#### Review of

# Fundamental Uncertainty, Portfolio Choice, and Liquidity Preference Theory

I am sorry to say that to this reviewer the conceptualization underlying the paper looks unclear and to some extent rather puzzling.

### 1 Purpose of the paper

The purpose of the paper is expressed in different ways:

- (i) "The aim of the approach is incorporate fundamental uncertainty into macro models" (p.15)
- (ii) a restatement of Keynes's liquidity preference theory based on "fundamental uncertainty"
- ( iii ) a new theory of "portfolio decisions" in which "fundamental uncertainty" plays a crucial role:
- "this paper ... provides an argument for a more bounded rational approach to portfolio choice between liquidity and risky assets" (p.1)
- (iv) "this paper critically assesses various formal representations of fundamental uncertainty" (p.1) and suggests a new "formal representation" of "fundamental uncertainty" considered superior to the existing ones.

It is clear from this reconstruction that the meaning, originality, and achievement of this paper crucially depend on the objective (iv). If the new "formal representation" is superior to the existing ones from a well-defined point of view, the author is right in pursuing the objective (iii) to show its fecundity in at least a specific field: "portfolio choice between liquidity and risky assets". The next stage could be the target (ii) but this further step is not even tried by the author. An attempt of this kind should consider the role that liquidity preference theory plays in the *General Theory* of Keynes and show that the restatement based on the new "formal representation" of fundamental uncertainty helps clarifying or improving this role. A success in the pursuit of target (ii) could raise reasonable hopes that the target (i) is possible and desirable. At the stage reached by the paper here reviewed this hope is not much more than wishful thinking.

For the reasons here explained, what follows focuses mainly on the suggested "new" formal representation of fundamental uncertainty.

#### 2 Conceptualization of "fundamental uncertainty"

As it should be clear from the preceding discussion on the purpose of the paper, the concept of "fundamental uncertainty" is the crucial pillar of the paper. The author suggests a "new" formal representation of "fundamental uncertainty" at the beginning of section 3 (p.7). We discuss this

"formal representation" in section 3 of this review. But this is a representation of what? The two preceding sections of the paper are supposed to give the answer. However, I am unable to find there a clear definition, explicit or implicit, of "fundamental uncertainty". From what the author says I draw the impression that whatever concept that deviates, even slightly, from standard additive probability theory is classified under the portmanteau concept of "fundamental uncertainty" (by the way, this minimalist definition is necessary to make sense of the formal representation suggested by the author (see section 3 of this review). So, it includes what is generally called "fundamental uncertainty" arising from the "uniqueness" of an event (p. 1) when "the agent has simply not objective basis to determine reasonable probability measures" (p.1) as well as much weaker hypotheses, often referred to as "strong" or "hard" uncertainty (from now on in this review, conventionally "hard uncertainty") when knowledge is "missing" (p.2) or "is inconclusive for probabilistic inferences" (p.1). From the conceptual point of view, the difference between "fundamental uncertainty" as usually defined and "hard uncertainty" is huge. In the case of "fundamental uncertainty" the agent believes that he has no relevant knowledge for his decision, while in the case of "hard" uncertainty he has some degree of relevant knowledge but he believes that his knowledge is incomplete or "ambiguous". Keynes's concept of weight of argument clarifies the issue. When the relevant knowledge on the probability distribution of a certain event is believed to be complete, the weight of argument w is maximum (w = 1) and the uncertainty, often called in this case "risk", may be aptly represented and dealt with by traditional probability theory that assumes additivity and full reliability of probability distributions. When the relevant knowledge is believed to be completely missing, the weight of argument is minimum (w = 0) and the uncertainty is generally called fundamental uncertainty. In all the other cases, the relevant knowledge is believed to be incomplete but not wholly absent, the weight of argument has an intermediate value ( 0< w < 1), and the uncertainty may be called, as we have suggested before, "hard". These three hypotheses must be kept rigorously distinct in order to choose the appropriate formal representation of uncertainty for the problem analyzed. This is not a terminological issue. The author may choose whatever terminology he likes, provided that he clearly defines his concept of uncertainty keeping distinct the three hypotheses above. The author, of course, is aware of the Keynes's concept of weight of argument as is shown by the following passage "the data may give some evidence, or weight, in Keynesian terms" (p.4) where the use of the concept is fully consistent with my use above but is referred to a different problem that will be discussed in section 3 of this review.

