that all of the changes are exaggerated for the last set of damage functions where damages accelerate with temperature.

Table 1 also shows, as expected, that increases in aversion to risk increase the value of stabilization – except for the the X_i (i = a; b; b*; c; and c*) damage variants. Again, why? Without some explanations about these results, the credibility of the damage functions that support these cases must be viewed with some skepticism.

References

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Table 1: Differences in SCC estimates from Kopp, et al. (2011) with and without stabilization expressed in terms of percentage change relative to the reference cases.

Case 1 ($\rho = 3$ and $\eta = 0$)

D	-8.1	-9.1	-12.5	0.0
We	-10.3	-14.3	-12.5	-6.7
L	-10.3	-14.3	-13.3	-5.1
SP	-11.5	-12.5	-8.3	-4.3
SP*	6.1	0.0	-8.7	20.9
Wa	50.0	47.8	25.0	66.1
K	-12.8	-14.3	-12.5	-4.2
AL	-2.4	-9.1	-12.5	0.0
ASB	-24.2	-73.3	-25.0	-21.1
Xa	-14.3	-26.7	-16.7	-41.1
Xb	-12.5	-22.2	0.0	-53.4
Xb*	-3.9	-22.2	-10.0	-21.3
Xc	-2.6	-25.0	0.0	32.5
Xc*	4.4	-33.0	0.0	49.1

Case 2 ($\rho = 0.95$ and $\eta = 1$)

D	-11.1	-10.3	-11.1	-7.5
We	-16.3	-14.6	-11.1	-16.7
L	-12.8	-12.5	-11.8	-10.9
SP	-11.1	-12.8	-11.1	-7.7
SP*	1.4	-3.2	-10.3	14.6
Wa	45.3	40.5	21.8	60.9
K	-14.6	-14.6	-11.1	-7.1
AL	-13.1	-10.3	-11.1	-7.5
ASB	-36.8	-33.9	-22.2	-42.6
Xa	-14.6	-33.3	-14.3	-37.8
Xb	-14.0	-31.6	-14.3	-35.4
Xb*	-6.0	-26.3	-7.7	1.1
Xc	-14.6	-20.0	0.0	30.5
Xc*	-9.4	-45.5	100.0	25.8

Case 3 (($\rho = 0.14$ and $\eta = 1.4$)

D	-13.5	-13.6	-15.0	-11.4
We	-19.0	-15.2	-15.0	-23.2
L	-15.1	-13.6	-11.1	-12.7
SP	-12.3	-13.0	-12.5	-11.0
SP*	-1.1	-7.7	-9.1	6.2
Wa	31.6	27.7	13.9	41.9
K	-14.5	-17.4	-15.0	-9.8
AL	-28.9	-13.6	-15.0	-11.4
ASB	-48.1	-35.4	-25.0	-54.3
Xa	-15.8	-30.0	-14.3	-22.7
Xb	-16.0	-34.5	-20.0	-10.6
Xb*	-7.8	-30.6	-12.5	17.5
Xc	-40.3	-30.8	0.0	14.1
Xc*	-39.2	-30.8	0.0	14.7