Consistently with the interpretation suggested above of the author's broad (and imprecise) concept of fundamental uncertainty, the author discusses a list of formal representations of "fundamental uncertainty" that generally refer to what I called here "hard" uncertainty and not

"fundamental" uncertainty. In the author's opinion: "all briefly discussed ways to represent fundamental uncertainty –multiple priors, non-additive measures, fuzzy-set representations- require heavy additional assumptions about non-observable entities, e.g. multiple priors, preferences about how to deal with ambiguity, ...etc" (p.5). The epistemological criticism raised by the author is based on a preceding work by Pasche (2008) where it is contended that "a theory loses explanatory power when imposing a rich and specific structure for non-observable antecedent conditions" (p.5). Although I could not read the original version of this criticism as I do not read German, this argument seems to me an updated version of the Occam's razor. This is a respectable empiricist argument. However, this argument may be a criterion of choice only between theories having an analogous theoretical and empirical scope. The theories criticized above are full-fledged axiomatized theories of decision under hard uncertainty, while the formal representation suggested by the author is just an alternative epistemological interpretation of a well-known fact (the dispersion of beliefs). In addition, it is not true that the antecedent conditions of the above theories are non-observable; in particular, an extensive experimental literature has given an empirical support to many of them. Finally, as we will see in the next section, also the author's representation does not escape the assumption of a crucial non-empirical counterfactual antecedent.

I conclude this point by observing that the criticism to the existing theories of decision under hard uncertainty are unconvincing, while the alternative conceptualization by the author is puzzling and its superiority based on the Pasche version of the Occam's razor looks at best inconclusive.

## 3. The author's formal representation of "fundamental uncertainty"

The author's formal representation of fundamental uncertainty assumes that individual beliefs are "rational" "in the sense that they are in the mean consistent with the ex post realized data" (p.7) but the agents are aware that they are boundedly rational since each of them knows that "...there exist other considerations about the underlying process than his own, and that his beliefs will differ from the ex post realizations with the same probability of any other agent" (p.7).

This formal representation of "fundamental uncertainty" is puzzling. If we refer this characterization to a representative agent, we are in a situation characterized by rational expectations where the representative agent is unboundedly rational and uncertainty is neither fundamental nor hard but "soft". In other words, from the point of view of the representative agent (and of the policy maker having to deal with such a macroeconomic process) the uncertainty implied by these assumptions is just "risk" in the sense of Knight and of the author. Dispersion of beliefs in this sense is allowed for by most textbooks on rational expectations.

However the author puts forward three reasons why this sort of aggregation is misleading (p.7). I accept this threefold argument that shows that dispersion of beliefs is inconsistent with the hypothesis of rational expectations of the representative agent but the reason has more to do with its faulty microfoundations, that are correctly shown to be inconsistent with methodological individualism, rather than with specific assumptions on individual and aggregate uncertainty. So the potential merit I see in this paper is not in the new formal representation of fundamental uncertainty that avoids non-observable antecedents, but in the criticism of the microfoundations of the rational expectations paradigm in macroeconomics. However, this is not the point of view taken by the author.

In addition, even this part that is the more interesting section of the paper (section 3.1) is haunted by further questionable assumptions and assertions.

A further example of questionable assumption is the assumption that the mean of individual beliefs are consistent with ex post realized data. This assumption is at variance not only with fundamental uncertainty and hard uncertainty but also with the microfoundations argument mentioned above. In addition, this assumption seems to me a non-observable and counter-factual antecedent as those criticized by the author.

A further example of questionable assertion is the assertion that the author's approach is preferable to the existing theories of decision under hard uncertainty because it is "more boundedly rational". In the absence of a well specified metric of rationality boundedness, it is difficult to understand what "more boundedly rational" means. In any case the deviation from unbounded rationality introduced by the author is too weak and vague to be appealing.

I stop here my comments because the applications of the author's formal representation of fundamental uncertainty to Portfolio decisions (section 3.2) and to the case of a debt-financed portfolio cannot be assessed in the absence of a systematic clarification of the conceptualization of "fundamental uncertainty" suggested by the author and of the formal representation of the latter. Final comment: there are many misprints and language errors that do not help the reader to understand the meaning of the arguments.